Mat 1160
WEEK 4

Dr. N. Van Cleave

Spring 2010

Student Responsibilities – Week 4

- **Note**: **Exam 1 on Thursday** — covers Chapter 1
- **Reading**:

  This week: Textbook, Sections 2.1 & 2.2
  Next week: Textbook, Sections 2.3 & 2.4
- Summarize Sections
- Work through Examples
- Recommended exercises:
  - Section 2.1: all 1-8, evens 10-50, evens 60-84, all 87-90
  - Section 2.2: all 1-6, evens 8-54, all 61-68

Chapter 2: Basic Concepts of Set Theory

2.1 Symbols & Terminology

- **Terminology**
  - **Set**: a collection of objects
  - **Element** or **Member** of a set: an object belonging to the set

- **Three ways to designate sets**:
  - **word description**
    ex: the set of odd counting numbers between 2 and 12
  - **listing method**
    ex: $\{3, 5, 7, 9, 11\}$
  - **set–builder notation**
    ex: $\{x \mid x \in \mathbb{N},\ x \textit{ is odd},\ \textit{and } x < 12\}$

Notes

- use curly braces – { } – to designate sets,

- use commas to separate set elements

- the variable in the set–builder notation doesn't have to be x.
  ex: $\{z \mid z \in \mathbb{N},\ z \textit{ is odd},\ \textit{and } z < 12\}$

- use ellipses (. . .) to indicate a continuation of a pattern established before the ellipses
  ex: $\{1, 2, 3, 4, \ldots, 100\}$

Important Number Sets

- $\mathbb{N}$ — Natural or Counting numbers: $\{1, 2, 3, \ldots\}$
- $\mathbb{W}$ — Whole Numbers: $\{0, 1, 2, 3, \ldots\}$
- $\mathbb{I}$ — Integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- $\mathbb{Q}$ — Rational numbers: $\{ \frac{p}{q} \mid p, q \in \mathbb{I}, q \neq 0 \}$
- $\Re$ — Real Numbers: $\{ x \mid x$ is a number that can be written as a decimal $\}$
- Irrational numbers: $\{ x \mid x$ is a real number and x cannot be written as a quotient of integers $\}$.

  Examples are: π, $\sqrt{2}$, and $\sqrt[3]{4}$
- $\emptyset$ — Empty Set: $\{ \}$

Notes

- The symbols $\{ \ x \mid x \ldots \}$ is read "x such that x *has some property...*"
- The symbol $\in$ means "is an element of"
- Any **rational** number can be written as either a TERMINATING decimal (like 0.5, 0.333, or 0.8578966) or a REPEATING decimal (like $0.\overline{333}$ or $123.3925\overline{45}$)
- The decimal representation of an **irrational** number **never terminates** and **never repeats**
- The set $\{ \ \emptyset \ \}$ is *not* empty, but is a set which *contains* the empty set (similar to an empty box within an empty box)

Set Cardinality

- **Cardinality** of a set: the number of distinct elements in the set
  - textbook: **n(A)** — or we can use $|A|$
  - If the cardinality of a set is a particular whole number, we call that set a **finite** set
  - If a set is too large to ever finish the counting process, it is called an **infinite** set
- **Well–Defined set**: one for which we can determine **membership** — given any arbitrary value we can determine **conclusively** whether or not that value is in the set

Set Membership

- **Well–Defined** means that given a set and an object, we can determine if the set contains that object
  - Is $2 \in \{ 0, 2, 4, 6 \}$?
  - Is $2 \in \{ 1, 3, 5, 7, 9 \}$?
  - Is $\emptyset \in \{ a, b, c \}$?
  - Is $\emptyset \in \{ \emptyset, \{ \emptyset \} \}$?
  - Is $\emptyset \in \{ \{ \emptyset \} \}$?
  - Is $\frac{1}{3} \notin \{ x \mid x = \frac{1}{p},\ p \in \mathbb{N} \}$

Set Equality

Set Equality: the sets A and B are equal (written $\mathrm{A} = \mathrm{B}$) provided:

- every element of A is an element of B, and
- every element of B is an element of A

  i.e., if they contain exactly the same elements

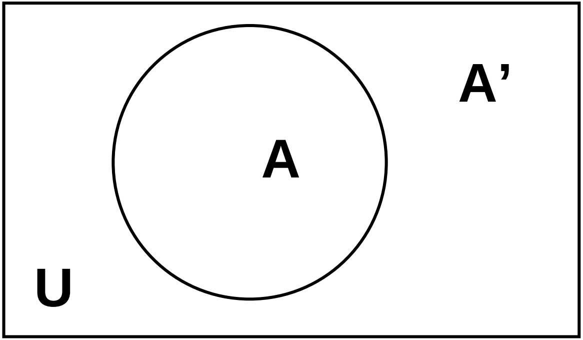
- Does $\{ a, b, c \} = \{ b, c, a \} = \{ a, b, a, b, c \}$?
- Does $\{ 3 \} = \{ x \mid x \in \mathbb{N} \text{ and } 1 < x < 5\}$?
- Does $\{ x \mid x \in \mathbb{N} \text{ and } x < 0\} =$
  $\{ y \mid y \in \mathbb{Q} \text{ and } y \text{ is irrational}\}$?

Sec 2.2 Venn Diagrams & Subsets

- **Universe of Discourse** – the set containing all elements under discussion for a particular problem.

  In math, this is called the **universal set** and is denoted by U

- **Venn Diagrams** can be used to represent sets and their relationships to each other.



- The "Universe" is represented with a rectangle
- Sets are represented with circles
- A' is the **complement** of set A

$$A' = \{ \ x \mid x \in U \text{ and } x \notin A \ \}$$

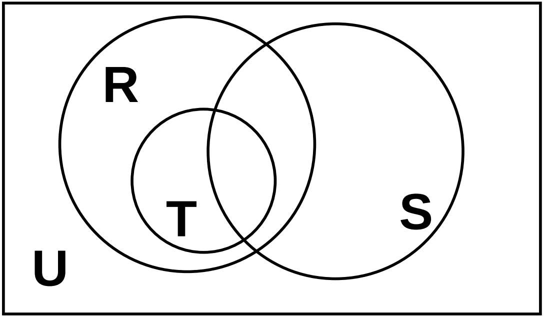
Let $\mathrm{U} = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$,

$\mathrm{R} = \{ 1, 2, 5, 6 \}$, and $\mathrm{S} = \{ 2, 4, 5, 7, 8 \}$

- What is: R', the complement of R ? ______________________
- What is: S', the complement of S ? ______________________
- What is: U', the complement of U ? ______________________
- What is: $\emptyset'$, the complement of $\emptyset$? ______________________

Subsets

- Set A is a **subset** of set B if every element of A is also an element of B, written $\mathrm{A} \subseteq \mathrm{B}$
- Of the sets U, R, S, and T shown in the Venn diagram below, which are subsets?



If $\mathrm{T} = \{ 2, 6 \}$, and the other sets are as given before, what elements are in the area where all the sets overlap?

Is $\mathrm{T} \subseteq \mathrm{S}$ in this case?

Is or Is Not a Subset?

Is the left set a subset of the set on the right?

$\{$ a, b, c $\}$	_____	$\{$ a, c, d, f $\}$
$\{$ a, b, c $\}$	_____	$\{$ c, a, b $\}$
$\{$ a, b, c $\}$	_____	$\{$ a, b, c $\}$
$\{$ a $\}$	_____	$\{$ a, b, c $\}$
$\{$ a, c $\}$	_____	$\{$ a, b, c, d $\}$
$\{$ a, c $\}$	_____	$\{$ a, b, d, e, f$\}$
set X	_____	set X
$\emptyset$	_____	$\{$ a, b, c $\}$
$\emptyset$	_____	$\emptyset$

Set Equality

- A second definition for **set equality**:

  $$\text{Set } \mathrm{A} = \mathrm{B} \text{ if } A \subseteq B \text{ and } B \subseteq A$$

- **Proper Subset**: $A \subset B$ if $A \subseteq B$ and $A \neq B$

- Is the left set **equal** to, a **proper subset** of, or **not a subset** of the set on the right?

{ 1, 2, 3 }	_____	$\mathbb{I}$, the integers
{ a, b }	_____	{ a }
{ a }	_____	{ a, b }
{ a, b, c }	_____	{ a, d, e, g }
{ a, b, c }	_____	{ a, b, c }
{ $\emptyset$ }	_____	{ a, b, c }
{ $\emptyset$ }	_____	{ }

Cardinality of the Power Set

Power Set: $\mathscr{P}(A)$ is the set of *all* possible subsets of the set A

For example, if $A = \{0, 1\}$, then

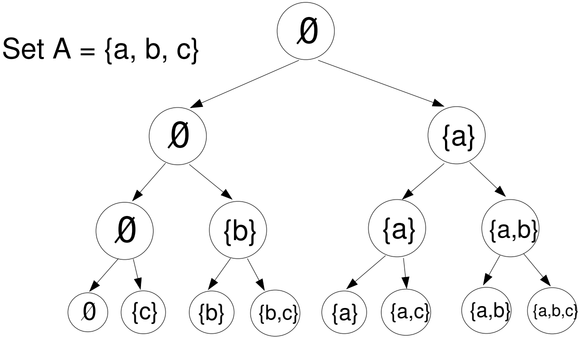
$$\mathscr{P}(A) = \{\varnothing, \{0\}, \{1\}, \{0, 1\}\}$$

Find the following Power Sets and determine their **cardinality**, or number of elements.

- $\mathscr{P}(\emptyset) \quad =$
- $\mathscr{P}(\{ \text{a} \}) \quad =$
- $\mathscr{P}(\{ \text{a, b} \}) \quad =$
- $\mathscr{P}(\{ \text{a, b, c} \}) \quad =$
- Is there a pattern?

Another Method for Generating Power Sets

- A **tree diagram** can be used to generate $\mathscr{P}(A)$. Each element of the set is either in a particular subset, or it's not.



- The number of subsets of a set with cardinality n is 2^n
- The number of **proper** subsets is $2^n - 1$ (Why?)