Sec 3.6 Analyzing Arguments with Truth Tables

Some arguments are more easily analyzed to determine if they are valid or invalid using **Truth Tables** instead of **Euler Diagrams**.

One example of such an argument is:

If it rains, then the squirrels hide. It is raining.

The squirrels are hiding.

Notice that in this case, there are no universal quantifiers such as all, some, or every, which would indicate we could use Euler Diagrams.

To determine the validity of this argument, we must first identify the component statements found in the argument. They are:

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p = it rains / is raining
q = the squirrels hide / are hiding
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Rewriting the Premises and Conclusion

Premise 1: $p \rightarrow q$ Premise 2: pConclusion: q

Thus, the argument converts to:

$$((p \rightarrow q) \land p) \rightarrow q$$

With Truth Table:

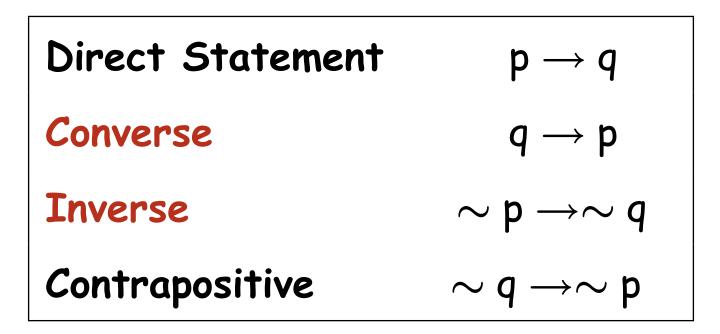
р	q	$((p \rightarrow q) \land p) \rightarrow q$
Т	Г	
Т	F	
F	Т	
F	F	

Are the squirrels hiding?

Testing Validity with Truth Tables

- 1. Break the argument down into component statements, assigning each a letter.
- 2. Rewrite the premises and conclusion symbolically.
- 3. Rewrite the argument as an **implication** with the **conjunction** of all the premises as the antecedent, and the conclusion as the consequent.
- Complete a Truth Table for the resulting conditional statement. If it is a tautology, then the argument is valid; otherwise, it's invalid.

Recall



Which are equivalent?

If you come home late, then you are grounded. You come home late. You are grounded. p = **q** = Premise 1: Premise 2: Conclusion: Associated Implication: р q

 P
 Q

 T
 T

 T
 F

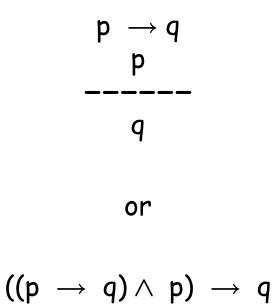
 F
 T

 F
 F

Are you grounded?

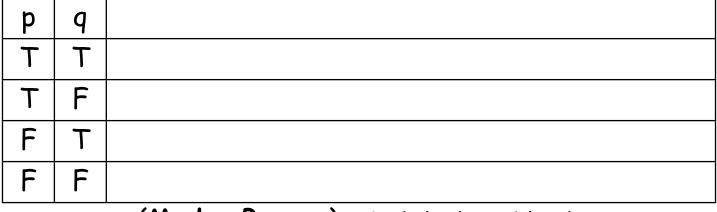
Modus Ponens — The Law of Detachment

Both of the prior example problems use a pattern for argument called modus ponens, or The Law of Detachment.



Notice that all such arguments lead to tautologies, and therefore are valid.

If a knee is skinned, then it will bleed. The knee is skinned. It bleeds. p = **q** = Premise 1: Premise 2: Conclusion: Associated Implication:



(Modus Ponens) - Did the knee bleed?

Modus Tollens — Example

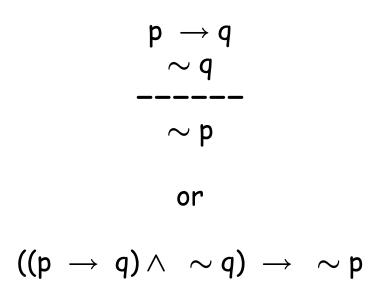
Thus, the argument converts to: $((p \rightarrow q) \land \sim q) \rightarrow \sim p$

р	q	$((p ightarrow q) \land \ \sim q) ightarrow \ \sim p$
T	Т	
Т	F	
F	Т	
F	F	

Did Frank sell his quota or not?

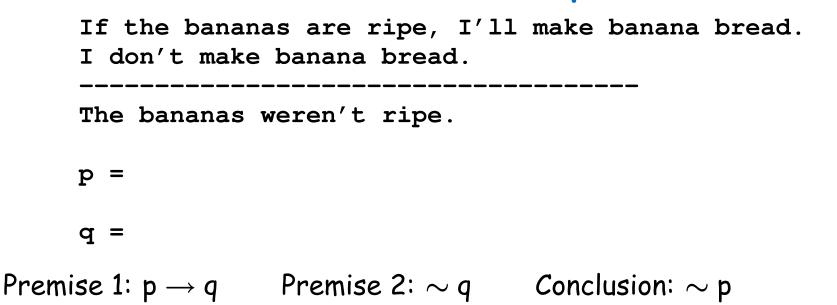
Modus Tollens

An argument of the form:



is called Modus Tollens, and represents a valid argument.

Modus Tollens — Example II



Thus, the argument converts to: $((p \rightarrow q) \land \sim q) \rightarrow \sim p$

р	q	$((p \to q) \land \ \sim q) \to \ \sim p$
T	Т	
Т	F	
F	Т	
F	F	

Were the bananas ripe or not?

Fallacy of the Inverse — Example

If it rains, I'll get wet. It doesn't rain. I don't get wet. p = q =Premise 1: $p \rightarrow q$ Premise 2: $\sim p$ Conclusion: $\sim q$

Thus, the argument converts to: $((p \rightarrow q) \land \sim p) \rightarrow \sim q$

р	q	$((p \rightarrow q) \land \sim p) \rightarrow \sim q$		
T	Т			
Т	F			
F	Т			
F	F			
Did I get wet?				

Fallacy of the Inverse

An argument of the form:

$$p \rightarrow q$$

 $\sim p$
 $\sim q$
or

$$((p \rightarrow q) \land \sim p) \rightarrow \sim q$$

is called the **Fallacy of the Inverse**, and represents an **invalid** argument.

Fallacy of the Inverse — Example II

Premise 1: $p \rightarrow q$ Premise 2: $\sim p$ Conclusion: $\sim q$

Thus, the argument converts to: $((p \rightarrow q) \land \sim p) \rightarrow \sim q$

р	q	$((p \to q) \land \ \sim p) \to \ \sim q$		
T	Т			
Т	F			
F	Т			
F	F			
1				



Another Type of (Invalid) Argument

If it rains, then the squirrels hide. The squirrels are hiding. It is raining. p = it rains / is raining q = the squirrels hide / are hidingPremise 1: $p \rightarrow q$ Premise 2: q Conclusion: p

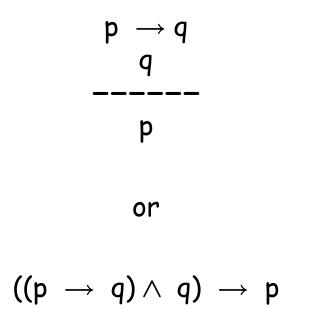
Thus, the argument converts to: $((p \rightarrow q) \land q) \rightarrow p$

р	q	$((p \rightarrow q) \land q) \rightarrow p$
Т	Т	
Т	F	
F	Т	
F	F	

(Fallacy of the Converse) — Is it raining?

Fallacy of the Converse

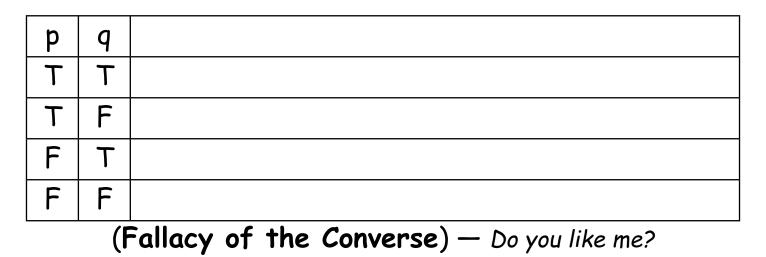
An argument of the form:



is sometimes called the **Fallacy of the Converse**, and represents an **invalid** argument.

If you like me, then I like you. I like you. You like me. p = q = Premise 1: Premise 2: Conclusion:

Associated Implication:



Disjunctive Syllogism — Example

Either you get home by midnight, or you're grounded. You aren't grounded.

You got home by midnight.

p =

q =

Premise 1: $p \lor q$ Premise 2: $\sim q$ Conclusion: p

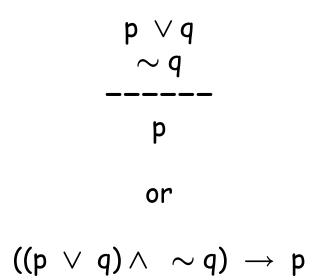
Thus, the argument converts to: $((p \lor q) \land \sim q) \rightarrow p$

р	q	$((p \lor q) \land \sim q) \rightarrow p$
T	Т	
Т	F	
F	Т	
F	F	

Did you get home by midnight?

Disjunctive Syllogism

An argument of the form:



is called a Disjunctive Syllogism, and represents a valid argument.

Disjunctive Syllogism — Example II

Either this milk has soured, or I have the flu. The milk has not soured.

I have the flu. p = q =

Premise 1: $p \lor q$ Premise 2: $\sim p$ Conclusion: q

Thus, the argument converts to: $((p \lor q) \land \sim p) \rightarrow q$

р	q	$((p \lor q) \land \sim p) \rightarrow q$
Т	Т	
Т	F	
F	Т	
F	F	



Reasoning by Transitivity — Example

If you're kind to people, you'll be well liked. If you're well liked, you'll get ahead in life.

If you're kind to people, you'll get ahead in life.

p = you're kind to people
q = you're well liked
r = you get ahead in life

Premise 1: $p \rightarrow q$ Premise 2: $q \rightarrow r$ Conclusion: $p \rightarrow r$ Thus, the argument converts to: $((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$

р	q	r	$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$
Т	Т	Т	
Т	Т	F	
Т	F	Т	
Т	F	F	
F	Т	Т	
F	Т	F	
F	F	Т	
F	F	F	

Reasoning by Transitivity

An argument of the form:

$$p \rightarrow q$$
$$q \rightarrow r$$
$$p \rightarrow r$$

$$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

is called **Reasoning by Transitivity**, and represents a valid argument.

Reasoning by Transitivity — Example

r =

Valid or Invalid?

Argument Forms

	VALID				
Modus Modus Disjunctive Reasoning b					
Ponens	Tollens	Syllogism	Transitivity		
$p \rightarrow q$	$p \rightarrow q$	p∨q	$p \rightarrow q$		
р	\sim q	\sim p	$q \rightarrow r$		
q	\sim p	q	$p \rightarrow r$		

INVALID				
Fallacy of	Fallacy of			
the Converse	the Inverse			
$p \rightarrow q$	$p \rightarrow q$			
q	\sim p			
р	\sim q			

Valid or Invalid?

If you stay in, your roommate goes out.
If your roommate doesn't go out, s/he will finish
 their math homework.
Your roommate doesn't finish their math homework.
Therefore, you do not stay in.

Determine a Valid Conclusion

- It is either day or night.
- If it is daytime, then the squirrels are scurrying.
- It is not nighttime.

Determine a Valid Conclusion

If it is cold, you wear a coat. If you don't wear a coat, you are dashing. You aren't dashing.

Valid or Fallacy? Which Form?

If you use binoculars, then you get a glimpse of the comet. If you get a glimpse of the comet, then you'll be amazed.

If you use binoculars, then you'll be amazed.

If he buys another toy, his toy chest will overflow. His toy chest overflows.

He bought another toy.

If Ursula plays, the opponent gets shut out. The Opponent does not get shut out.

Ursula does not play.

If we evolved a race of Isaac Newtons, that would be progress. (A. Huxley) We have not evolved a race of Isaac Newtons.

That is progress.

Alison pumps iron or Tom jogs. Tom doesn't jog.

Alison pumps iron.

Jeff loves to play gold. If Joan likes to sew, then Jeff does not love to play golf. If Joan does not like to sew, then Brad sings in the choir. Therefore, Brad sings in the choir. If the Bobble head doll craze continues, then Beanie Babies will remain popular. Barbie dolls continue to be favorites or Beanie Babies will remain popular. Barbie dolls do not continue to be favorites. Therefore, the Bobble head doll craze does not continue. If Jerry is a DJ, then he lives in Lexington. He lives in Lexington and is a history buff. Therefore, if Jerry is not a history buff, then he is not a DJ. If I've got you under my skin, then you are deep in the heart of me. If you are deep in the heart of me, then you are not really a part of me. You are deep in the heart of me, or you are really a part of me. Therefore, if I've got you under my skin, then you are really a part of me.