Perceptions of Elementary Preservice Teachers’ Mathematical Knowledge and Number Sense

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Abstract

The challenge and impact of in-service and preservice teachers’ lack of mathematical knowledge has been well documented. This article expands the discussion with examples from our experience working with preservice teachers enrolled in a math methods course for elementary teachers. Examples that relate the perceptions of preservice teachers’ prior experiences and preconceived attitudes to their lack of number sense are discussed.
Background

Over the last 20 years, research in the field of mathematics education has been heavily marked with studies highlighting the importance of teacher knowledge for practicing teachers and preservice teachers alike. At the forefront of these studies are the deficiencies in mathematical knowledge that have the potential to impede learning (Ma, 1999). A serious look at this subject begins with the premise that a deep understanding of mathematics is necessary in order to be effective in the teaching of mathematics (Ball, 1990; Ma, 1999; Conference Board of the Mathematical Sciences, 2001; Brabeck & Shirley, 2003). The National Council of Teachers of Mathematics (NCTM, 2000) took this a step further with its Teaching Principle suggesting “teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility” (p.17).

In general, preservice teachers lack in their understanding of mathematics (Ball, 1990; Ma, 1999). Hill et al. (2008) also noted a positive correlation between teachers’ levels of knowledge of content and teaching (KCT), which combines knowing about teaching and knowing about mathematics, and the quality of mathematical instruction teachers provided for their students. Similarly, Van Dooren, Verschaffel, and Onghena (2002) proposed that a lack of mathematical knowledge results in inability to recognize “good teaching.” Borko and Eisenhart (1992) found that preservice teachers often did not have the content knowledge to teach some of the mathematical concepts that teaching required. Looking specifically at the concept of division of fractions, Ball (1990) noted that preservice teachers lacked both the conceptual knowledge and the ability to explain this concept in meaningful ways. Simon (1993) examined preservice teachers’ understanding of both partitive division, a problem type where the total of objects is known with the number of groups unknown, and quotative division, a problem type where the
number of groups is known and the total in each is unknown, of whole numbers. He also noted inadequate conceptual knowledge and an inability to understand number relationships. Though, in reality, the problem is much deeper than a lack of mathematical knowledge. Moreover, Ma (1999) found that though U.S. elementary teachers had been exposed to more advanced mathematics and in general had more education; Chinese elementary teachers had a better and deeper understanding of the mathematics that they taught. Although U.S. teachers generally possessed procedural knowledge for math, as did their Chinese counterparts, the Chinese also possessed conceptual knowledge.

In response to these issues, merely knowing mathematics may not be the answer. Ball, Thames and Phelps (2008) supported this sentiment, stating that it is important to not only deeply know the content, but also hold the specialized knowledge for the teaching of math. Hill, Schilling, and Ball (2004) also pointed out that this problem is further convoluted in that there is a lack of consensus on what a teacher needs to know in order to teach elementary mathematics.

The lack of preservice teachers’ mathematical conceptual knowledge and weak number sense does not happen in a vacuum. Many preservice teachers enter teacher education programs with preconceived notions they formed as elementary students about the teaching and learning of mathematics (Lubinski & Otto, 2004; Brown & Borko, 1992;). And, based upon their perceived success as elementary students, many preservice teachers also often see themselves as teaching mathematics as they were taught, mostly through the verbal transmission of knowledge and pre-existing methods and procedures (Coffey, 2004; Smith, 1996; Thompson, 1992; McDiarmid, 1990). A common misconception regarding math instruction at the elementary school level is the idea that math instruction involves merely basic arithmetic and is void of any mathematical complexities (Strawhecker, 2004). However, studies conducted by Ma (1999) and Ball (1990)
indicated that a deep understanding of mathematical concepts is necessary for the effective teaching of mathematics. Many preservice teachers have never acquired this conceptual knowledge because their own mathematical experiences lacked meaning. These habits, once established, can be difficult to change leading to resistance by preservice teachers to learning and teaching mathematics in this manner (Lubinski & Otto, 2004; Timmerman, 2004; Mewborn, 1999; McDiarmid, 1990).

**Hope on the horizon.** Even with all of the issues regarding teacher knowledge in mind, there is hope on the horizon and it lies in teacher preparation. Anderson and Piazza (1996) proposed that professional teacher educators are in the best position to promote reform, and found that preservice teachers’ attitudes toward mathematics were positively influenced when involved in math courses, both content and methods, with a constructivist approach. Similarly, Steele and Widman (1997) noted extreme attitudinal changes in preservice teachers for whom a methods course, based upon constructivist learning principles, was developed. For the first time in their mathematical careers, many of the preservice teachers in this study reported a clearer understanding of the procedures they were using to solve problems as well as why the procedures worked. They also became more willing to take risks in their learning and confident in their ability to defend solutions. In short, their image of teaching mathematics was transformed toward a more constructivist philosophy. In a related study, Lowery (2002) found that preservice teachers whose content and methods courses were taught in a school-based setting and with a constructivist-instructional model made gains in positive attitude and confidence in teaching mathematics. Moreover, Strawhecker (2004) reported that concurrent enrollment in a math methods course, a field experience, and a math content course resulted in significant gains in mathematical content knowledge, as well as preservice teachers’ views of
mathematics becoming more consistent with that of reform mathematics. In reform mathematics classrooms, the constructivist approach to learning is emphasized with an outcome of conceptual knowledge for the learner (Wood, Nelson, & Warfield, 2001). The reform approach to teaching and learning differs substantially from most preservice teachers’ prior mathematical experiences and established beliefs (Romberg, 2002).

Heaton (2000) also recommended some specific direction, suggesting that rather than merely focusing on learning more mathematics one should focus on looking differently at the mathematics one feels that one knows. Ma (1999) provided a foundation for mathematical understanding with 1) a depth of topic, or the connecting of a specific topic “with more conceptually powerful ideas” in math; 2) a breadth of topic, or connecting “a specific topic with similar or less conceptual power”; 3) as well as with an emphasis on thoroughness or the capability to weave all of the topics together (2000, p. 121). Heaton (2000) summed all of this up, encouraging teachers to “have the experience of learning a few areas of mathematics in depth and to apply what was learned from that experience to learning something new” (p.158). This emphasis on the interconnectedness is also echoed in the National Council for the Teachers of Mathematics’ process standard of Connections (NCTM, 2000).

Yet another area deemed important in terms of helping develop preservice teacher knowledge is reflection. McDiarmid (1990) noted that purposeful experiences paired with reflection opportunities help preservice teachers attend to what is important while establishing a more immediate need for the knowledge. And Soto-Johnson, Iiams, Oberg, Boschmans, and Hoffmeister (2008), who had preservice teachers reading and writing about Ma’s (1999) book, Knowing and Teaching Elementary Mathematics, found that the active reflection appeared to help preservice elementary teachers reinforce their conceptual understanding of mathematics,
articulate their teaching intentions for their future classroom, and confront a range of strong emotional reactions that accompanied their reading.

**Teacher mathematical knowledge: Number sense.** A more specific area to raise concern for researchers is that of teachers’ number sense. Shumay (2011) contends that all other strands of mathematics are “rooted” in number sense. Additionally, processes such as communicating, reasoning, and problem solving are facilitated through number sense. Tsao (2005) reported those with good number sense have a solid understanding of number meanings and numerical relationships and are flexible in thinking in regard to numbers. Number sense also includes the ability and tendency for using this understanding in flexible ways when making mathematical judgments as well as utilizing functional strategies for handling numbers and operations (Schneider & Thompson, 2000; McIntosh et al, 1999; Reys & Yang, 1998). Shumway (2011) supported this stating, “As students build their number sense, mathematics takes on greater meaning. It becomes more about reaching understanding than following rigid sets of rules” (p. 8). Kaminski (1997) proposed the importance of number sense, arguing that it can assist learners in their understanding of and computing in mathematics. And the National Council of Teachers of Mathematics (2000), whose stance on the significance of teacher knowledge and the ability to use that knowledge flexibly as suggested under its Teaching Principle, also notes the importance of developing number sense. Shumway (2011) best summed up the importance of number sense stating, “It is the key to understanding all math” (p. 8).

Like other areas of mathematical knowledge, teachers’ number sense is an area of concern. Johnson (1998) found that the general number sense of preservice teachers was inadequately developed, with Tsao (2005) reporting that many preservice teachers were not prepared to be immersed into a number sense focused curriculum, one which emphasizes mental
math and estimation over memorized algorithms. In a study that investigated the number sense strategies used by preservice teachers, Yang (2007) found that preservice teachers were much more apt to apply rule-governed explanations rather than number sense based explanations.

Also following suit with teacher knowledge in general, Yang and Reys (2001a, 2001b) emphasized the classroom teacher’s role in developing elementary students’ number sense. Schneider and Thompson (2000) define number sense as “a good understanding of numbers meanings and numerical relationships” (p. 146). For children to develop a strong number sense, teachers must possess a deep understanding of number sense. With the deficit in number sense of preservice teachers, Tsao (2005) advocates for prioritization on the part of colleges and universities so as to produce future classroom teachers for whom number sense focused classrooms will be the norm.

**Reflections from the University Classroom**

Observations and informal feedback from the elementary preservice teachers at one midwestern university yielded similar findings, in that many of the preservice teachers struggled with general mathematical knowledge. Additionally, they failed to demonstrate a strong number sense for many of the topics presented in an Elementary Math Methods course.

Feelings of inadequacy became known to the instructor on the first day of class when the preservice teachers were asked what subject they would select if they could teach only one content area. Fewer than 30% of the 60 preservice teachers chose mathematics. Based on an informal poll question approximately 30 of the same group of preservice teachers reported holding ill feelings, in general, with regard to mathematics. In the same math methods class, the majority of the preservice teachers openly disclosed their inability to apply mathematical concepts effectively and efficiently in a writing assignment that was shared with the instructor.
On the contrary, these same preservice teachers displayed interest in helping their future students learn mathematics in a conceptual-based, constructivist manner, as opposed to a more procedural-based, behaviorist approach.

In spite of this honest yet rough beginning to the semester, many appear to enjoy the initial focus of the course. At the beginning of the course, the preservice teachers learn about concepts pertaining to early number sense while experiencing more than a few eye-opening moments as they begin to understand the foundation of several math concepts, such as counting. More often than not, preservice teachers enjoy success as they prove what they already know procedurally with their newly developed conceptual understanding. Additionally, preservice teachers are put in a position to actually learn a new concept in a manner that parallels the work of Baek and Flores (2005).

In this activity, preservice teachers are put in a position to progress through three levels of strategies, similar to those that are constructed by young children for whom addition and subtraction with single-digit numbers is new. The activity presents the preservice teachers word problems to solve where letters represent quantities. For example, a preservice teacher might solve $C + D$ by counting out “$A, B, C$ and $A, B, C, D$” with his manipulatives or fingers, replicating the direct-modeling phase experienced by children (Baek & Flores, 2005). As the topics in the methods class become increasingly difficult, in that these topics require the preservice teachers to unpack the known procedures to better understand the concepts, initial optimism seems to blur as levels of self-doubt increase.

Midway through the semester, the instructor introduced several thinking strategies for basic math facts for addition, subtraction, and multiplication. Reys, Lindquist, Lambdin, and Smith (2009) define these as “efficient methods for determining answers on the basic facts” (p.
The extent at which these preservice teachers begin to show their lack of ability to use strategies for handling numbers and operations, or number sense (Tsao, 2005) becomes most evident as they try to learn these basic fact thinking strategies (Reys et al., 2009, pp. 197-202):

<table>
<thead>
<tr>
<th>Addition: Basic Fact Thinking Strategies</th>
<th>Degree of Complexity</th>
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</thead>
<tbody>
<tr>
<td>Commutativity</td>
<td>Low</td>
</tr>
<tr>
<td>Adding One and Zero</td>
<td>Low</td>
</tr>
<tr>
<td>Adding Doubles</td>
<td>Low</td>
</tr>
<tr>
<td>Counting On</td>
<td>Low</td>
</tr>
<tr>
<td>Combinations to 10</td>
<td>Low</td>
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<tr>
<td>Adding to 10 and Beyond</td>
<td>Medium to High</td>
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Less complex thinking strategies for addition facts, such as Commutativity, Adding One and Zero, Adding Doubles, and Counting On are comprehended more readily and accepted by the preservice teachers without resistance. In class discussions, many are able to articulate a level of appreciation for the relevance of Combinations to 10, however, when learning the strategy Adding to 10 and Beyond, a disconnect becomes more evident. At this point, nearly half of the preservice teachers begin to struggle with accepting and embracing the strategy. Perhaps more notable, their weak number sense prohibits them from understanding how the strategy works in that numbers can be decomposed. For example, for the basic fact 8 + 6, one would process or verbalize the Adding to 10 and Beyond strategy as “I know 8 + 2 = 10 so I will use 2 from the 6. The remaining 4 from the 6 will then be added to 10 and 10 + 4 = 14. So 8 + 6 = 14.” The comments most often heard include, “Why would you go through all those steps?”

When viewing this strategy, in particular, from their procedural-based background it is quite easy to see that barriers prevent them from true understanding. And though they initially appeared to be convinced of the importance of a strategy-based learning environment for
children, the handful of preservice teachers with weak abilities in terms of their number sense, almost seem to dismiss the notion of teaching mathematics in a conceptual-based, constructivist manner altogether.

After much persistence by the instructor in providing a variety of experiences, some of the once negative attitudes shifted to a more positive light. Most of the preservice teachers came to understand “how” the strategy works, yet remained skeptical about “why” this strategy would be necessary in an elementary setting. With preservice teachers’ inability to make mathematical connections, learning the thinking strategies for the basic facts for addition and subtraction, which has the potential to impact future elementary students, was a time-consuming challenge in the math methods classroom.

Learning of the basic fact thinking strategies for multiplication facts unfold in a similar fashion (Reys, et al., 2009, pp. 204-207):

<table>
<thead>
<tr>
<th>Multiplication: Basic Fact Thinking Strategies</th>
<th>Degree of Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutativity</td>
<td>Low</td>
</tr>
<tr>
<td>Skip Counting</td>
<td>Low</td>
</tr>
<tr>
<td>Repeated Addition</td>
<td>Low</td>
</tr>
<tr>
<td>Splitting the Product into Known Parts</td>
<td>Medium to High</td>
</tr>
<tr>
<td>One More Set</td>
<td>Medium to High</td>
</tr>
<tr>
<td>Twice as Much as a Known Fact</td>
<td>Medium to High</td>
</tr>
<tr>
<td>Working from Known Facts of 5</td>
<td>Medium to High</td>
</tr>
</tbody>
</table>

Once again, the more general strategies, such as Commutativity, Skip Counting, and Repeated Addition are accepted as many preservice teachers may have been introduced to them as elementary students. The problems again start to surface as students focus on the strategy of Splitting the Product into Known Parts, which includes One more set, Twice as much as a known
fact, and *Working from known facts of five*. In essence, the aforementioned strategies are based on the distributive property of multiplication. And though the preservice teachers generally view them as more interesting and novel than *Adding to 10 and Beyond*, again, some of them fail to see how elementary students could possibly understand and apply the thinking strategies when they themselves struggle to understand them. As instructors for the course, the picture is quite clear, however, these preservice teachers are unable to acknowledge that their future students may not have the formidable obstacle of a procedure-based mind if number sense is in place. Moreover, the elementary preservice teachers are unable to fathom how the rigor of the Common Core Standards, coupled with instructional strategies outlined in standards-based math textbooks may maximize children’s development of number sense (Reys et al., 2009) as long as the teacher is able to provide effective instruction to support students’ thinking.

**Conclusion**

In an effort to close the gap in mathematics achievement, engaging elementary preservice teachers in constructivist-based courses may promote higher levels of number sense development. The contention that elementary students’ number sense takes years of rich and varied experiences to be developed (Reys et al., 2009), implies that teacher educators’ efforts to enhance preservice teachers’ number sense may be a great challenge. Continued studies, both formal and informal, of elementary preservice teachers to determine how number sense can be evaluated, as well as what kinds of mathematical experiences promote gains in number sense are recommended.
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