Exercise 1. Consider the language $L = \{1^n 2^n : n > 0\}$. Is the string 122 in L? Justify your answer.

Solution.

Exercise 2. Let $L_1 = \{a^n b^n : n > 0\}$. Let $L_2 = \{c^n : n > 0\}$. For each of the following strings, state whether or not it is an element of $L_1 L_2$. Justify your answer.

a) ϵ

b) $aabbcc$

c) $abbcc$

d) $aabbcccc$

Solution.

Exercise 4. Let $L = \{w \in \{a, b\}^* : |w| \equiv_3 0\}$. (In words, the length of w is congruent to zero, modulo three.) List the first six elements in a lexicographic enumeration of L.

Solution.

Exercise 5. Consider the language L of all strings drawn from the alphabet $\{a, b\}$ with at least two different substrings of length 2.

b) List the first six elements of a lexicographic enumeration of L.

Solution.

Exercise 6. For each of the following languages L, give a simple English description. Show two strings that are in L and two that are not (unless there are fewer than two strings in L or two not in L, in which case show as many as possible.)

a) $L = \{w \in \{a, b\}^* :$ exactly one prefix of w ends in $a\}$.

b) $L = \{w \in \{a, b\}^* :$ all prefixes of w end in $a\}$.

c) $L = \{w \in \{a, b\}^* : \exists x \in \{a, b\}^+$ such that $w = axa\}$.

Solution.

Exercise 8. For each of the following statements, state whether it is true or false. Prove your answer. *Hint:* To prove two sets X and Y are equal, show the two subset inclusions $X \subseteq Y$ and $Y \subseteq X$. To show set inequality, show that there is some element $x \in X$ with $x \notin Y$ or some $y \in Y$ with $y \notin X$. (Choose specific languages which allow you to find such an element.)

a) $\forall L_1, L_2\, (L_1 = L_2 \text{ iff } L_1^* = L_2^*)$

c) Every infinite language is the complement of a finite language

d) $\forall L\, ((L^R)^R = L)$

h) $\forall L_1, L_2, L_3\, ((L_1 \cup L_2)L_3 = L_1 L_3 \cup L_2 L_3)$

i) $\forall L_1, L_2, L_3\, ((L_1 L_2) \cup L_3 = (L_1 \cup L_3)(L_2 \cup L_3))$

k) $\forall L\, (\varnothing L^* = \{\epsilon\})$

Solution.