Mat 3770: Combinatorial Computing

Week 1

Spring 2014

Evaluation

In this course there will be:

- Weekly quizzes
- Daily homework
- Three class exams, and
- A comprehensive final exam

The relative weights of these components are:

Exam I	15 %
Exams II & III	20 % each
Homework & Quizzes	10 %
Final (Comprehensive)	35 %

Academic Integrity

- The Office of Student Standards provides guidelines for expectations of all EIU students.
- Violations include **cheating** on examinations, **collusion**, and conduct which **disrupts** the academic environment.
- All work turned in, including homework submissions, is to be your own work. Trim off any rough edges, and staple.
- Never copy from another student, nor allow other students to copy your solutions.

Miscellaneous Notes

- Attendance is required at all class and exam sessions.
- Make-up exams are available only if agreed upon **before** the regular exam is given.
- Homework
  - Homework is due the class meeting following the last lecture for a particular section.
  - Every homework is to be turned in on a separate sheet(s) of paper and labeled by book (Tucker or Rosen) and section number.
  - Homework will be credited for completion and thoroughness.

- Your **cell phone and all other electronic devices are to be turned off**, **put away**, and **kept out of sight** during lectures.
- Please ask questions when you experience problems. Ask in class or see me outside of the regularly scheduled meeting times. Feel free to use email as well.

Week 1

- Syllabus and General Course Guidelines
- Schedule (Tentative, but note exam dates)
- Homework
- Chapter 1. Elements of Graph Theory

Week 1 — Student Responsibilities

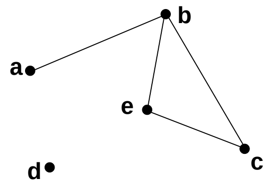
- Hwk from **Rosen**
  Sec 9.1: 1, 13ab
  Sec 9.2: 27

- Hwk from **Tucker**
  Sec. 1.1: 4, 7, 14 (all **maximum** matchings),
  20, 24, 25, 38, 39

- Attendance **Sunnily** Encouraged

Graph Models

- A **graph**, $G = (V, E)$, is a set of vertices or points (V) and the set of edges (E) connecting them. In this example,
  $V = \{a, b, c, d, e\}$, and $E = \{ab, bc, be, ce\}$.

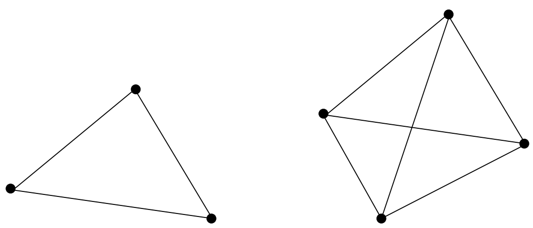


- The **degree** of a vertex is the number of **incident** edges. **What is the degree of each vertex in the figure above?**
- Two vertices are **adjacent** if there is an edge connecting them. **Which pairs of vertices are adjacent in this figure?**

Complete Graphs

- If all possible edges are in E (i.e., every pair of vertices is connected), then G is called a **complete** graph.

- A complete graph over n vertices is called K_n.

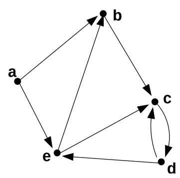


These are examples of the complete graphs K_3 and K_4.

Directed Graphs

- If order of endpoints is important, then the edges of a graph are said to be **directed** edges.

- A **directed** graph is one in which all edges are directed. (Also known as a *digraph*.)



- Vertex **indegree**: the number of directed edges coming in.
- Vertex **outdegree**: the number of directed edges going out.

Paths and Circuits

- A **path** P from vertex a to vertex b is a sequence of **distinct** vertices:
  $$P = \{a = x_1, x_2, \ldots, x_n = b\}$$
  such that consecutive vertices are adjacent
  (i.e., $< x_i, x_{i+1} > \in E$).

- The number of vertices in the path P is n;
  then length of P (the number of edges in the path) is $n - 1$.

- A **circuit** is a path than ends where it starts (i.e., $x_n = x_1$).
  (This single repetition is allowed, otherwise vertices are distinct.)

Connected Components

- An undirected graph is **connected** if every pair of vertices is connected by a path.

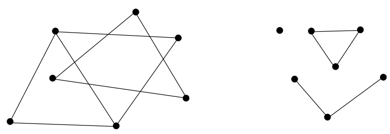


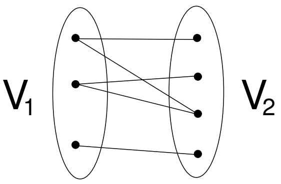
fig a. **fig b.**

- The **connected components** of a graph are the equivalence classes of vertices under the *"is reachable from"* relation.
  How many components are in each figure above?

- A graph is connected if it has exactly one connected component.

Bipartite Graphs

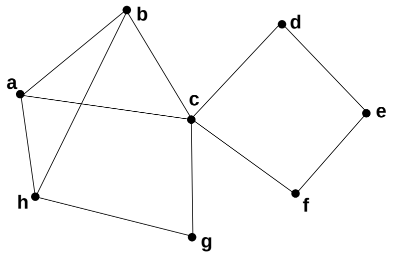
- A **bipartite** graph is an undirected graph $G = \{V, E\}$ in which V can be **partitioned** into two sets, V_1 and V_2 such that $< u, w > \in E$ implies either $u \in V_1$ and $w \in V_2$, or $u \in V_2$ and $w \in V_1$.



- Note: the result is that all edges have endpoints in both V_1 and V_2.

Matching

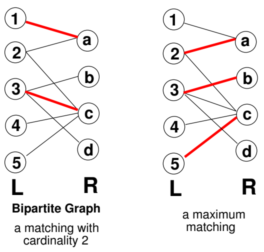
- Given an undirected graph, $G = (V, E)$, a **matching** is a subset of edges $M \subseteq E \ni \forall$ vertices $v \in V$, **at most** one edge of M is incident on v (i.e., no two edges in M share an endpoint.)



- A vertex $v \in V$ is **matched** by the matching M if some edge in M is incident on v; otherwise, v is **unmatched**.

Maximum Matching

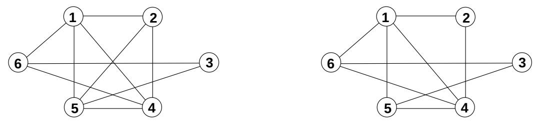
- A **maximum** matching is a matching of maximum cardinality, that is, a matching $M \ni \forall$ matchings $M', |M| \geq |M'|$.



- Suppose L is a set of machines with a set R of tasks to be performed simultaneously. We define an edge $< u, w > \in E$ to mean a particular machine $u \in L$ is capable of performing a particular task $w \in R$.

Edge Cover

- An **edge cover** is a set C of vertices in a graph G with the property that every edge of G is incident to at least one vertex in C (i.e., C contains at least one endpoint of every edge).

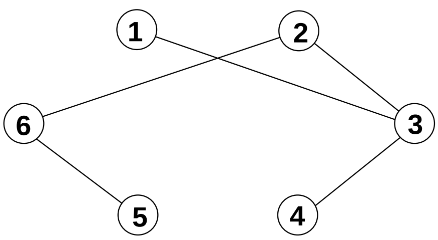


- An **edge cover** of an undirected graph $G = (V, E)$ can also be defined as the subset $V' \subseteq V, \ni$ if $< u, w > \in E$, then $u \in V'$ or $w \in V'$, or both.

  In other words, each vertex "covers" its incident edges, and an edge cover for G is a set of vertices that covers all the edges in E.

Minimum Edge cover

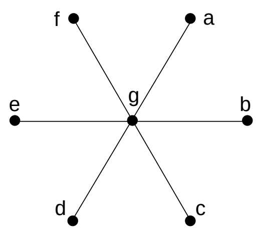
- The **size of a cover** is the number of vertices in it.



$C_1 = \{6, 3\}$ (minimum) $C_2 = \{1, 2, 4, 5\}$

Independent Sets

- An **independent set** of a graph $G = (V, E)$ is a subset $V' \subseteq V \ni$ each edge in E is incident on at most one vertex in V' (i.e., a set of vertices without an edge between any two of them).

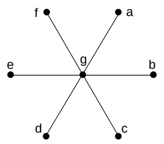


Maximal vs Maximum Independent Sets

- A **maximal** independent set is a an independent set $V' \ni \forall\, u \in V - V'$, the set $V' \cup \{u\}$ is not independent (i.e., every vertex not in V' is adjacent to some vertex in V'. (An easy problem: can't add any more vertices and remain independent.)

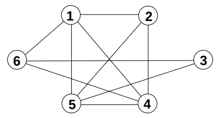
- A **maximum** independent set is an independent set of largest size in a general graph — a very different and much harder problem!.

`maximal:{g}    maximum: {a, b, c, d, e, f}`



Independent Set and Edge Cover Connection

- **Theorem**. Let graph $G = (V, E)$, then S will be an independent set of vertices **IFF** $V - S$ is an edge cover.
  - ($\Rightarrow$) If there are no edges between two vertices in S, then every edge involves (at least) one vertex not in S, i.e., that is, in $V - S$.
  - ($\Leftarrow$) If C is an edge cover (and thus all edges have at least one end vertex in C), then there is no edge joining two vertices in $V - C$. Thus, $V - C$ is an independent set.



`S = {1,3}  V-S = {2,4,5,6}`

Try these: `R = {2,3}  T = {5,6}`