MAT 2345 — Proof Worksheet 2

Solve each of the following on a separate sheet of paper, clearly answering any question(s) given.

1. Prove or Disprove each of the following. What type of proof did you use?
   - (a) The sum of two irrational numbers is irrational
   - (b) If a and b are rational numbers that are not integers, then a^b is rational
   - (c) If $f(n) = n^2 - n + 17$, then $f(n)$ is prime for all positive integers n
   - (d) If p and q are primes (> 2), the $pq + 1$ is never prime
   - (e) 30! ends in exactly seven 0s
2. Let $a|b$ mean "a divides b" (evenly, i.e., with a remainder of zero). Prove or Disprove each of the following. What type of proof did you use?
   - (a) If $a|b$ and $c|d$, then $(ac)|(b+d)$
   - (b) If $a|c$ and $b|c$, then $(ab)|c^2$
   - (c) If $a|bc$ then $a|b$ or $a|c$
3. Give a direct proof: "If x is an odd integer and y is an even integer, then $x + y$ is odd."
4. Give a proof by contradiction: "If n is an odd integer, then n^2 is odd."
5. Give (a) a direct proof and (b) a proof by contradiction: "If x and y are odd integers, then $x + y$ is even."
6. Consider the theorem: If x is an odd integer, then $x + 2$ is odd.
   - (a) Give a direct proof
   - (b) Give an indirect proof
   - (c) Give a proof by contradiction
7. Consider the theorem: If n is an even integer, then $n + 1$ is odd.
   - (a) Give a direct proof
   - (b) Give an indirect proof
   - (c) Give a proof by contradiction
8. Prove: n is even if and only if n^2 is even.
9. Prove: If m and n are even integers, then $4|mn$.
10. Prove or disprove: there are six consecutive composite integers.