MAT 3770—Exam 1 Topics
Tucker Sections 1.1–2.4, inclusive
with corresponding sections from Rosen

The following is meant to be a guide, not an exhaustive nor all–inclusive outline (CMA). Review the handouts and homework assignments for more details. You also need to know vocabulary and induction proofs.

- 1.1 **Graph Models**:
  graph, vertex, edge, adjacent, incident, directed, distinct, path, circuit, connected, bipartite, matching (maximum—largest cardinality), edge cover, independent set (maximum—largest cardinality, maximal—cannot add another vertex)
  Theorem. S is an independent set IFF $V - S$ is an edge cover.

- 1.2 **Isomorphism**:
  isomorphism, degree, subgraph, complete graph (K_n), complement of a graph, in–degree, out–degree, determining whether two graphs are (show a 1-1 onto mapping between vertices) or are not (show contradiction) isomorphic
  Theorem. G_1 is isomorphic to G_2 IFF their complements are isomorphic.

- 1.3 **Edge Counting**
  range graphs, length of circuits / paths, bipartite graphs
  Theorem. Sum of the degrees of all vertices is twice the number of edges.
  Corollary. In any graph, the number of vertices of odd degree is even.
  Theorem. G is bipartite IFF every circuit in G has even length.

- 1.4 **Planar Graphs**
  planar, planar graph, map coloring, circle–chord method for drawing planar graphs, determining planarity or non–planarity, $K_{3,3}$ configuration, K_5 configuration
  Theorem. A graph is planar IFF it does not contain a subgraph that is a K_5 or $K_{3,3}$ configuration.
  Theorem. (Euler's Formula) $r = e - v + 2$ regions if G is connected and planar.
  Corollary. $e \leq 3v - 6$ if $e > 1$ and G connected and planar.

- Chap. 1 **Supplement**
  representing graphs in computer programs: adjacency matrix, adjacency list

- 2.1 **Euler Cycles**
  cycle, Euler cycle, multigraphs, trail, Euler trail
  Theorem. An undirected multigraph has an Euler cycle IFF it is connected and all vertices have even degree.
  Corollary. A multigraph has an Euler trail, but not an Euler cycle, IFF it is connected and has exactly two vertices of odd degree.

Continued on reverse.

- 2.2 **Hamilton Circuits**
  Hamilton circuits and paths; harder to determine existence than Euler circuits — can produce or show construction must fail using three rules:

  1. If a vertex x has degree 2, both edges incident to x must be part of any HC.
  2. No proper sub-circuit (circuit not containing all vertices) can be formed when constructing a HC.
  3. Once the HC is forced to use 2 edges at a vertex x, all other (unused and unusable) edges incident to x can be discarded.

  Theorem. Connected graph G with $n > 2$ vertices has a HC if the degree of each vertex is at least $\frac{n}{2}$.

  Theorem. Connected graph G with n vertices, $x_1, x_2, \ldots, x_n$, ordered by degree from smallest to largest, has a HC if for each $k \leq \frac{n}{2}$, either $\deg(x_k) > k$ or $\deg(x_{n-k}) \;\geq\; n-k$

  Theorem. Planar graph G has a HC if $\sum_i (i-2)(r_i - r'_i) \;=\; 0$
  Tournaments, $K - n$ with arbitrary directed edges

  Theorem. Every tournament has a directed Hamilton Path.
  Gray Codes

- 2.3 **Graph Coloring**:
  coloring, chromatic number (χ), chromatic polygon, wheel graphs

- 2.4 **Coloring Theorems**:
  triangulation of a polygon, edge chromatic number

  Theorem.1. The vertices in a triangulation of a polygon can be 3–colored.

  Corollary. The Art Gallery Problem with n walls requires at most $\lfloor \frac{n}{3} \rfloor$ guards.

  Theorem. 2. (Brook's) If the graph G is not an odd circuit or a complete graph, then χ(G) $\leq d$, where d is the maximum degree of a vertex of G.

  Theorem. 3. For any positive integer k, there exists a triangle–free graph G with $\chi(\text{G}) = k$.

  Theorem. 4. (Vizing's) If the maximum degree of a vertex in a graph G is d, then the edge chromatic number of G is either d or $d + 1$.

  Theorem. 5. Every planar graph can be 5–colored.