Worldwide Youth in Science and Engineering

2012 Academic Challenge

MATHEMATICS TEST - REGIONAL

This Test Consists of 40 Questions

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ⊕ , ◑ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

WYSE Academic Challenge
Mathematics Test (Regional) - 2012

1. You are to select 6 cookies from a cookie jar that contains 8 chocolate chip cookies, 4 Oreos, 4 sugar cookies and 3 shortbread cookies. Find the probability of choosing 4 chocolate chip cookies in the selection of your 6 cookies. Round to 3 decimal places.

   a) 0.002 b) 0.009 c) 0.142 d) 0.201 e) 0.343

2. Given the function $f(x)$ below, what is $\lim_{x\to 3} f(x)$?

   Let $f(x) = \begin{cases} \dfrac{x^2 - 5x + 6}{x - 3}, \text{ if } x \neq 3 \\ 7, \text{ if } x = 3 \end{cases}$.

   a) -2 b) 0 c) 1 d) 2 e) 7

3. Find the polar coordinate of the point that is found on the opposite side of the pole from the given point $\left(2, \frac{2\pi}{3}\right)$.

   a) $\left(2, \frac{4\pi}{3}\right)$ b) $\left(-2, \frac{5\pi}{3}\right)$ c) $\left(2, -\frac{2\pi}{3}\right)$ d) $\left(-2, \frac{2\pi}{3}\right)$ e) $\left(-2, -\frac{2\pi}{3}\right)$

4. When rolling five fair, six-sided dice, how many possible ways are there to get a full house (three dice show one denomination, and the other two show one denomination)?

   a) 10 b) 30 c) 50 d) 60 e) 300

5. Given the equation $A = \dfrac{B + C}{B + D}$, solve for *B*.

   a) $B = \dfrac{C - D}{A}$ b) $B = \dfrac{A + C}{A + D}$ c) $B = \dfrac{C + AD}{A + 1}$ d) $B = \dfrac{C - AD}{A}$ e) $B = \dfrac{C - AD}{A - 1}$

6. Let S_n represent the sum of *n* terms in the arithmetic sequence 3 , 11/3 , 13/3 , 5 , ... If S_n = 80, find *n*.

   a) 12 b) 13 c) 14 d) 15 e) 16

7. Your bike's position on a graph after t hours is given by x = 5 + 4t and y = 7 + 8t, with x and y in miles, and the origin is equivalent to home. After how many hours, to the nearest hour, are you 500 miles from home?

   a) 21 hrs b) 55 hrs c) 57 hrs d) 38 hrs e) 51 hrs

8. Two positive numbers add up to a sum of 56 and multiply to get a product of 65. What is the larger of these two numbers?

   a) $\sqrt{3640}$ b) $28 - \sqrt{719}$ c) $28 + \sqrt{719}$ d) $28 + \sqrt{849}$ e) $56 - \sqrt{849}$

9. Given the equations $x = 4t + 2$; $y = 5t + 7$, find $4y - 5x$ in terms of t.

a) 38 b) 18 c) $-9t - 27$ d) $-9t + 9$ e) 9

10. What is $i^9 + i^{13} + i^{10} + i^{12}$?

a) 2i b) 4i c) 2i + 2 d) 0 e) i

11. Given the equation $16x^2 - 96x = 9y^2 + 36y + 36$, Find the eccentricity e.

a) 0 b) $\frac{3}{4}$ c) 1 d) $\frac{4}{3}$ e) $\frac{5}{3}$

12. The newly-discovered substance By-34 has a half-life of x years. If a sample of 200g of By-34 takes 53 years to decay to 80g, what is x, to the nearest hundredth of a year?

a) 31.88 yrs b) 40.09 yrs c) 44.17 yrs d) 50.66 yrs e) 57.12 yrs

13. A worker received a paycheck at the beginning of the month. One fifth of the paycheck went toward deductions. Of the amount left after deductions, one half went to bills. Of the amount left after bills, one fourth was spent on food. The amount left after food was $600. How much was the original paycheck?

a) $1,230 b) $1,350 c) $2,000 d) $12,000 e) $24,000

14. If a 3 by 4 matrix is multiplied by a matrix to produce another 3 by 4 matrix, then describe the other matrix used in the multiplication.

a) The matrix is a 3 by 4 matrix
b) The matrix is a 4 by 3 matrix
c) The matrix is a square matrix with 3 rows
d) The matrix cannot be a square matrix and has 4 columns
e) The matrix is a square matrix with 4 columns

15. What is the volume, in cubic meters, of a pool whose length is 100 meters, width is 20 meters, shallow end is 3 meters deep and deep end is 13 meters down? Assume that the pool depth decreases at a uniform rate from the deep to the shallow end.

a) 6000 m^3 b) 8000 m^3 c) 15000 m^3 d) 16000 m^3 e) 26000 m^3

16. A regular hexagon is cut in half to form two trapezoids. If one of these trapezoids has a perimeter of 300 inches, what is the area of the trapezoid? Round your answer to the nearest whole square inch.

a) 1559 sq in b) 2700 sq in c) 3118 sq in d) 4677 sq in e) 5625 sq in

17. Given $\begin{vmatrix} -6 & -1 & 3 \\ 2 & k & -3 \\ 10 & 1 & k \end{vmatrix} = 50$, find the integer *k*.

a) $-\frac{8}{3}$ b) -2 c) 2 d) a and b e) no solution

18. Nine dogs and six cats can create fifteen furballs in five days. Eight dogs and eleven cats can create fifty-five furballs in ten days. How many furballs can one cat create in ten days?

a) 1 b) 2 c) 3 d) 4 e) 5

19. Find the degree difference of the vertex angle sum of a right trapezoid and the vertex angle sum of a rhombus.

a) 60° b) 30° c) 0° d) -20° e) -30°

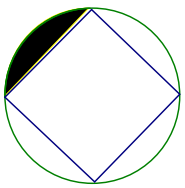
20. What is the oblique asymptote of $y = \frac{x^3}{x^2 + 3x + 1}$?

a) y = 0 b) y = 3x c) y = 7x + 4 d) y = 3 - x e) y = x - 3

21. A sphere has a volume of 100 cubic inches. Determine the surface area of the sphere Round your answer to the nearest whole square inch.

a) 22 sq in b) 26 sq in c) 35 sq in d) 71 sq in e) 104 sq in

22. Given a rhombus inscribed in a circle with radius 2 centimeters, find the area of the shaded region shown in the diagram below.



a) $4\pi - 8\,\text{cm}^2$ b) $\pi - 2\,\text{cm}^2$ c) $\pi - 4\ \text{cm}^2$ d) $8 - 4\pi\,\text{cm}^2$ e) $4\pi - 2\,\text{cm}^2$

23. What is the cosine of the smallest angle of a right triangle whose long leg is 40 inches and whose hypotenuse is 41 inches?

a) $\frac{41}{40}$ b) $\frac{40}{11}$ c) $\frac{7}{41}$ d) $\frac{40}{41}$ e) $\frac{9}{41}$

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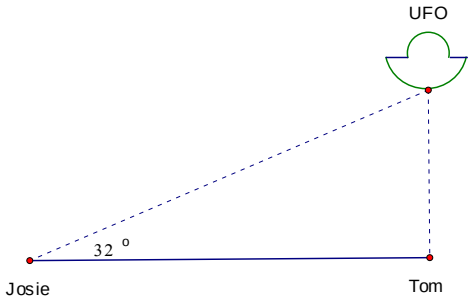
24. Angles A and B are complementary, angles A and C are supplementary, and angles B and C are supplementary. What must be true about the sum of the angles A, B, and C?

a) 225° b) 240° c) 270° d) 360° e) Situation can't exist

25. (Linda, trig, easy) Josie and Tom, who are directly across from one another, spot a UFO. Josie is 11,800 ft from the UFO at an angle of elevation of $32°$. Tom is directly below the UFO. How far apart are these two observers from one another? See Diagram below. Note: Diagram is not drawn to scale. Round to the nearest foot.



a) 10007 ft b) 8926 ft c) 7373 ft d) 7084 ft e) 6253 ft

26. In how many ways can one rearrange the letters of the word "Kennebunkport"?

a) 483,840
b) 39,916,800
c) 259,459,200
d) 479,001,600
e) 6,227,020,800

27. Solve for x in the logarithmic equation $\log_8 27 \bullet \log_3 x = 9$

a) $8^{\frac{9}{\log 9}}$ b) $8^{\frac{3\log 8}{\log 9}}$ c) $3^{9\log_8 27}$ d) $3^{9\log_{27} 8}$ e) no solution

28. The length of a rectangle is six inches more than eight times its width. The area of the rectangle is 97,460 square inches. What is its perimeter?

a) 1992 in. b) 996 in. c) 1248 in. d) 3544 in. e) 1870 in.

29. In a group of 100 people, 60 are male, 50 have blue eyes, and 40 are males with blue eyes. If we randomly select a female from the group, what is the probability that she has blue eyes?

a) 10% b) 25% c) 50% d) 80% e) not enough info given

30. Given the equation $\sqrt{x^n y^m}\ \sqrt[3]{x^{m+2} y^{2n}} = x^2 y^3$, solve for m.

a) -12 b) -2 c) 22 d) 32 e) no solution

31. If we circumscribe a circle about a triangle whose sides are 11, 60, and 61 to the nearest square inch, what is its area?

a) 2922 in^2 b) 5844 in^2 c) 330 in^2 d) 2592 in^2 e) 10290 in^2

32. Simplify the expression $\cos(\tan^{-1} x)$ into an expression of x that does not include any trigonometric functions.

a) x^2+1 b) $\sqrt{1-x^2}$ c) $\sqrt{x^2+1}$ d) $\frac{1}{\sqrt{1-x^2}}$ e) $\frac{1}{\sqrt{x^2+1}}$

33. Find the surface area in terms of the radius, in square units, of a solid cone balloon weight with a height that is $\frac{\sqrt{3}}{2}$ times the diameter of the cone.

a) $3\pi r^2$ b) $\frac{\sqrt{3}}{3}\pi r^2$ c) $\frac{\sqrt{3}}{3}\pi r^3$ d) $\pi r^2 + \frac{\sqrt{3}}{2}\pi r$ e) $\frac{\sqrt{3}}{2}\pi r^3$

34. How many right triangles are there where two sides are 3 and 4 inches a piece, respectively?

a) 0 b) 1 c) 2 d) 3 e) 4

35. Joe's coin collection consists of dimes and quarters. If there are six more than four times as many quarters as dimes and there are 106 coins in the collection, find the worth of Joe's coin collection.

a) \$13.60 b) \$16.40 c) \$17.80 d) \$23.50 e) \$24.20

36. Two observers stand on either side of an airplane that is directly above a straight section of the interstate, one mile marker apart. One of them notices that the angle of elevation to the plane is 45 degrees. The other notices that the angle of elevation to the plane is 26 degrees. How high is the airplane above the ground, to the nearest foot?

a) 3,791 ft. b) 1,489 ft. c) 3,489 ft. d) 1,731 ft. e) 3,640 ft.

37. A population of bacteria started with 100 bacteria when it was prepared. After five hours of exponential growth, the population had doubled to 200 bacteria. If the bacteria continue to grow exponentially, how many bacteria will there be in the population 24 hours after it was prepared? Round your answer to the nearest whole number of bacteria.

a) 960 b) 1520 c) 2560 d) 2786 e) 2880

38. Find the range of the function $f(x) = \dfrac{2x^2 + 2x - 4}{3x^2 + 13x + 14}$.

a) $y \in (-\infty, -2) \cup (-2, \infty)$

b) $y \in (-\infty, 0) \cup (0, \infty)$

c) $y \in (-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

d) $y \in (-\infty, -\frac{7}{3}) \cup (-\frac{7}{3}, -2) \cup (-2, \infty)$

e) $y \in (-\infty, -6) \cup (-6, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

39. What is the amplitude of 11 sin 3x + 48 cos 3x, to the nearest integer?

a) 11 b) 49 c) 48 d) 37 e) 47

40. Allison recently took a flight from Los Angeles to New York that started at LA, flew to Denver, stopped at Chicago, and finally flew to New York. She experienced delays at all four airports. The following was true about the delays:

I. The shortest delay was 20 minutes, and the longest was 60 minutes
II. The delay at New York was 50% longer than the delay at LA
III. The delay at Denver was 50% longer than the delay at New York
IV The combined Chicago/LA delay was 5 minutes longer than the Denver/NY delay

What was the total amount of delay that Allison experienced? Round your answer to the nearest whole minute.

a) 155 min b) 160 min c) 165 min d) 175 min e) not enough information