

MAT 3570: Homework Assignment 2

Due: Wednesday, January 16

Background

A **Taylor series**[†] provides a way to represent a function as an infinite sum of terms. Using a finite number of terms of this sum, we can obtain an approximation of the function. The practical value of Taylor series is appealing: for computational purposes, we replace a “complicated” function with a “simpler” one.

The purpose of this homework is twofold: first, to review the mechanical aspects of deriving a Taylor series and second, explore MATLAB’s graphing capabilities.

Mathematical Review

Read Theorem 0.8 (pages 21–22) in our textbook, then find the Taylor polynomial of degree 6 about the point $x = 0$ for each of the following functions.

- a.) $\cos 2x$
- b.) $\sin^2 x$
- c.) $\ln(1 + x)$

You should be comfortable with the mechanics of finding these Taylor polynomials, though you may want to use www.wolframalpha.com to check your answers. (You will not need to submit the supporting work for this part of the assignment, but I will expect you to be able to know how to do this for the exams.)

Plotting with MATLAB

Throughout this semester, we will find it helpful to be able to graph functions with MATLAB. The basic idea is quite simple: to obtain a graph of $y = f(x)$, we will collect a sample of points $(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_n, f(x_n))$. It is convenient to choose the values x_1, x_2, \dots, x_n to be linearly spaced on some desired interval of the x -axis.

For example, let’s graph two functions: $f_1(x) = \sin x$ and $f_2(x) = x - x^3/3! + x^5/5!$ over the interval $[-2\pi, 2\pi]$. Figure 1 provides the MATLAB commands; the resulting graph is shown in Figure 2. This graph will appear in a separate “figure” window. To save the plot, select **File**, then **Save As**. Use the “EPS” (Encapsulated PostScript) file format.

At the Computer

Your goal is to produce three graphs: one for each of the three functions above, along with the corresponding Taylor series approximations. Your graphs should be named `plotA`, `plotB`, and `plotC`. For the first two functions, use the x interval $[-2\pi, 2\pi]$ and choose an appropriate y range that gives a “pleasing” pair of graphs. For the third function, use the interval $[-\frac{1}{2}, \frac{3}{2}]$.

When you have obtained all three plots, submit a folder named `hw02` which contains a diary of your work, any `.m` files you used, and the three plots.

[†]These series are named after the English mathematician Brook Taylor, who gets credit for the idea (1715). Taylor series centered about 0 are also known as **Maclaurin series**, after the Scottish mathematician Colin Maclaurin. As an interesting bit of trivia, Colin began his university studies at the age of 11!

```
% Define the Taylor polynomial
f = inline('x - x.^3/factorial(3) + x.^5/factorial(5)');

% Sample 100 points on the interval [-2pi, 2pi]
x = linspace(-2*pi, 2*pi, 100);

% Compute y values for sin x and its Taylor polynomial
y1 = sin(x);
y2 = f(x);

% Plot the two functions with a specified "window"
hold on;
plot(x, y1, 'blue', x, y2, 'red');
axis([-2*pi 2*pi -3 3]);

% Overlay the graphs with a grid
grid on;

% Annotate the graph
title('Taylor polynomial of degree 5 for sine');
xlabel('x');
ylabel('y');
legend('sine', 'Taylor polynomial');
legend('Location', 'Northwest');
```

Figure 1: MATLAB commands which will produce a graph of $\sin x$ and its degree 5 Taylor series approximation. Of particular interest is how functions can be defined with `inline`.

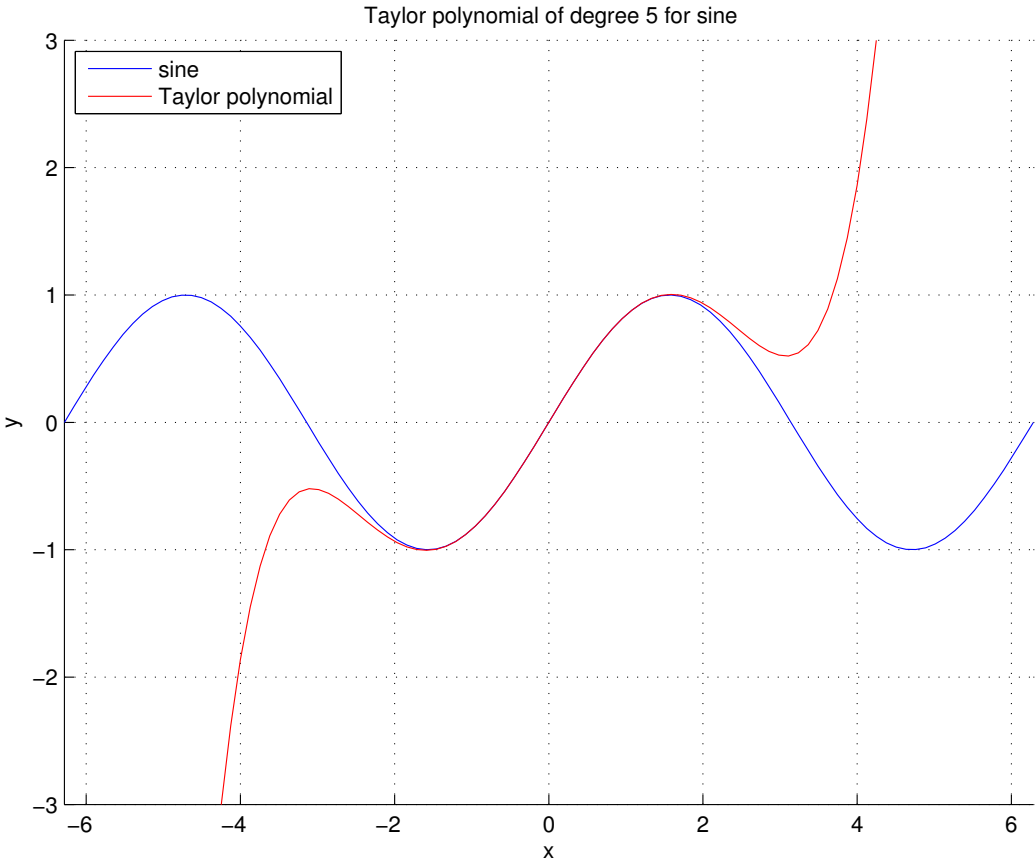


Figure 2: Plot produced from MATLAB.