Math That Blooms (Forever)!!

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WELCOME!

Overview

- Dr. Van Cleave: A Little About Me
- Emergent Behavior
- Fractals
- Hexagons
- Natural Phenomenon
- Complex Systems and Chaos

The Farm Where I Grew Up

In The Middle Of Five Kids

Graduated From Arcola HS

Graduated From EIU — Teacher Cert: Math And Art

Taught At Hutsonville

Returned to EIU For An MA in Math

MS and PhD in Computer Science from UK

Visiting Position At Williams College in MA

Then to Texas Tech University

A Tribal & Community College in Minnesota

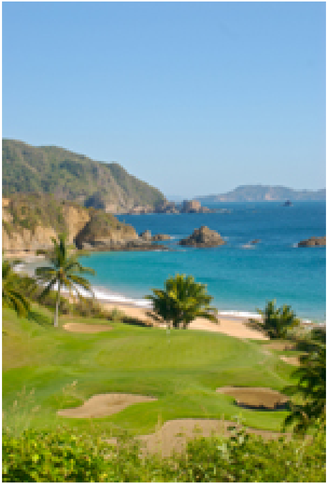
And Back To EIU

I've Gotten to Travel: England & France

Puerto Rico & Belgium

Switzerland & Alaska

Mexico & Italy



The Point is

Never limit yourself by

Thinking you **can't** do something!

You'll never know unless you **try**.

Emergent Behavior

- A complex system can give rise to **emergent behavior**.
- **Emergence** is the way **patterns** arise out of relatively simple interactions.
- An **emergent behavior** or **property** can appear when a number of simple entities operate in an environment, forming more **complex behaviors** as a **collective**.

For Example: Termites

- Complex social systems may exhibit behaviors that are **emergent**.
- The mound building of termites is a property that emerges from the collection of termites.

Termite Mounds

In Australia and Africa — can reach over 30 feet in height.

And Then There Are the Ants...

First you see one...

Then a few more...

Then the deluge!

10 Tons of Concrete and 40 Tons of Dirt Later. . .

Fungi Farms and Refuse Bins ⇒ Air Circulation

Natural Phenomenon: Ice Crystals

Water crystals forming on glass

is an emergent natural process — a

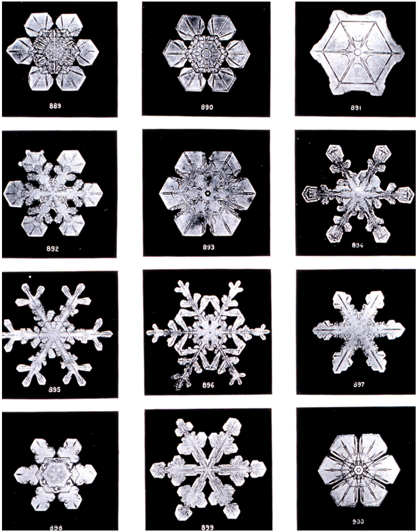
high level of organizational structure

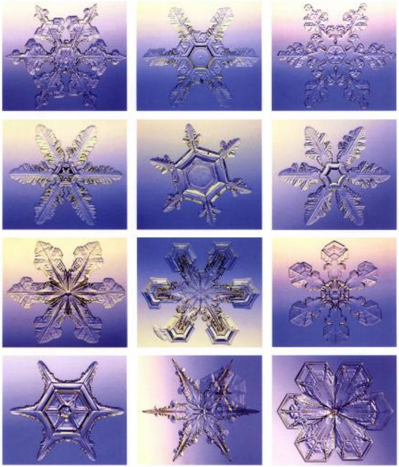
crafted directly by the

random motion

of water molecules.

Nature: Snowflakes





Fractal

a geometric figure consisting of an

identical pattern

which repeats on an ever–reducing scale.

Fractals are everywhere in nature

Fractals in Nature: *chou Romanesco*

Fractal forms are complex shapes which look more or less the same at a wide variety of scale factors.

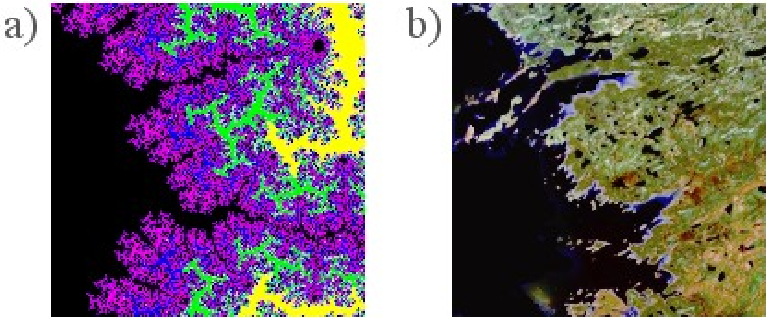
Diving Into Veggies

Look Closely — The Pattern Repeats!

Nature: Frost

Fractal: Frost

Synthesizing Nature: Coastlines

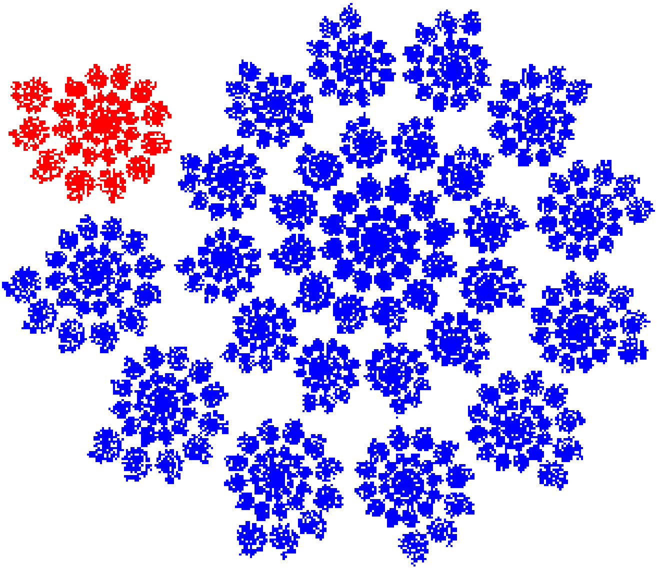


a) Part of the Mandelbrot set.

b) Part of the North American coastline near Hudson Bay.

Ferns

A Spiral of Spirals of Spirals. . .



Chaos and Hexagons

The most interesting stable patterns appear

right at the edge of chaos...

...a honeycomb arrangement of **hexagons**.

John Gribbin
Deep Simplicity

Hexagons Are Everywhere!

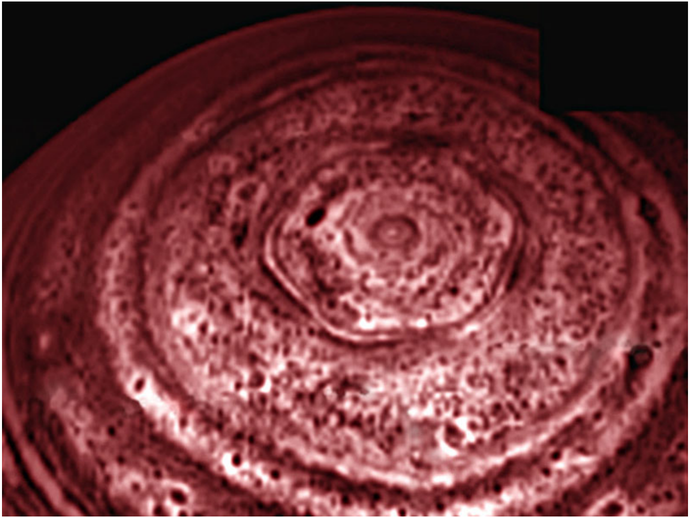
Hexagons Appear in Unexpected Places

Giant's Causeway, Ireland

About 40,000 interlocking basalt columns, the result of an ancient volcanic eruption.

Fingal's Cave, Scotland

Even in Space!



A hexagonal "feature" around Saturn's North Pole

Creating a Hexagon in Fluid (Aliens Not Included)

Similar effect with buckets of water

A pentagonal "hole" in the center of water swirling against the edge of a rotating bucket.

A **hexagon** appears when the bucket is spun at a higher speed.

Hexagons We Can Eat. . .

(More emergent behavior)

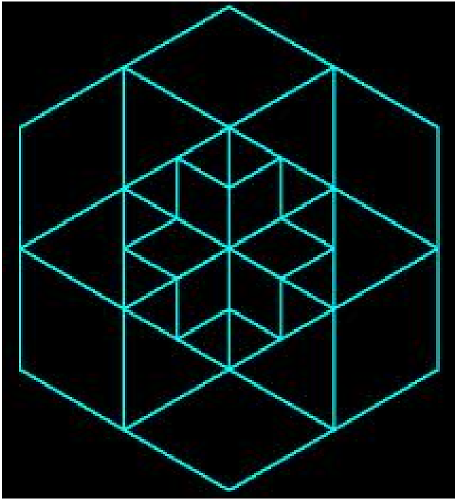
Some of Us Sport Our Own!

Hexagons in the Home

Crochet Pattern

Crochet Colorfully Combined,
Tiling the Plane

Quilt Block Patterns

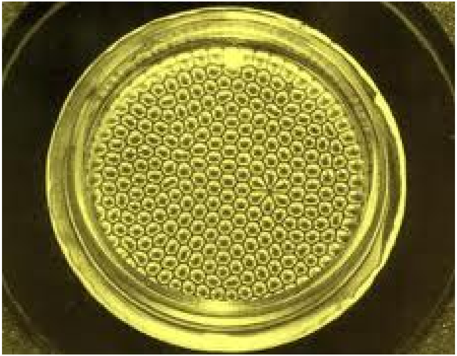


Hexagons in the Kitchen

Bénard Convection

Starting to form . . .

Even More Pronounced



Bénard Convection

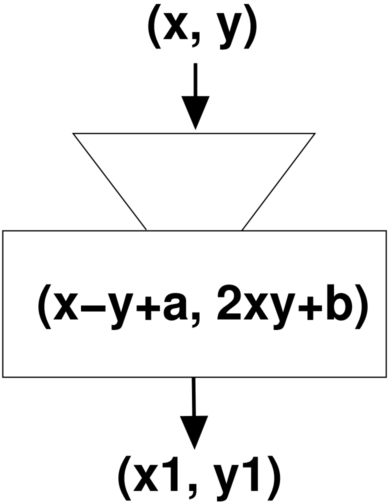
One can observe the formation of Bénard cells
by heating a thin layer of oil in a pan.

Julia Set

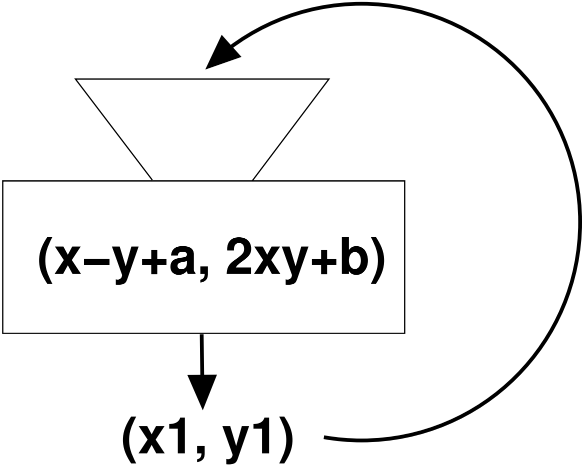
Map (x, y) coordinates in the plane
using the following formula repeatedly:

$$F_{a,b}(x, y) = (x^2 - y^2 + a, \;\; 2xy + b).$$

Process A Point

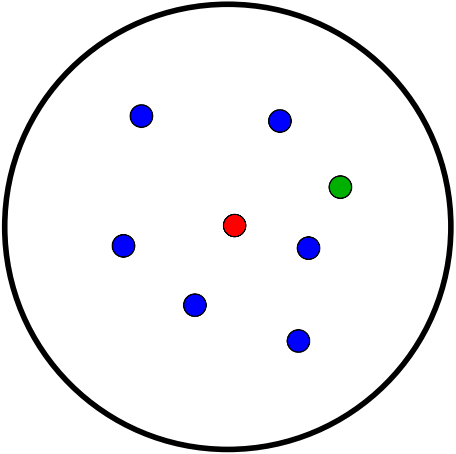


Then Process The Output



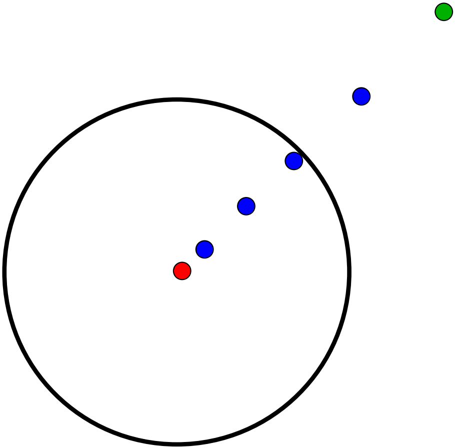
The Points May Be Trapped

Close to where they start. . .



Or They May Escape

And move away from the original point. . .

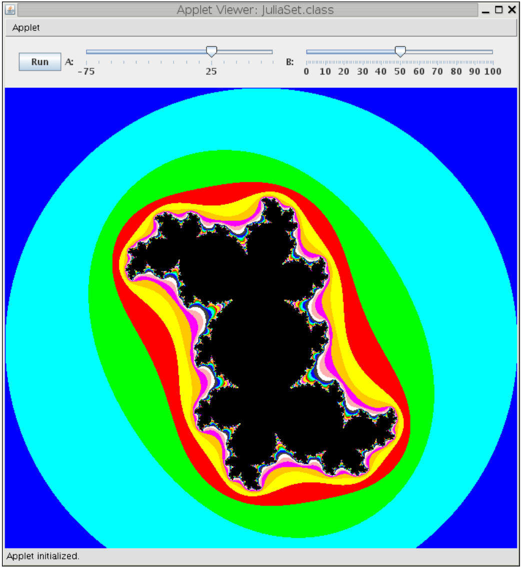


Give The Final Point A Color!

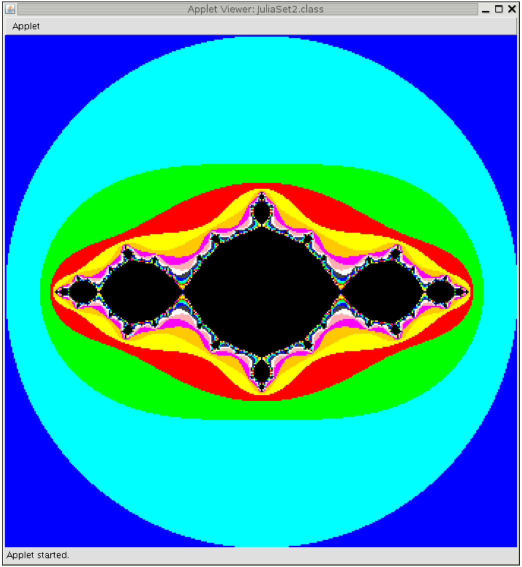
Those that stay close to home, we can color black.

Those that escape, we can color according to how many iterations it takes to get outside the threshold circle.

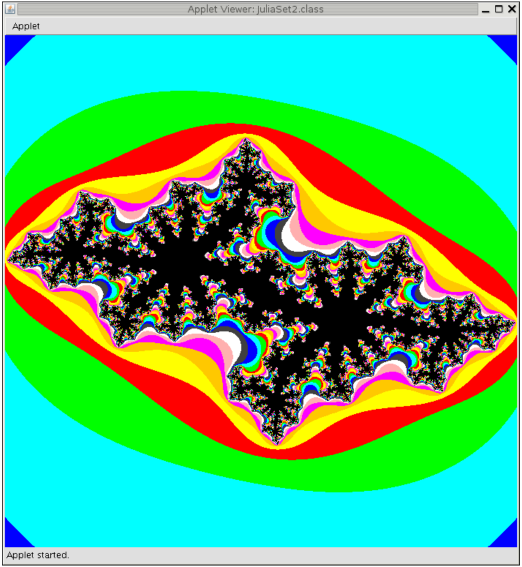
Julia Set Examples — From CS I



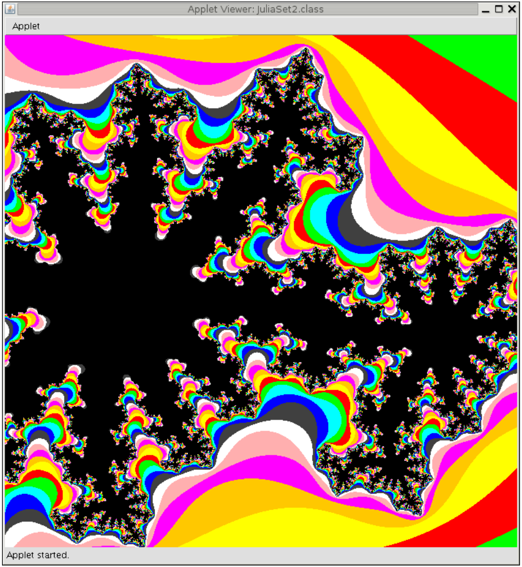
Turtles All The Way Down



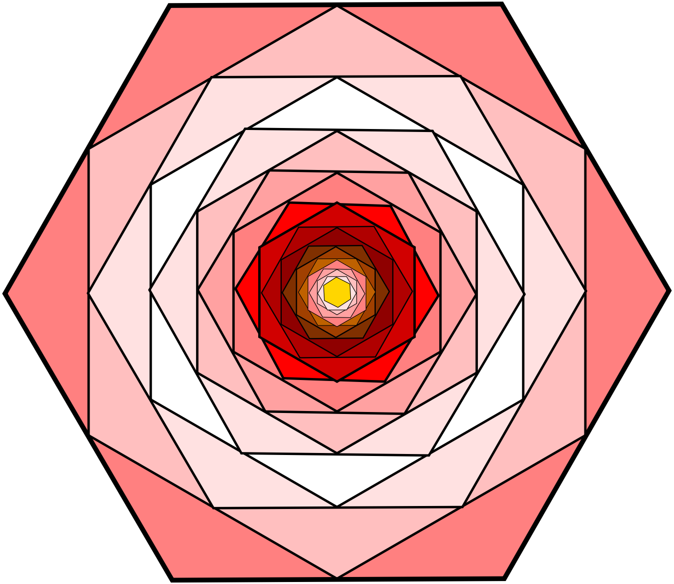
Another Set Of Values For a And b



Zooming In



Another Fractal Pattern From Hexagons



Complex Systems and Chaos

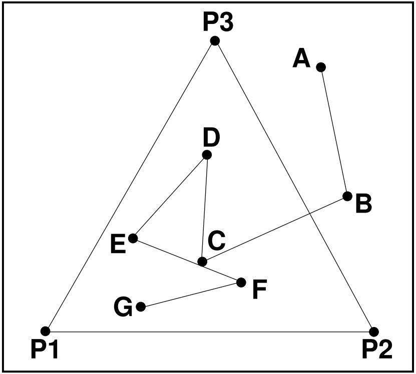
- A **complex system**: **interconnected parts** that **as a whole** exhibit properties not obvious from the **individual's** properties.
- One type of complex, dynamical system: **chaotic system**

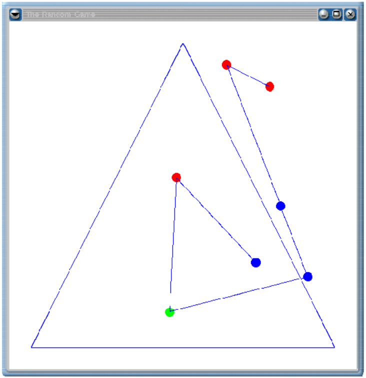
  From chaos sometimes comes order!
- Simple Rule(s) $\Rightarrow$ Complex Result.

Sierpinski Gasket – also from CS I

The Chaos Game

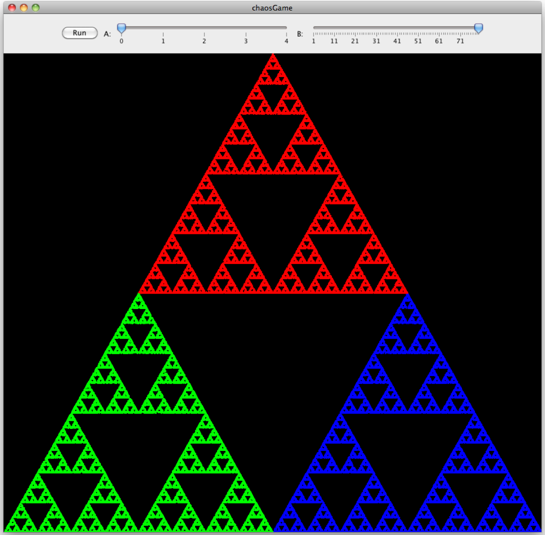
Rule: Randomly pick a vertex and move halfway toward it.





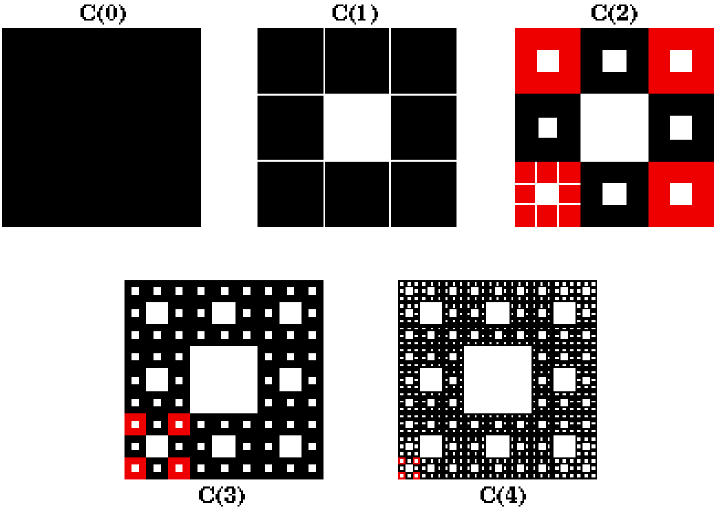
Color the new point according to which vertex was chosen.

The Result:



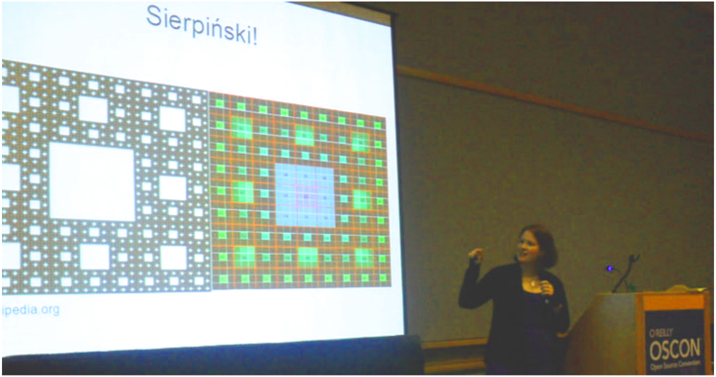
Crochet: Sierpinski Pattern

Sierpinski Carpet

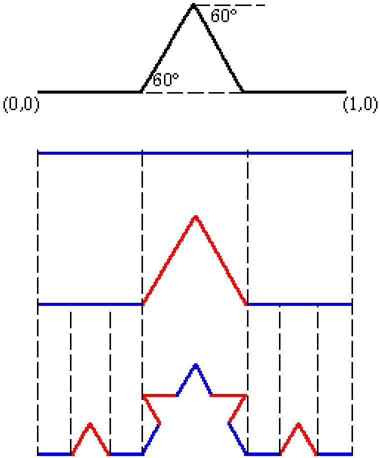


Rule: Subdivide square into 3×3 squares, remove middle; Repeat.

Sierpinski Carpet — and Quilt (OSCON, Australia)

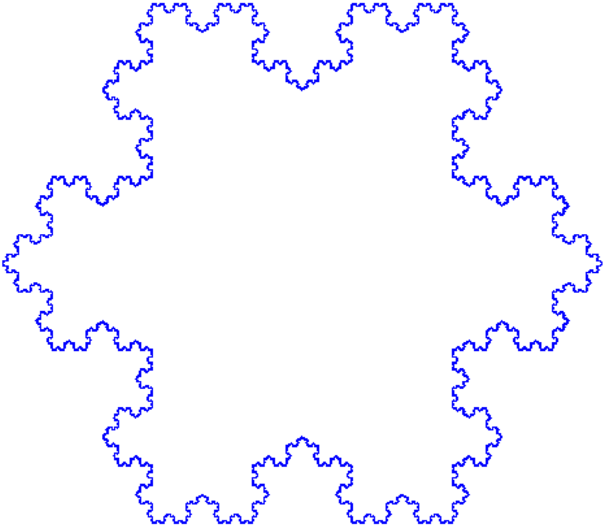


The Koch Snowflake

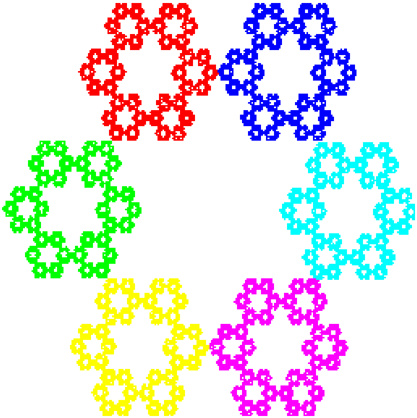


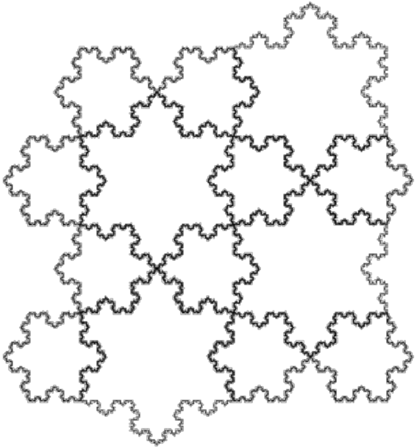
Simple Rule: Replace the middle third of each line segment with 2 sides of an equilateral triangle.

The Koch Snowflake – Starting from a Triangle

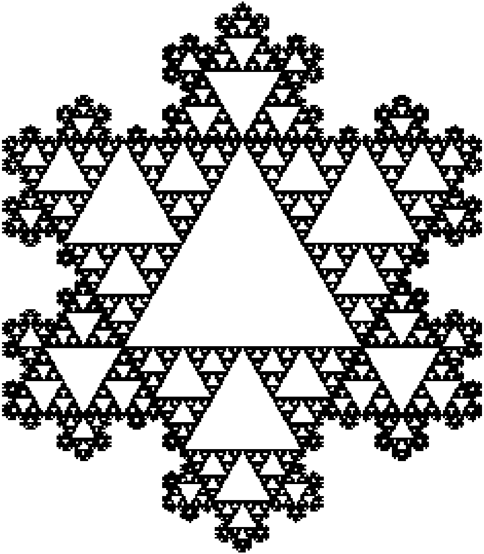


Koch Snowflake Tilings. . .

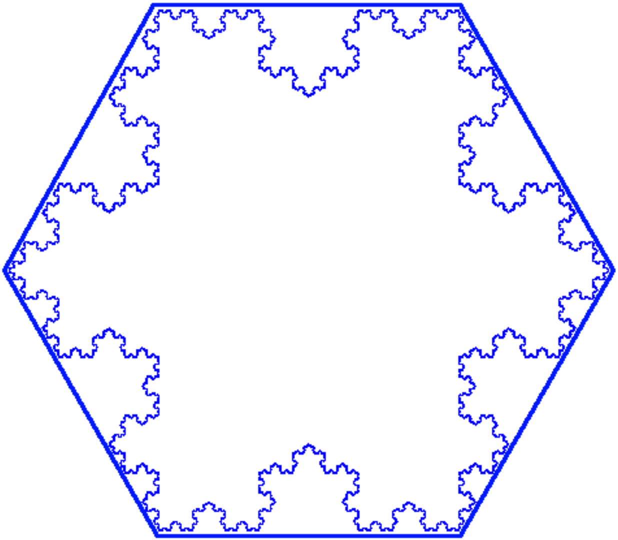




Back to the Sierpinski Triangle!



But wait! It Forms Another Hexagon!



Thank You!

Any Questions?