On separate paper, provide induction proofs for each of the following, using the same technique presented in lecture. Be sure to **label** all parts and include **reasons** for each step. Staple solutions together in order given.

1. $5|(n^5 - n) \ \ \forall\ n \geq 0$

2. $8|(n^2 - 1) \ \ \forall\ \ odd\ n > 0$

3. $\displaystyle\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} \ = \ \frac{n}{2n+1} \ \ \forall\ n \geq 1$

4. Use induction to prove: $n^3 \geq (n^2 + 3) \ \ \forall\ n \geq 2$

5. For which non–negative integers n is $2n + 3 \leq 2^n$? Prove your answer correct using mathematical induction.

6. Prove by induction that a set with n elements has $\frac{n(n-1)}{2}$ subsets containing exactly two elements whenever n is an integer greater than or equal to 2.

7. Prove $\forall\ n \geq 1$, that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets such that $A_j \subseteq B_j$ for $j = 1, 2, 3, \ldots, n$, then
$$\cup_{j=1}^{n} A_j \subseteq \cup_{j=1}^{n} B_j$$

8. Prove by induction that $T(n) \ = \ 3n + 2$ if
$$T(n) \ = \ \begin{cases} 2 & n = 0 \\ 3 + T(n-1) & n > 0 \end{cases}$$