**MAT 3770—Exam 1 Topics**
Tucker Sections 1.1–2.2, inclusive
with corresponding sections from Rosen

The following is meant to be a guide, not an exhaustive nor all–inclusive outline (CMA). Review the handouts and homework assignments for more details. You also need to know vocabulary and induction proofs.

- 1.1 **Graph Models**:
  graph, vertex, edge, adjacent, incident, directed, distinct, path, circuit, connected, bipartite, matching (maximum—largest cardinality), edge cover, independent set (maximum—largest cardinality, maximal—cannot add another vertex)
  **Theorem**. $S$ is an independent set IFF $V - S$ is an edge cover.

- 1.2 **Isomorphism**:
  isomorphism, degree, subgraph, complete graph ($K_n$), complement of a graph, in–degree, out–degree, determining whether two graphs are (show a 1-1 onto mapping between vertices) or are not (show contradiction) isomorphic
  **Theorem**. $G_1$ is isomorphic to $G_2$ IFF their complements are isomorphic.

- 1.3 **Edge Counting**
  range graphs, length of circuits / paths, bipartite graphs
  **Theorem**. Sum of the degrees of all vertices is twice the number of edges.
  **Corollary**. In any graph, the number of vertices of odd degree is even.
  **Theorem**. $G$ is bipartite IFF every circuit in $G$ has even length.

- 1.4 **Planar Graphs**
  planar, planar graph, map coloring, circle–chord method for drawing planar graphs, determining planarity or non–planarity, $K_{3,3}$ configuration, $K_5$ configuration
  **Theorem**. A graph is planar IFF it does not contain a subgraph that is a $K_5$ or $K_{3,3}$ configuration.
  **Theorem**. (Euler's Formula) $r = e - v + 2$ regions if $G$ is connected and planar.
  **Corollary**. $e \leq 3v - 6$ if $e > 1$ and $G$ connected and planar.

- Chap. 1 **Supplement**
  representing graphs in computer programs: adjacency matrix, adjacency list

- 2.1 **Euler Cycles**
  cycle, Euler cycle, multigraphs, trail, Euler trail
  **Theorem**. An undirected multigraph has an Euler cycle IFF it is connected and all vertices have even degree.
  **Corollary**. A multigraph has an Euler trail, but not an Euler cycle, IFF it is connected and has exactly two vertices of odd degree.

*Continued on reverse.*

- 2.2 **Hamilton Circuits**

  Hamilton circuits and paths; harder to determine existence than Euler circuits — can produce or show construction must fail using three rules:

  1. If a vertex $x$ has degree 2, both edges incident to $x$ must be part of any HC.
  2. No proper sub-circuit (circuit not containing all vertices) can be formed when constructing a HC.
  3. Once the HC is forced to use 2 edges at a vertex $x$, all other (unused and unusable) edges incident to $x$ can be discarded.

  **Theorem**. Connected graph $G$ with $n > 2$ vertices has a HC if the degree of each vertex is at least $\frac{n}{2}$.

  **Theorem**. Connected graph $G$ with $n$ vertices, $x_1, x_2, \ldots, x_n$, ordered by degree from smallest to largest, has a HC if for each $k \leq \frac{n}{2}$, either $\deg(x_k) > k$ or $\deg(x_{n-k}) \; \geq \; n - k$

  **Theorem**. Planar graph $G$ has a HC if $\sum_i (i-2)(r_i - r'_i) \; = \; 0$
  Tournaments, $K - n$ with arbitrary directed edges

  **Theorem**. Every tournament has a directed Hamilton Path.
  Gray Codes