Exercise 2. Use Theorem 1.6 to determine whether Fixed Point Iteration of $g(x)$ is locally convergent to the given fixed point r.

(a) $g(x) = \dfrac{2x-1}{x^2}, r = 1$

(b) $g(x) = \cos x + \pi + 1, r = \pi$

(c) $g(x) = e^{2x} - 1, r = 0$

Solution.

Exercise 4. Find each fixed point and decide whether Fixed Point Iteration is locally convergent to it.

(a) $g(x) = x^2 - \dfrac{3}{2}x + \dfrac{3}{2}$

(b) $g(x) = x^2 + \dfrac{1}{2}x - \dfrac{1}{2}$

Solution.

Exercise 8. Prove that the method of Example 1.6 will calculate the square root of any positive number.

Solution.

Exercise 9. Explore the idea of Example 1.6 for cube roots. If x is a guess that is smaller than $A^{1/3}$, then A/x^2 will be larger than $A^{1/3}$, so the average of the two will be a better approximation than x. Suggest a Fixed Point Iteration on the basis of this fact, and use Theorem 1.6 to decide whether it will converge to the cube root of A.

Solution.

Computer Problem 4. Calculate the cube roots of the following numbers to eight correct decimal places, by using Fixed Point Iteration with $g(x) = \dfrac{2x + \frac{A}{x^2}}{3}$, where A is:

(a) 2

(b) 3

(c) 5

State your initial guess and the number of steps needed. Explain why a fixed point of g is a cube root of A.

Solution.

Computer Problem 4*. Repeat Computer Problem 4, using the iteration function you considered for Exercise 9.

Solution.