MAT 3570: Homework Assignment 1

Due: Friday, January 11

Introduction

The primary purpose of this assignment is to familiarize you with MATLAB[†], the software system we will use throughout this course.

A Computational Problem

Mathematicians have been intrigued with the constant π for thousands of years. One aspect of the intrigue is to find ever better techniques for estimating the value of π. Suppose you want to produce an estimate of π and you have a calculator capable only of fairly basic arithmetic operations. How could you produce a good estimate? We will use MATLAB and some mathematical history from the 16th century to answer this question.

Using an infinite product and nested square roots, the French mathematician François Viète (also known as Franciscus Vieta) is credited with the following identity:

$$\frac{\pi}{2} = \frac{2}{\sqrt{2}} \times \frac{2}{\sqrt{2+\sqrt{2}}} \times \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \times \cdots$$

Clearly, we cannot carry out this product with a computer since there are an infinite number of factors involved. However, we can replace the infinite product with a finite product by "truncating" it. By varying the number of factors used—1, 2, 3, and so forth—we can produce approximations to $\pi/2$, which we will denote $a_1, a_2, a_3, \ldots$

Let d_i denote the denominator of the ith factor in the Viète product. Observe that these denominators can be defined inductively as follows:

$$\begin{aligned} d_1 &= \sqrt{2} \\ d_{i+1} &= \sqrt{2+d_i} \text{ for } i = 1, 2, 3, \ldots \end{aligned}$$

With this notation, we can restate the Viète product as follows:

$$\frac{\pi}{2} = \prod_{i=1}^{\infty} \frac{2}{d_i}$$

and the truncated product with n factors is just:

$$a_n = \prod_{i=1}^{n} \frac{2}{d_i}$$

The ultimate goal for this assignment is to tabulate the values of a_n and $e_n = |a_n - \frac{\pi}{2}|$ for $1 \leq n \leq 20$. The quantity e_n is known as the **absolute error** and is a measure of the difference between a computed value and a known value.

This may be a good time to pause: can you devise an algorithm based on Viète's identity which will compute and output the values of a_n and e_n, for $1 \leq n \leq 20$?

[†]MATLAB is one of the leading software systems for numerical computation. This software is available for your use in either of the Macintosh labs in Old Main. There is a free system, Octave, that is (mostly) compatible with MATLAB. If you are interested in getting a copy for your own computer, see `http://www.gnu.org/software/octave`. A student version of MATLAB is available for purchase at `http://www.mathworks.com`.

Away from the Computer

Design a sequence of MATLAB statements which will compute and output a table of values with three columns: n, a_n, and e_n. There should be 20 rows of numerical values: one for each value of n from 1 to 20. Each column should be appropriately labeled. Values of a_n should be output with 12 digits after the decimal point and values of e_n should be shown with scientific notation, with three places after the decimal.

This task will take approximately 20 lines of code and should make use of arrays, simple loops, and formatted print (`fprintf`) statements. Use the MATLAB examples shown in class as a guide.

At the Computer

1. Using your EIU credentials, login to one of the Macintosh computers in Old Main.
2. Open the Finder application and create a folder named `3570` in your "home" folder.
3. Create a folder named `hw01`, within your `3570` folder.
4. Using the Finder, navigate to the `Applications` folder and locate the $\frac{\text{SUB}}{\text{MIT}}$ application. Drag this icon onto the upper portion of the Finder window. A small green plus sign will appear within the icon when you position it in the gray portion of the Finder window. You will need this to submit most of the homework you complete for this class.
5. On the Macintosh dock, locate the MATLAB icon, which looks like this:  . Click on this icon to launch it. The first time it runs, it will create a folder named `MATLAB`, within your `Documents` folder. It is important to be aware of this folder, since this is where you will store all of the m-files you will use.
6. In the MATLAB command window, enter the following command:

   ```
   com.mathworks.mlwidgets.html.HtmlComponentFactory.setDefaultType('HTMLRENDERER');
   ```

   This only needs to be done once.
7. MATLAB has a *diary* feature which records your interactions. At the `>>` command prompt, enter:

   ```
   diary('~/3570/hw01/mywork')
   ```

   This will give you a permanent record of your MATLAB session. (The tilde in the string above refers to your home folder.) For this homework exercise, you will be submitting this diary.

   As a side note, your diary will include every aspect of your session, including any mistakes you might make. Don't worry about turning in these mistakes — I will only be interested to see that you eventually produced a correct solution.
8. For online help, click on **Help**, then select **Documentation**. For an example, in the help window that appears, replace `Search Documentation` with `sign` to search for information about the signum function.
9. Sometimes, you might know the name of a MATLAB feature, but you won't know the details. Let's say you've heard about `fprintf`, but don't know how to use it. Try the following two commands at the MATLAB prompt to learn more:

   ```
   help fprintf
   doc fprintf
   ```

10. At a command prompt, enter the command `format long` to change the default output format. Enter `help format` to learn about other options.

11. Enter a few expressions which will help verify the results of the table you will produce later:

    (a) $\dfrac{2}{\sqrt{2}}$

    (b) $\dfrac{2}{\sqrt{2}} \cdot \dfrac{2}{\sqrt{2+\sqrt{2}}}$

    (c) $\dfrac{2}{\sqrt{2}} \cdot \dfrac{2}{\sqrt{2+\sqrt{2}}} \cdot \dfrac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}}$

12. Experiment with MATLAB, using examples shown in class or ones of your own invention. Include examples to demonstrate your understanding of looping, arrays, and the `fprintf` statement.

13. Create a file, `vieta.m` by giving the following command:

    ```
    edit vieta
    ```

    In the editor window which appears, add your MATLAB code which will generate the desired table. Save your work using the `Save` button within the editor window.

14. To execute the commands within the `vieta.m` file, give the following command at the MATLAB prompt:

    ```
    vieta
    ```

    Ensure that the values of a_1,a_2, and a_3 agree with the results previously computed. If you need to debug your code, make changes within the editor window, save the file, then execute the statements by reissuing the `vieta` command.

15. When you are ready to quit MATLAB, turn off the diary:

    ```
    diary off
    ```

16. Quit MATLAB.

17. Using the Finder, add a copy of `vieta.m` to your `hw01` folder. (Recall that by default, `.m` files are stored in the `MATLAB` folder within your `Documents` folder.) This folder should now have two files: your diary and the table-generating MATLAB code.

18. To submit your work, drag your `hw01` folder onto the submit icon.