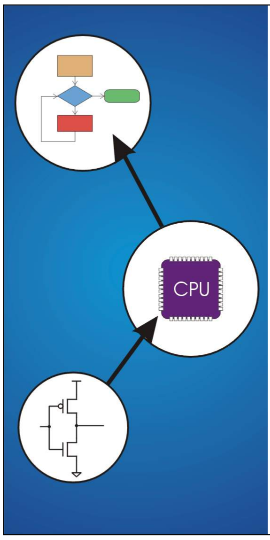


Chapter 3
Digital Logic Structures

Transistor: Building Block of Computers

Microprocessors contain millions of transistors
- Intel Pentium 4 (2000): **48 million**
- IBM PowerPC 750FX (2002): **38 million**
- IBM/Apple PowerPC G5 (2003): **58 million**

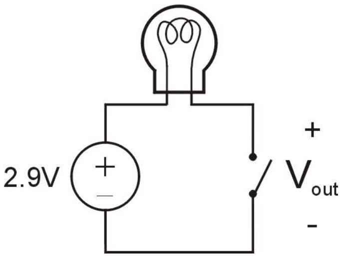
Logically, each transistor acts as a switch

Combined to implement logic functions
- **AND, OR, NOT**

Combined to build higher-level structures
- **Adder, multiplexer, decoder, register, …**

Combined to build processor
- **LC-3**

Simple Switch Circuit



Switch open:
- **No current through circuit**
- **Light is off**
- **V_{out} is +2.9V**

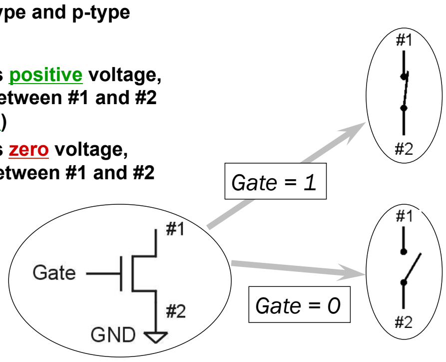
Switch closed:
- **Short circuit across switch**
- **Current flows**
- **Light is on**
- **V_{out} is 0V**

Switch-based circuits can easily represent two states: on/off, open/closed, voltage/no voltage.

n-type MOS Transistor

MOS = Metal Oxide Semiconductor
- **two types: n-type and p-type**

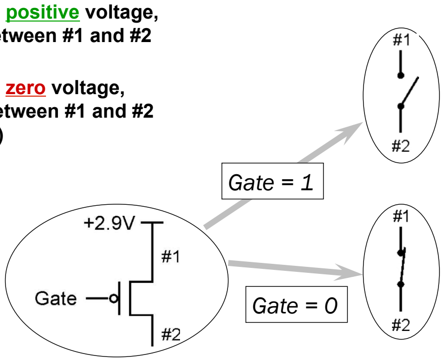
n-type
- **when Gate has positive voltage, short circuit between #1 and #2 (switch closed)**
- **when Gate has zero voltage, open circuit between #1 and #2 (switch open)**



Terminal #2 must be connected to GND (0V).

p-type MOS Transistor

p-type is *complementary* to n-type
- **when Gate has positive voltage, open circuit between #1 and #2 (switch open)**
- **when Gate has zero voltage, short circuit between #1 and #2 (switch closed)**

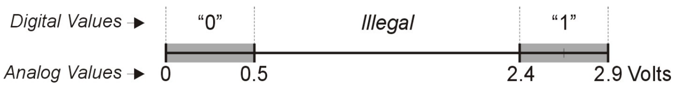


Terminal #1 must be connected to +2.9V.

Logic Gates

Use switch behavior of MOS transistors to implement logical functions: AND, OR, NOT.

Digital symbols:
- **recall that we assign a range of analog voltages to each digital (logic) symbol**



- **assignment of voltage ranges depends on electrical properties of transistors being used**
  - **typical values for "1": +5V, +3.3V, +2.9V**
  - **from now on we'll use +2.9V**

CMOS Circuit

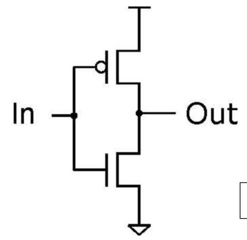
Complementary MOS

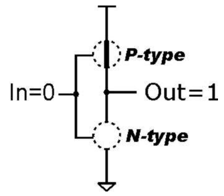
Uses both n-type and p-type MOS transistors

- **p-type**
  - ➢ **Attached to + voltage**
  - ➢ **Pulls output voltage UP when input is zero**
- **n-type**
  - ➢ **Attached to GND**
  - ➢ **Pulls output voltage DOWN when input is one**

For all inputs, make sure that output is either connected to GND or to +, but not both!

Inverter (NOT Gate)

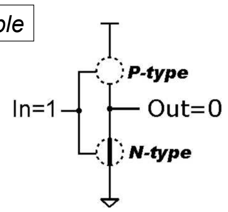




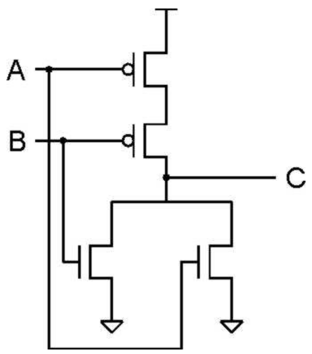
Truth table

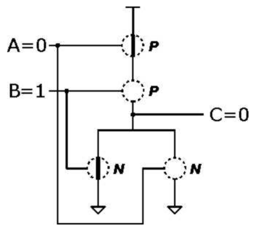
In	Out
0 V	2.9 V
2.9 V	0 V

In	Out
0	1
1	0



NOR Gate

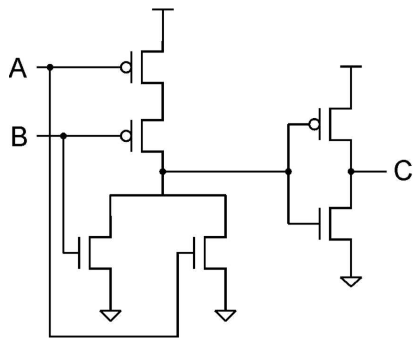




A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Note: Serial structure on top, parallel on bottom.

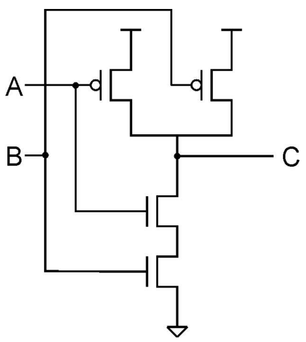
OR Gate

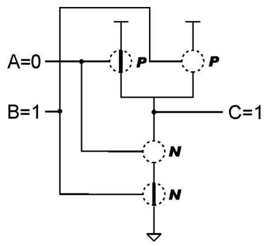


A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

Add inverter to NOR.

NAND Gate (AND-NOT)

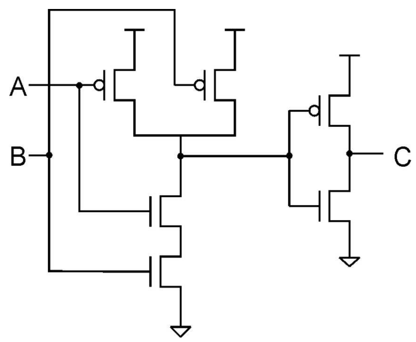




A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

Note: Parallel structure on top, serial on bottom.

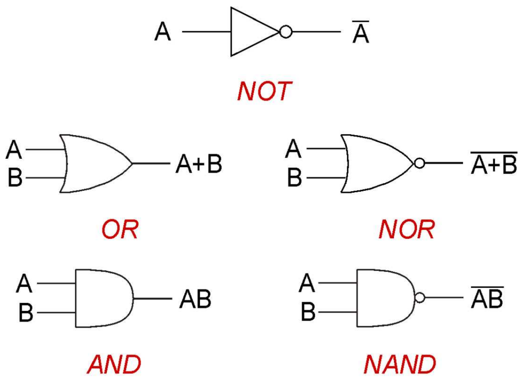
AND Gate



A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Add inverter to NAND.

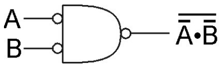
Basic Logic Gates



DeMorgan's Law

Converting AND to OR (with some help from NOT)

Consider the following gate:



A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

Same as A+B!

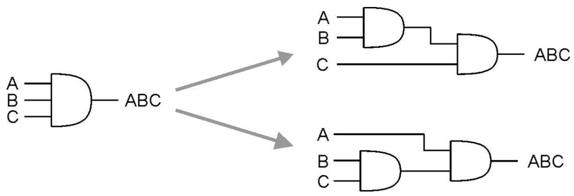
To convert AND to OR (or vice versa), invert inputs and output.

More than 2 Inputs?

AND/OR can take any number of inputs.

- **AND = 1 if all inputs are 1.**
- **OR = 1 if any input is 1.**
- **Similar for NAND/NOR.**

Can implement with multiple two-input gates, or with single CMOS circuit.



Summary

MOS transistors are used as switches to implement logic functions.

- **n-type: connect to GND, turn on (with 1) to pull down to 0**
- **p-type: connect to +2.9V, turn on (with 0) to pull up to 1**

Basic gates: NOT, NOR, NAND

- **Logic functions are usually expressed with AND, OR, and NOT**

DeMorgan's Law

- **Convert AND to OR (and vice versa) by inverting inputs and output**

Building Functions from Logic Gates

Combinational Logic Circuit

- **output depends only on the current inputs**
- **stateless**

Sequential Logic Circuit

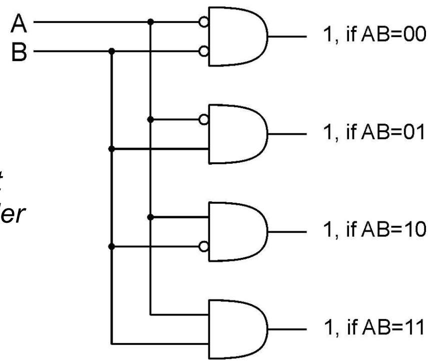
- **output depends on the sequence of inputs (past and present)**
- **stores information (state) from past inputs**

We'll first look at some useful combinational circuits, then show how to use sequential circuits to store information.

Decoder

n **inputs, 2^n outputs**

- **exactly one output is 1 for each possible input pattern**

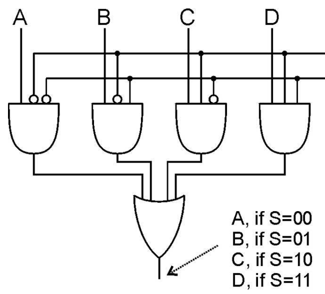


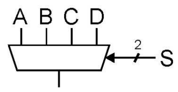
2-bit decoder

Multiplexer (MUX)

***n*-bit selector and *2ⁿ* inputs, one output**

- **output equals one of the inputs, depending on selector**

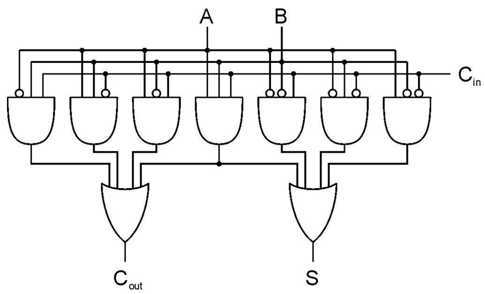




4-to-1 MUX

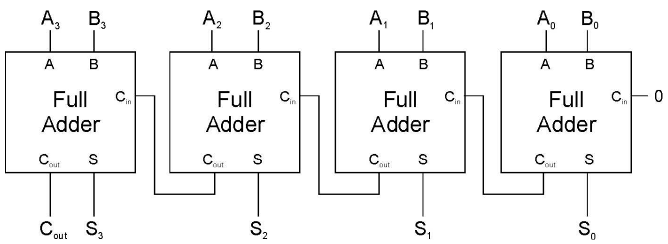
Full Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.



A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

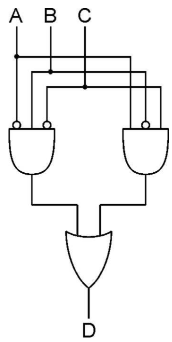
Four-bit Adder



Logical Completeness

Can implement ANY truth table with AND, OR, NOT.

A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



1. AND combinations that yield a "1" in the truth table.
2. OR the results of the AND gates.

Combinational vs. Sequential

Combinational Circuit

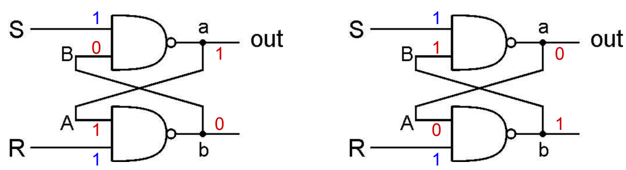
- **always gives the same output for a given set of inputs**
  - **ex: adder always generates sum and carry, regardless of previous inputs**

Sequential Circuit

- **stores information**
- **output depends on stored information (state) plus input**
  - **so a given input might produce different outputs, depending on the stored information**
- ***example:* ticket counter**
  - **advances when you push the button**
  - **output depends on previous state**
- **useful for building "memory" elements and "state machines"**

R-S Latch: Simple Storage Element

R is used to "reset" or "clear" the element – set it to zero.
S is used to "set" the element – set it to one.

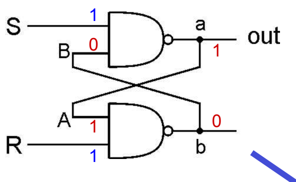


If both R and S are one, out could be either zero or one.

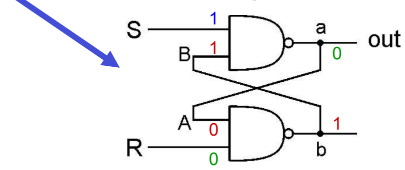
- **"quiescent" state -- holds its previous value**
- **note: if a is 1, b is 0, and vice versa**

Clearing the R-S latch

Suppose we start with output = 1, then change R to zero.



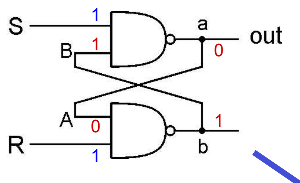
Output changes to zero.



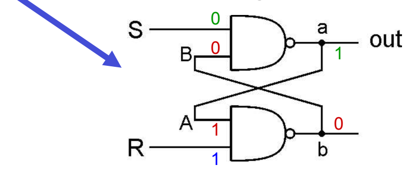
Then set R=1 to "store" value in quiescent state.

Setting the R-S Latch

Suppose we start with output = 0, then change S to zero.



Output changes to one.



Then set S=1 to "store" value in quiescent state.

R-S Latch Summary

R = S = 1
- **hold current value in latch**

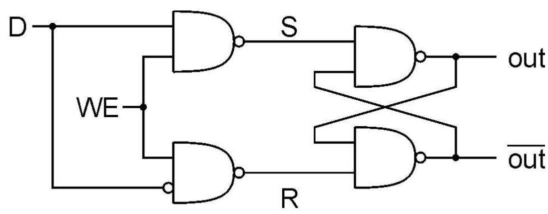
S = 0, R=1
- **set value to 1**

R = 0, S = 1
- **set value to 0**

R = S = 0
- **both outputs equal one**
- **final state determined by electrical properties of gates**
- *Don't do it!*

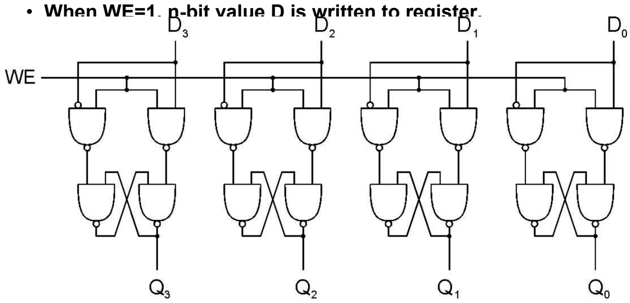
Gated D-Latch

Two inputs: D (data) and WE (write enable)
- **when WE = 1, latch is set to value of D**
  - **S = NOT(D), R = D**
- **when WE = 0, latch holds previous value**
  - **S = R = 1**



Register

A register stores a multi-bit value.
- **We use a collection of D-latches, all controlled by a common WE.**
- **When WE=1, n-bit value D is written to register.**



Representing Multi-bit Values

Number bits from right (0) to left (n-1)
- **just a convention -- could be left to right, but must be *consistent***

Use brackets to denote range:
D[l:r] **denotes bit** l **to bit** r**, from *left* to *right***

15 0
A = **0101001101010101**

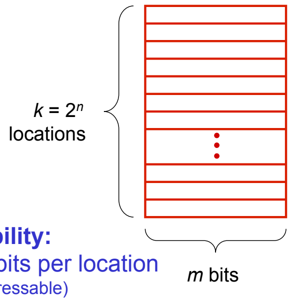
A[14:9] = **101001**   A[2:0] = **101**

May also see A<14:9>**,**
especially in hardware block diagrams.

Memory

Now that we know how to store bits, we can build a memory – a logical *k* × *m* array of stored bits.

Address Space:
number of locations
(usually a power of 2)



Addressability:
number of bits per location
(e.g., byte-addressable)

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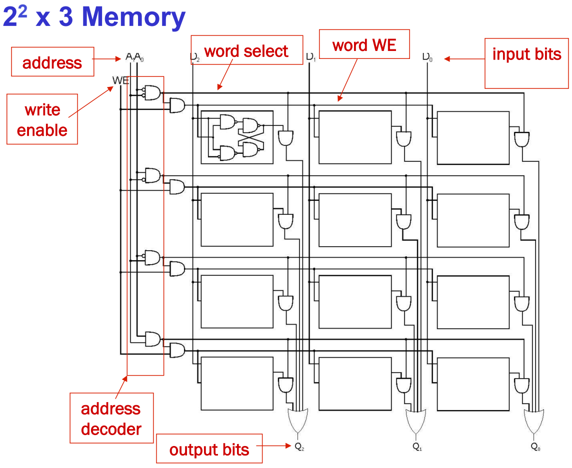
2^2 x 3 Memory



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More Memory Details

This is a not the way actual memory is implemented.
- **fewer transistors, much more dense, relies on electrical properties**

But the logical structure is very similar.
- **address decoder**
- **word select line**
- **word write enable**

Two basic kinds of RAM (Random Access Memory)

Static RAM (SRAM)
- **fast, maintains data as long as power applied**

Dynamic RAM (DRAM)
- **slower but denser, bit storage decays – must be periodically refreshed**

Also, non-volatile memories: ROM, PROM, flash, …

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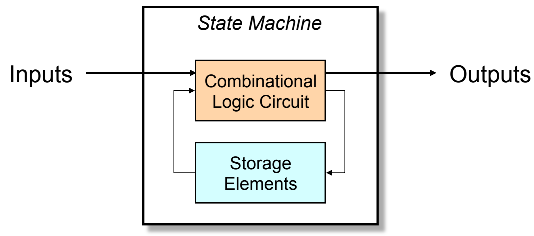
State Machine

Another type of sequential circuit
- **Combines combinational logic with storage**
- **"Remembers" state, and changes output (and state) based on inputs and current state**



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Combinational vs. Sequential

Two types of "combination" locks

Combinational
Success depends only on the values, not the order in which they are set.

Sequential
Success depends on the sequence of values (e.g, R-13, L-22, R-3).

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State

The state of a system is a snapshot of all the relevant elements of the system at the moment the snapshot is taken.

Examples:
- **The state of a basketball game can be represented by the scoreboard.**
  - **Number of points, time remaining, possession, etc.**
- **The state of a tic-tac-toe game can be represented by the placement of X's and O's on the board.**

State of Sequential Lock

Our lock example has four different states, labelled A-D:

A: The lock is not open, and no relevant operations have been performed.

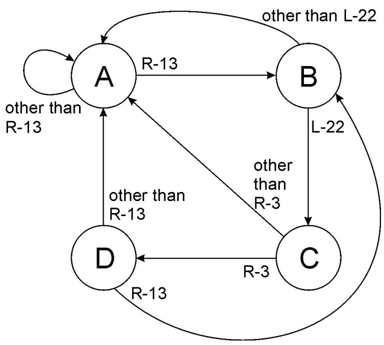
B: The lock is not open, and the user has completed the R-13 operation.

C: The lock is not open, and the user has completed R-13, followed by L-22.

D: The lock is open.

State Diagram

Shows states and actions that cause a transition between states.



Finite State Machine

A description of a system with the following components:

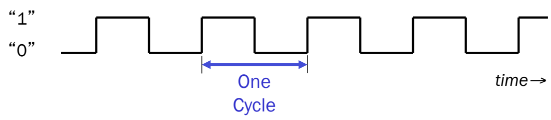
1. **A finite number of states**
2. **A finite number of external inputs**
3. **A finite number of external outputs**
4. **An explicit specification of all state transitions**
5. **An explicit specification of what determines each external output value**

Often described by a state diagram.

- **Inputs trigger state transitions.**
- **Outputs are associated with each state (or with each transition).**

The Clock

Frequently, a clock circuit triggers transition from one state to the next.



At the beginning of each clock cycle, state machine makes a transition, based on the current state and the external inputs.

- Not always required. In lock example, the input itself triggers a transition.

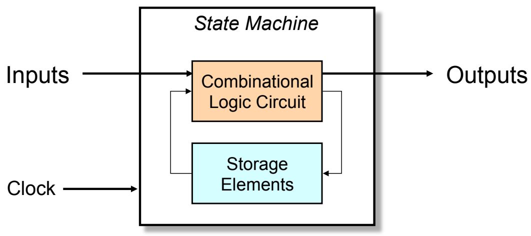
Implementing a Finite State Machine

Combinational logic

- **Determine outputs and next state.**

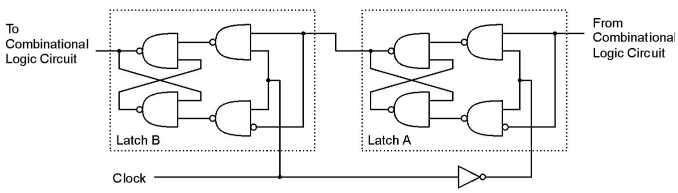
Storage elements

- **Maintain state representation.**



Storage: Master-Slave Flipflop

A pair of gated D-latches, to isolate *next* state from *current* state.



During 1st phase (clock=1), previously-computed state becomes *current* state and is sent to the logic circuit.	During 2nd phase (clock=0), *next* state, computed by logic circuit, is stored in Latch A.

Storage

Each master-slave flipflop stores one state bit.

The number of storage elements (flipflops) needed is determined by the number of states (and the representation of each state).

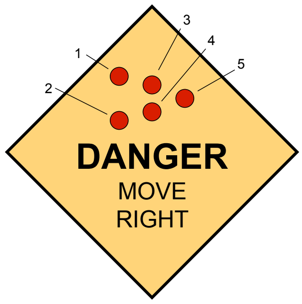
Examples:

- **Sequential lock**
  - **Four states – two bits**
- **Basketball scoreboard**
  - **7 bits for each score, 5 bits for minutes, 6 bits for seconds, 1 bit for possession arrow, 1 bit for half, …**

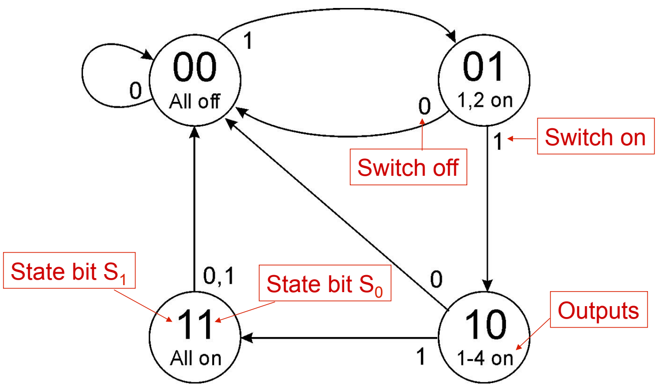
Complete Example

A blinking traffic sign

- **No lights on**
- **1 & 2 on**
- **1, 2, 3, & 4 on**
- **1, 2, 3, 4, & 5 on**
- **(repeat as long as switch is turned on)**



Traffic Sign State Diagram



Transition on each clock cycle.

Traffic Sign Truth Tables

Outputs
(depend only on state: S_1S_0)

Z: Lights 1 and 2; Y: Lights 3 and 4; X: Light 5

S_1	S_0	Z	Y	X
0	0	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1

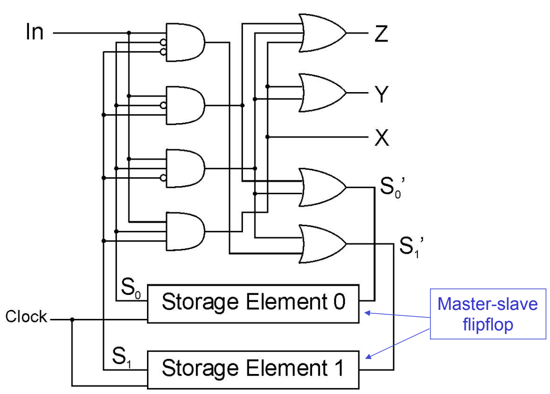
Next State: $S_1' S_0'$
(depend on state and input)

In: Switch

In	S_1	S_0	S_1'	S_0'
0	X	X	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

Whenever In=0, next state is 00.

Traffic Sign Logic



From Logic to Data Path

The data path of a computer is all the logic used to process information.

- **See the data path of the LC-3 on next slide.**

Combinational Logic

- **Decoders -- convert instructions into control signals**
- **Multiplexers -- select inputs and outputs**
- **ALU (Arithmetic and Logic Unit) -- operations on data**

Sequential Logic

- **State machine -- coordinate control signals and data movement**
- **Registers and latches -- storage elements**

LC-3 Data Path

Combinational Logic

Storage

State Machine