Regular expressions: example 1

Expression:

$$01$$

Language:

$$\{01\}$$

Regular expressions: example 2

Expression:

$$01 \cup 10 \cup 010$$

Language:

$$\{01, 10, 010\}$$

Any finite set can be described by a (boring) regular expression

Regular expressions: example 3

Expression:

$$01^*$$

Language:

$$\{0, 01, 011, 0111, \dots\} = \{01^n : n \geq 0\}$$

Expression:

$$01^+$$

Language:

$$\{01, 011, 0111, \dots\} = \{01^n : n \geq 1\}$$

Like arithmetic expressions, operators have precedence
Note: 01^* vs. $0(1^*)$ vs. $(0)1^*$ vs. $(01)^*$

Regular expressions: example 4

Expression:

$$(0 \cup 1)^*$$

Language:

$$\{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, \dots\}$$

To create a string:

Choose an exponent n, then make n **independent** *choices*

Regular expressions: example 5

Expression:

$$(0 \cup 1)^* 110$$

Language:

$$\{w \in \{0,1\}^* : w \textit{ has suffix } 110\}$$

Expression:

$$(0 \cup 1)^* 110 (0 \cup 1)^*$$

Language:

$$\{w \in \{0,1\}^* : w \textit{ has substring } 110\}$$

Subdivide the regular expression; think about each piece separately

Regular expressions: example 6

Expression:

$$(0^*1^*)^*$$

Language:

$$\{0,1\}^*$$

Why is this true?

Regular expressions: example 7

Expression:

$$(0 \cup 1)^*0(0 \cup 1)^*0(0 \cup 1)^*$$

Language:

$$\{w \in \{0,1\}^* : w \textit{ has at least two } 0\textit{'s}\}$$

Subdivide the regular expression; think about each piece separately

Regular expressions: example 8

Expression:

$$0^*1^*$$

Language:

$$\{\epsilon, 0, 1, 00, 01, 11, 000, 001, \dots\} = \{0^i 1^j : i, j \geq 0\}$$

Each * is **independent** from the other

Regular expressions: example 9

Expression:

$$((0 \cup 1)(0 \cup 1))^*$$

Language:

$$\{w \in \{0,1\}^* : |w| \textit{ is even}\}$$

To create a string:

- Choose an exponent $n \geq 0$
- For each iteration, write one bit, then another bit

Regular expressions: example 10

Expression:

$$(0 \cup 1)^* 0 (0 \cup 1)(0 \cup 1)$$

Language:

$$\{w \in \{0,1\}^* : w \textit{ has a } 0 \textit{ as the third bit from the end}\}$$

Regular expressions: example 11

Expression:

$$(0 \cup 1)^* \varnothing (0 \cup 1)^*$$

Language:

$$\{\}$$

Expression:

$$(\varnothing)^*$$

Language:

$$\{\epsilon\}$$

Is $\varnothing$ useful?
What is the point of allowing it as a regular expression?

How about these?

(a) 0^*10^*

(b) $(0 \cup 1)^*1(0 \cup 1)^*$

(c) $(0 \cup 1)^*001(0 \cup 1)^*$

(d) $((0 \cup 1)(0 \cup 1)(0 \cup 1))^*$

(e) $(0 \cup \epsilon)1^*$

(f) $1^*\varnothing$

Designing regular expressions

(a) Strings with an odd number of 1's

(b) Strings with exactly two 0's

(c) Strings that do not end with 01