MAT 2345 — Worksheet 2 — Proofs, Part 1

Solve each of the following on a separate sheet of paper, clearly answering any question(s) given.

Justify your answers .

1. Which rule of inference is used in each of the following:

   (a) If it snows today, the university will be closed.
   The university will not be closed today.
   Therefore, it did not snow today.

   (b) If I work all night on this homework, then I can answer all the exercises.
   If I answer all the exercises, I will understand the material.
   Therefore, if I work all night on this homework, then I will understand the material.

2. Determine whether the following arguments are valid:

   (a) $$\begin{array}{l} p \to q \\ \underline{\neg p \quad\quad} \\ \neg q \end{array}$$

   (b) $$\begin{array}{l} p \to r \\ q \to r \\ \underline{\neg(p \vee q) \quad\quad} \\ \neg r \end{array}$$

   (c) $$\begin{array}{l} p \to r \\ q \to r \\ \underline{q \vee \neg r \quad\quad} \\ \neg p \end{array}$$

3. Determine whether each of the following arguments is valid. Name the rule of inference or the fallacy if applicable.

   (a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

   (b) If n is a real number such that $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

4. Write each of the following arguments in symbolic form, then determine whether they are valid or invalid.

   (a) She is a Math Major or a Computer Science Major.
   If she does not know discrete math, she is not a Math Major.
   If she knows discrete math, she is smart.
   She is not a Computer Science Major.
   Therefore, she is smart.

   (b) Rainy days make gardens grow.
   Gardens don't grow if it is not hot.
   It always rains on a day that is not hot.
   Therefore, if it is not hot, then it is hot.

   (c) If you are not in the tennis tournament, you will not meet Ed.
   If you aren't in the tennis tournament or if you aren't in the play, you won't meet Kelly.
   You meet Kelly or you don't meet Ed.
   It is false that you are in the tennis tournament and in the play.
   Therefore, you are in the tennis tournament.

   (d) It is not rainy and it is not cold.
   If it is not July, then it is rainy or cold.
   If it is July or August, then it is rainy.
   Therefore, it is rainy.

5. Show these arguments are valid, citing the relevant Rule(s) of Inference:

   (a) Every student in this class passed the first exam.
   Alvina is a student in this class.
   Alvina passed the first exam.

   (b) Jean is a student in my class.
   No student in my class is from England.
   Jean is not from England.

   (c) My daughter visited Europe last week.
   Someone visited Europe last week.

6. Suppose you wish to prove a theorem of the form "if p then q".

   (a) If you give a direct proof, what do you assume and what do you prove?

   (b) If you give an indirect proof, what do you assume and what do you prove?

   (c) If you give a proof by contradiction, what do you assume and what do you prove?

7. Prove or Disprove each of the following. What type of proof did you use?

   (a) The sum of two primes is a prime

   (b) If p and q are primes (> 2), then $p + q$ is composite

   (c) There exist two consecutive primes, each greater than 2

8. Let a, b, c, d, and k in this question and the next be integers. Let $a|b$ mean "a divides b" (evenly, i.e., with a remainder of zero). Here is a sample proof that contains an error. Explain why the proof is not correct.

   Theorem. If $a|b$ and $b|c$, then $a|c$.

   Proof: Since $a|b$, $b = ak$. Since $b|c$, $c = bk$.

   Therefore $c = bk = (ak)k = ak^2$, and hence $a|c$.

9. Prove or Disprove each of the following. What type of proof did you use?

   (a) If $a|b$ and $c|d$, then $(a + c)|(b + d)$

   (b) If $a|b$ and $b|c$, then $a|c$

   (c) If $a|c$ and $b|c$, then $(a + b)|c$