

Exercise 2. Write a regular expression to describe each of the following languages. Provide reasons for your answers.

- b) $\{w \in \{a, b\}^* : w \text{ does not end in } ba\}$.
- d) $\{w \in \{0, 1\}^* : w \text{ corresponds to the binary encoding, without leading 0s, of natural numbers evenly divisible by 4}\}$.
- e) $\{w \in \{0, 1\}^* : w \text{ corresponds to the binary encoding, without leading 0s, of natural numbers that are powers of 4}\}$.
- f) $\{w \in \{0, 1, 2, \dots, 9\}^* : w \text{ corresponds to the decimal encoding, without leading 0s, of an odd natural number}\}$.
- g) $\{w \in \{0, 1\}^* : w \text{ has } 001 \text{ as a substring}\}$.
- h) $\{w \in \{0, 1\}^* : w \text{ does not have } 001 \text{ as a substring}\}$.
- k) $\{w \in \{a, b\}^* : w \text{ has both } aa \text{ and } aba \text{ as substrings}\}$.
- p) $\{w \in \{a, b\}^* : \#_a(w) \leq 3\}$.

Solution.

Exercise 20. From among the following proposed identities, find two which are true and two which are false. Give reasons for your answers.

- (a) $(ab)^*a = a(ba)^*$.
- (b) $(a \cup b)^*b(a \cup b)^* = a^*b(a \cup b)^*$.
- (c) $(a \cup b)^*b(a \cup b)^* \cup (a \cup b)^*a(a \cup b)^* = (a \cup b)^*$.
- (d) $(a \cup b)^*b(a \cup b)^* \cup (a \cup b)^*a(a \cup b)^* = (a \cup b)^+$.
- (e) $(a \cup b)^*ba(a \cup b)^* \cup a^*b^* = (a \cup b)^*$.
- (f) $a^*b(a \cup b)^* = (a \cup b)^*b(a \cup b)^*$.
- (g) If α and β are any two regular expressions, then $(\alpha \cup \beta)^* = \alpha(\beta\alpha \cup \alpha)^*$.
- (h) If α and β are any two regular expressions, then $(\alpha\beta)^*\alpha = \alpha(\beta\alpha)^*$.

Solution.