

March 19, 2014

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## John Machin

roximating



- ▶ 1680-1751
- English mathematician and astronomer
- ► Private tutor to Brook Taylor
- Best known for formulas he invented for calculating  $\pi$

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Line drawing from MacTutor History of Mathematics archive



## Mathematical underpinnings

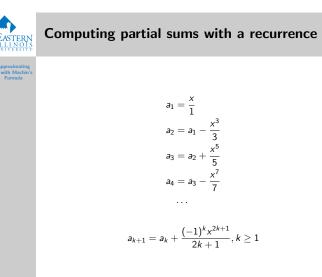
Taylor's series for arctangent

$$\begin{aligned} \arctan x &= \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \\ &= \sum_{j=0}^{\infty} \frac{(-1)^j x^{2j+1}}{2j+1} \end{aligned}$$

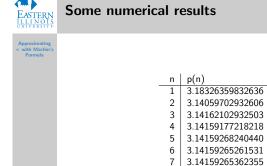
Machin's formula

$$\pi = 16\arctan\frac{1}{5} - 4\arctan\frac{1}{239}$$

Partial sums  
arctan 
$$x = \sum_{j=0}^{\infty} \frac{(-1)^j x^{2j+1}}{2j+1}$$
  
 $a_1 = \frac{x}{1}$  *j* runs from 0 to 0  
 $a_2 = \frac{x}{1} - \frac{x^3}{3}$  *j* runs from 0 to 1  
 $a_3 = \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5}$  *j* runs from 0 to 2  
 $a_4 = \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$  *j* runs from 0 to 3  
...  
 $a_{k+1} = \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + \frac{(-1)^k x^{2k+1}}{2k+1}$ 



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ing iin's	<pre>% Arguments for atan() xA = 1/5; xB = 1/239; % Total number of desired approximations n = 10; % atan approximations for xA and xB using just one term a(1) = xA; b(1) = xB; %and the corresponding approximation for pi p(1) = 16*a(1) - 4*b(1); % Improve the approximation by increasing the number of terms used for k = 1:n-1</pre>	
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3.14159265358860

3.14159265358984 10 3.14159265358979

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## Summary

- For centuries, mankind has been fascinated with  $\pi$ .
- How can we compute accurate approximations of  $\pi$ ?
- ▶ We have observed Machin's formula leads to fast convergence.