MAT 1160 — Mathematics, A Human Endeavor

- **Syllabus**: office hours, grading
- **Schedule** (note exam dates)
- **Academic Integrity Guidelines**
- **Homework** & **Quizzes**
- **Course Web Site** :

  www.eiu.edu/~mathcs/mat1160/

Course Overview

Development of mathematical reasoning and problem solving through concentrated study of a limited variety of topics

Course Objectives

This course should encourage and promote:

1. **a positive attitude toward math**,
2. **successful experiences with math**, and
3. **greater clarity and precision when writing mathematics. . .**

Suggestions for Success

- **Attend all lectures** (and exams!)
- Do the assignments, hand in on time — count on spending *at least* **3 hours studying for every hour in lecture**
- Seek help at first sign of trouble — don't wait!
- Math tutors available – see posted places and times
- Obtain a 3-ring binder to organize class handouts, notes, and homework

What Causes Good and Bad Grades?
Scientific American, January 2005

In one study, researchers had students write down what "went through their minds" when they were trying to get better grades.

Students who improved with each test were thinking:

- "I need to work harder"
- "I can learn this material if I apply myself"
- "I can control what happens to me in this class"
- "I have what it takes to do this"

Students who did not improve were thinking:

- "It's not my fault"
- "This test was too hard"
- "I'm not good at this"

Bottom line: Take personal control of your performance.

Course Topics

- Chap. 1 : The Art of **Problem Solving**
- Chap. 2 : The Basic Concepts of **Set Theory**
- Chap. 3 : Introduction to **Logic** (with supplements)
- **Graph Theory** — handouts

Student Responsibilities – Week 1

- **Reading**:

  This week: Textbook, Sections 1.1 & 1.2
  Next week: Textbook, Sections 1.3 & 1.4

- **Homework Sec 1.1** is due in class on Thursday, 1/15

- **Homework Sec 1.2** is due in class next Tuesday, 1/20

Section 1.1: Solving Problems by Inductive Reasoning

Conjecture: a conclusion drawn from repeated observations of a particular process or pattern. The conjecture may or may not be true.

Inductive Reasoning: drawing a **general** conclusion or conjecture from observing **specific** examples.

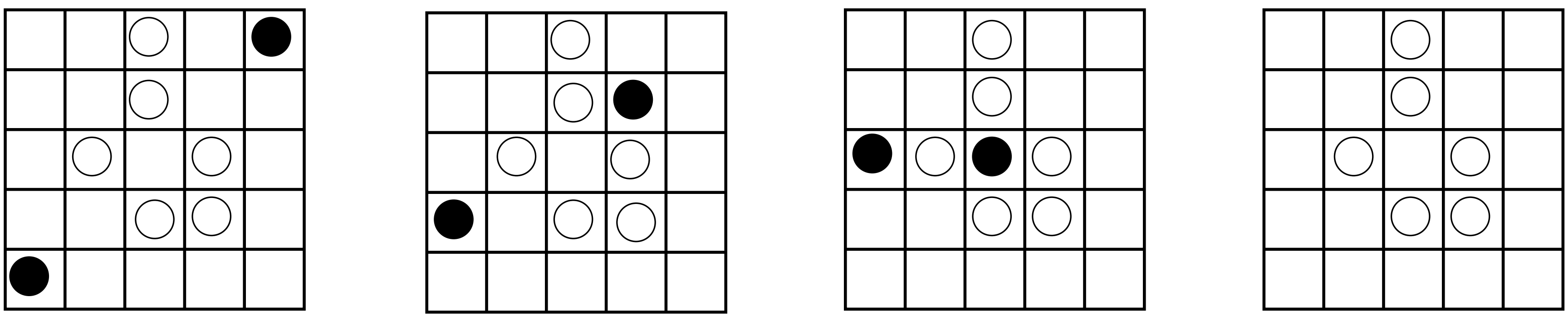
Counterexample: an example or case which disproves a conjecture.

Deductive Reasoning: applying **general** principles to **specific** examples.

Inductive Reasoning Examples

- Inductive: from specific observations to general conclusion

One type of problem which requires inductive reasoning is attempting to determine the next value in a pattern. For example:



What arrangement of the black dots should be placed in the fourth grid?

Inductive Reasoning Application: Number Patterns

Number sequence: an ordered list of numbers having a first number, a second number, a third number, and so on.

Example: 2, 4, 6, 8, 10, . . .

Term: one of the numbers in a sequence.

Ellipsis: the three dots . . .

Arithmetic sequence: a number sequence which has a common **difference** between successive terms.

Example: 1, 5, 9, 13, 17, 21, . . .

Geometric sequence: a number sequence which has a common **ratio** between successive terms.

Example: 2, 4, 8, 16, 32, . . .

Number Pattern Examples

Determine the *probable* next number in each list:

- 3, 7, 11, 15, 19, 23, . . .
- 2, 6, 18, 54, . . .
- 1, 1, 2, 3, 5, 8, 13, 21, . . . (Fibonacci Sequence)
- 1, 3, 9, 27, 81, . . .
- 3, 6, 9, 12, . . .

Predict the next equation:

$$\begin{array}{rcrcr} 37 & \times & 3 & = & 111 \\ 37 & \times & 6 & = & 222 \\ 37 & \times & 9 & = & 333 \\ 37 & \times & 12 & = & 444 \end{array}$$

$$\begin{array}{rcrcl} 34 & \times & 34 & = & 1156 \\ 334 & \times & 334 & = & 111{,}556 \\ 3334 & \times & 3334 & = & 11{,}115{,}556 \end{array}$$

$$\frac{1}{2} = 1 - \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{4} = 1 - \frac{1}{4}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 - \frac{1}{8}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1 - \frac{1}{16}$$

How many days in February?

January						
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

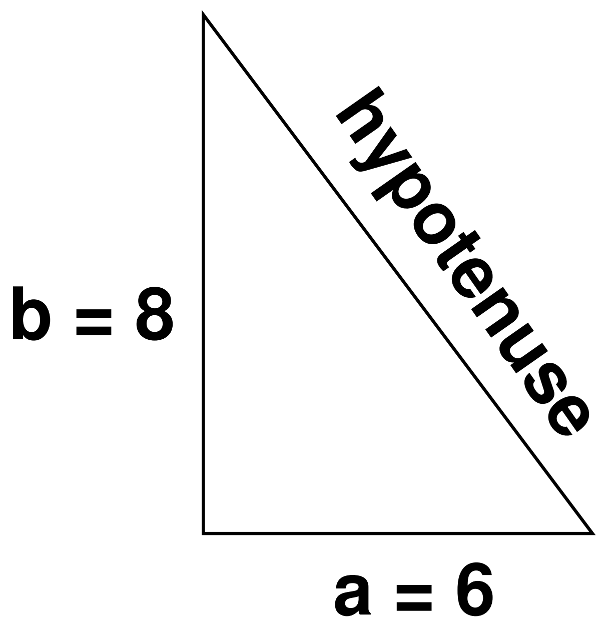
May						
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

February						
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	??	??	??	

Deductive Reasoning

Deductive Reasoning: the process of applying general principles (or rules) to specific examples.

Apply the Pythagorean Theorem to the right triangle with short sides of length 6 and 8.



$$
\begin{aligned}
a^2 + b^2 &= \text{hypotenuse}^2 \\
6^2 + 8^2 &= \text{hypotenuse}^2 \\
36 + 64 &= \text{hypotenuse}^2 \\
100 &= \text{hypotenuse}^2 \\
10 &= \text{hypotenuse}
\end{aligned}
$$

Inductive or Deductive?

- **Inductive**: from specific observations to general conclusion
- **Deductive**: from general principle to specific application

1. It has been cold the past five days, and is cold today as well. So it will also be cold tomorrow.
2. Mandy has 9 stuffed toys. Bert gives her 5 more for her birthday. Therefore she now has 14 of them.
3. In the sequence 0, 3, 6, 9, 12,. . . , the most probably next number is 15.
4. My house is painted white. Both my neighbors houses are painted white. Therefore all the houses in my neighborhood are painted white.
5. The 3-inch cube of wood has a volume of 27 cubic inches.

Logical Arguments

Premise: an assumption, law, rule, widely held idea, or observation. The basis for our case or **argument**.

Conclusion: the result of applying inductive or deductive reasoning to the premise.

Together, the **premise** and **conclusion** make up a **logical argument**.

Premise(s), Conclusion(s), and Type of Reasoning

- All men are mortal. Socrates is a man.
  Therefore, Socrates is mortal.

- If you take your medicine you'll feel better.
  You take your medicine. You should feel better.

- It has been cold the past five days, and is cold today as well. So it will also be cold tomorrow.

- Mandy has 9 stuffed toys. Bert gives her 5 more for her birthday. Therefore she now has 14 of them.

- It is a fact that every student who ever attended this university has been fabulously successful. I am attending this university, so I can expect to be fabulously successful, too.

- If you build it, they will come. You build it. Hence, they will come.

Sec 1.2 More on Number Patterns

- **Arithmetic sequence**: a number sequence which has a common *difference* between successive terms.

- **Geometric sequence**: a number sequence which has a common *ratio* between successive terms.

Method of successive differences: an algorithm to help determine the pattern in a number sequence. The steps are:

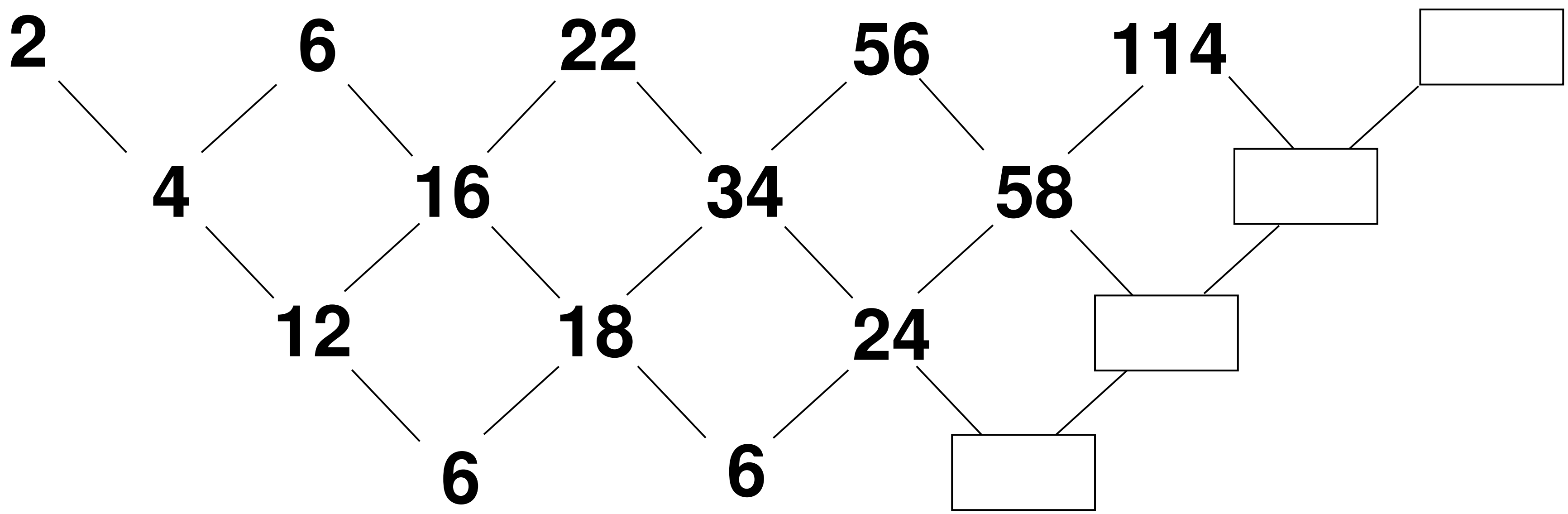
- Find the differences between the first and second, second and third, third and fourth, . . . , terms of the sequence

- If the resulting numbers are not the same (constant) value, repeat the process on these resulting numbers

- Once a line of constant values is obtained, work backward by adding until the desired term of the given sequence is obtained.

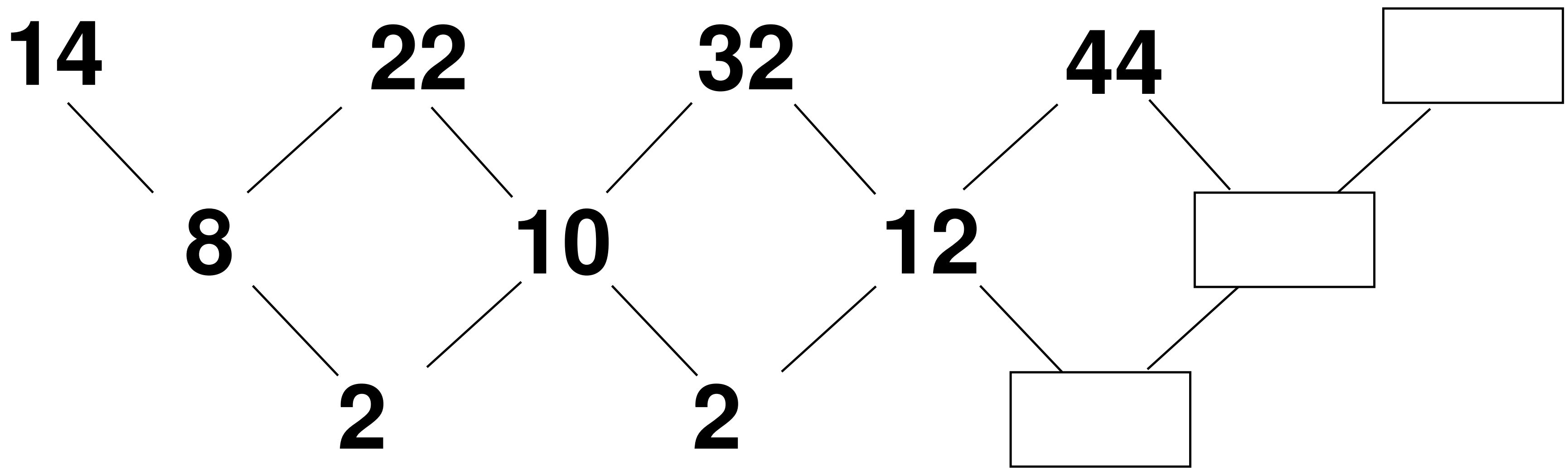
Lattice Example I

Given the sequence 2, 6, 22, 56, 114, . . . , find the next number in the sequence:



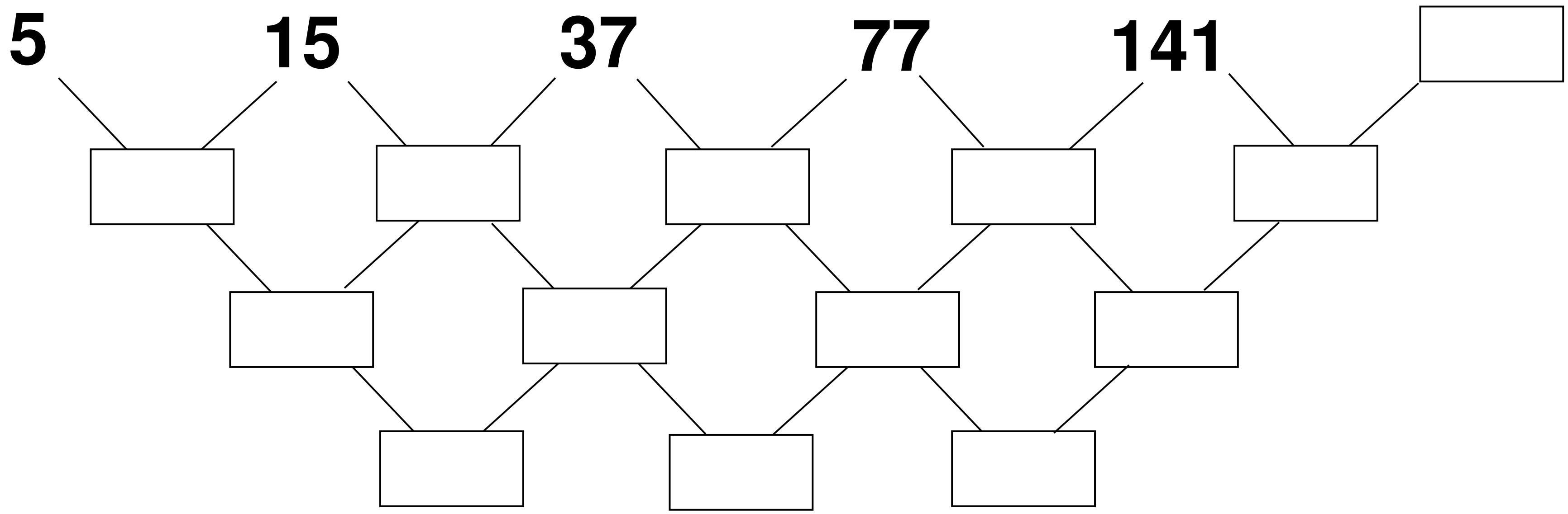
Lattice Example II

Sequence: 14, 22, 32, 44, . . .



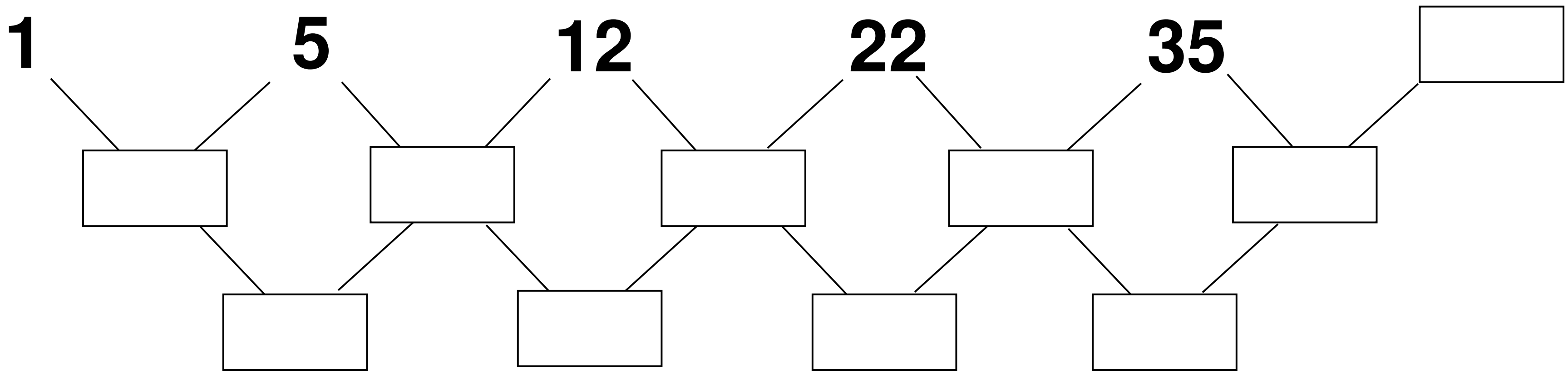
Lattice Example III

Sequence: 5, 15, 37, 77, 141, . . .



Lattice Example IV

Sequence: 1, 5, 12, 22, 35, . . .



Formulas

- **Sum of the First n Odd Counting Numbers**:

  If n is any counting number,

  the sum of the first n odd numbers is n^2:

  $$1 + 3 + 5 + \ldots + (2n - 1) = n^2.$$

  Example. If $n = 4$, then $1 + 3 + 5 + 7 = 16$, **and** $4^2 = 16$.

❖ **Two Special Sum Formulas**:

For any counting number n:

the square of the sum of the first n **counting numbers is the sum of the cubes of the numbers**:

$$(1 + 2 + 3 + \ldots + n)^2 = 1^3 + 2^3 + 3^3 + \ldots + n^3$$

Example. If $n = 4$, then $(1 + 2 + 3 + 4)^2 = 10^2 = 100$, **and** $1 + 8 + 27 + 64 = 100$

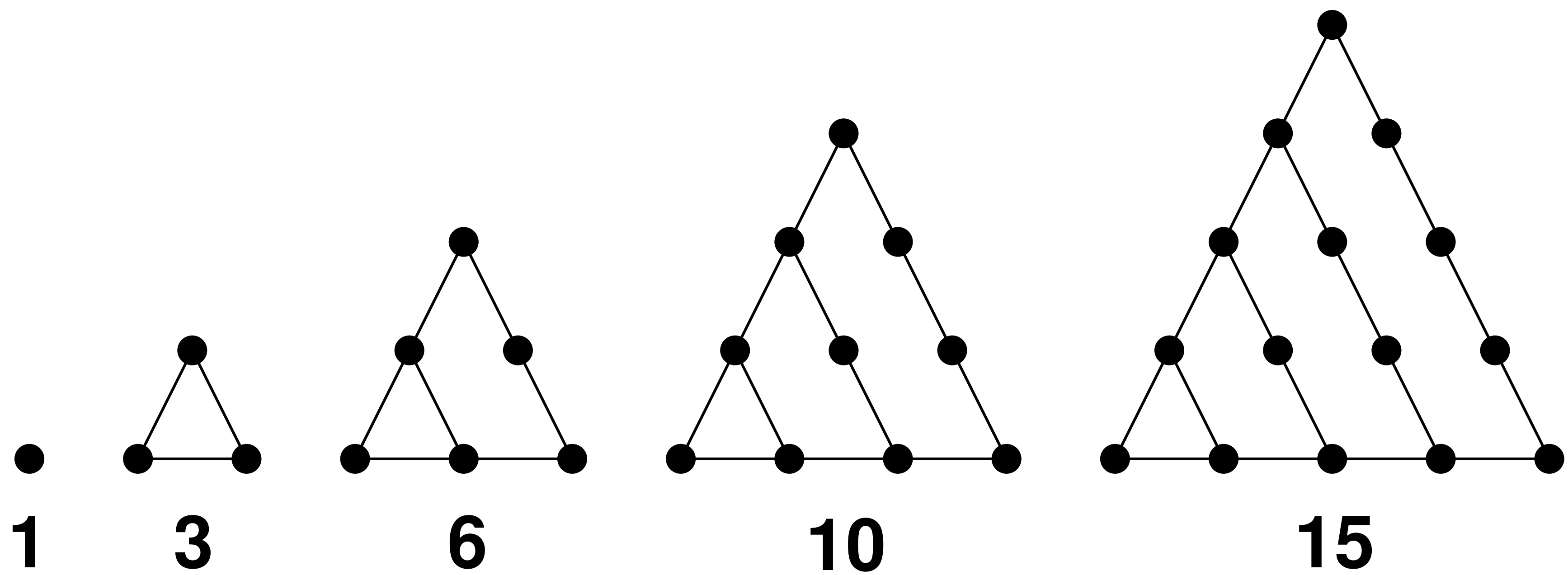
The sum of the first n **counting numbers is**:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$

Example. If $n = 4$, then $1 + 2 + 3 + 4 = 10$, **and** $\frac{4(5)}{2} = \frac{20}{2} = 10$

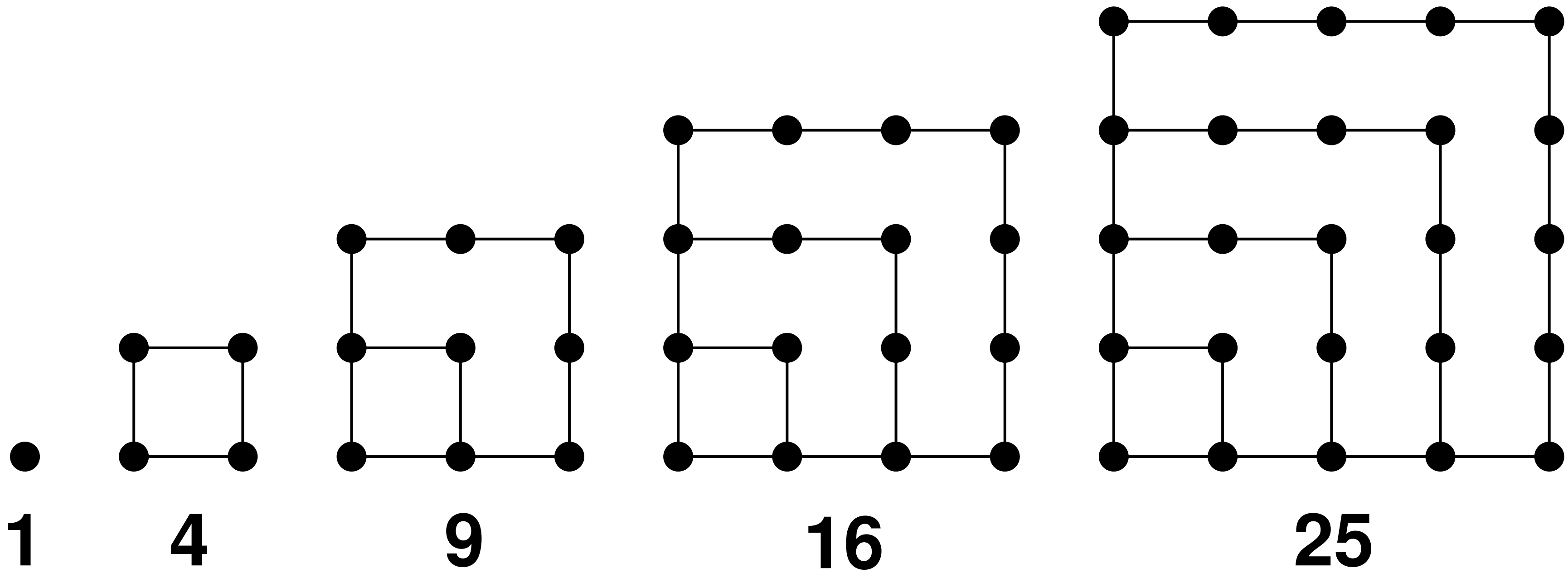
Figurate Numbers
Geometric Arrangements of Points

Pythagoras & Greek mathematicians (c. 540 B.C.) studied properties of numbers and music.



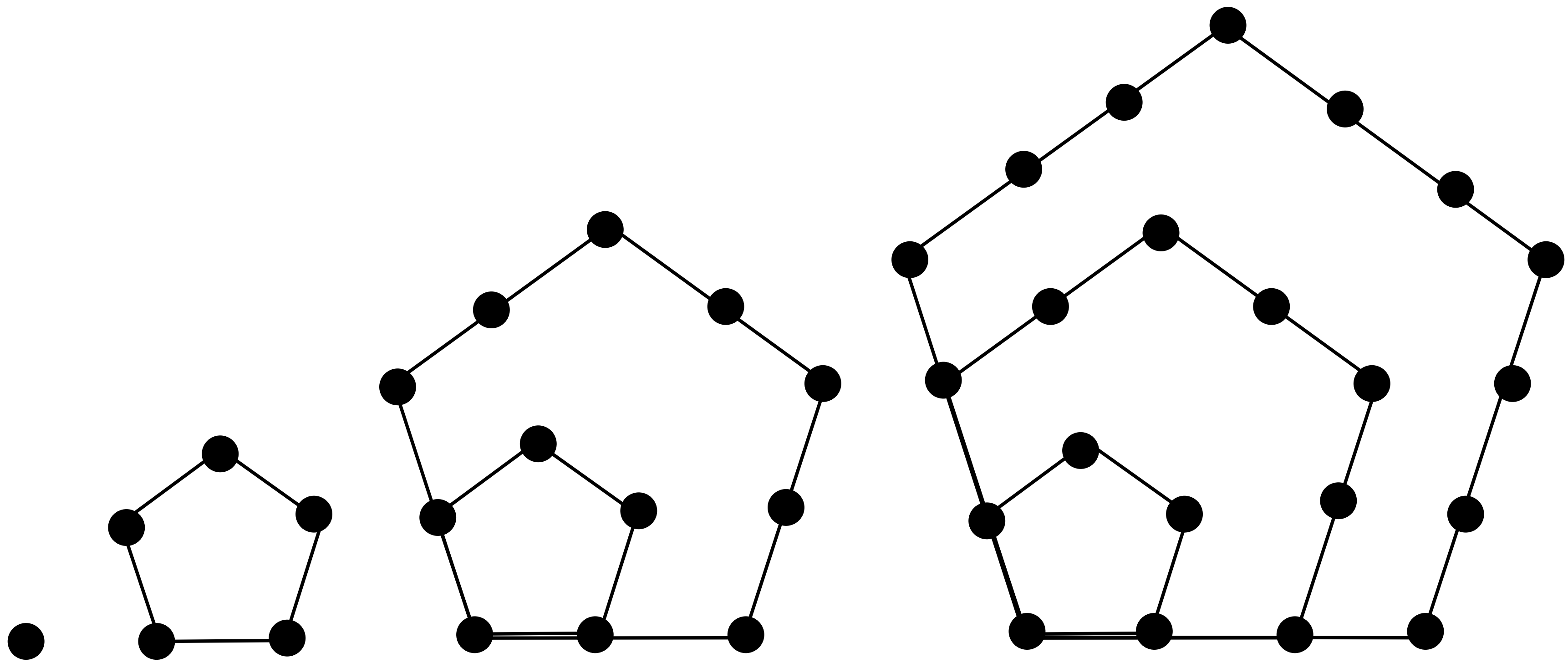
Triangular Numbers

What is the 6^{th} triangular number?
Can you draw the corresponding figure?



Square Numbers

What is the 6th square number?
Can you draw the corresponding figure?



1 5 12 22

Pentagonal Numbers

What is the 5th pentagonal number?
Can you draw the corresponding figure?

Figurate Number Formulas

For any natural number n, . . .

- the nth **triangular** number is given by: $T_n = \frac{n(n+1)}{2}$
- the nth **square** number is given by: $S_n = n^2$
- the nth **pentagonal** number is given by: $P_n = \frac{n(3n-1)}{2}$
- the nth **hexagonal** number is given by: $H_n = \frac{n(4n-2)}{2}$
- the nth **heptagonal** number is given by: $Hp_n = \frac{n(5n-3)}{2}$
- the nth **octagonal** number is given by: $O_n = \frac{n(6n-4)}{2}$

What is the tenth heptagonal number? __________
The ninth pentagonal number? __________
The third octagonal number? __________