



# 2012 Academic Challenge

## MATHEMATICS TEST - SECTIONAL

This Test Consists of 40 Questions

### Mathematics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader

Linda Wiggins, Illinois State University – Author

Matthew Childers, Illinois State University – Author

Sandra Cox, Kaskaskia College – Reviewer

Mary Weaver, WYSE – Coordinator of Test Production

### GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

**\*\*\* TIME: 40 MINUTES \*\*\***

**DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!**

© 2012 Worldwide Youth in Science and Engineering

“WYSE”, “Worldwide Youth in Science and Engineering” and the “WYSE Design” are service marks of and this work is the Copyright © 2012 of the Board of Trustees of the University of Illinois at Urbana - Champaign.

All rights reserved.

WYSE Academic Challenge  
Mathematics Test (Sectional) - 2012

1. What is the derivative, with respect to  $x$ , of the function  $f(x) = 10xe^{2x}$ ?

- a)  $20e^{2x}$       b)  $10e^{2x}$       c)  $10 + 2e^{2x}$       d)  $\frac{5}{2}x^2e^{2x}$       e)  $10e^{2x} + 20xe^{2x}$

2. Of the systems given below, choose the one whose  $x$  solution is given by the determinant

$$\text{set up } x = \frac{\begin{vmatrix} 1 & 10 \\ -22 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 10 \\ -4 & 2 \end{vmatrix}}.$$

a)  $x + 10y = 1$   
 $-4x + 2y = -22$

b)  $x + y = 10$   
 $4x + 22y = -2$

c)  $x + 10y = -1$   
 $4x - 2y = 22$

d)  $x + 10y = 1$   
 $-22x + 2y = -4$

e)  $x + 10y = 10$   
 $4x - 2y = -2$

3. When graphed in the  $xy$ -coordinate plane, what is the shape of the graph of  $x = \sin t$ ,  $y^2 = \sin^2 t$ ?

- a) Circle  
b) Single line  
c) Parabola or parabolae  
d) Hyperbola  
e) Dual lines

4. Given the function  $f(x) = 2^{x+3} - 7$ , find  $f^{-1}(x)$ .

a)  $f^{-1}(x) = \frac{\log_2 7 - x}{3}$

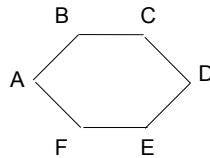
b)  $f^{-1}(x) = \log_2(x + 7) - 3$

c)  $f^{-1}(x) = \log_2\left(\frac{x+7}{8}\right)$

d)  $f^{-1}(x) = \frac{\log_2(x + 7)}{\log_2 8}$

e)  $f^{-1}(x) = \frac{\log_2(7x)}{3}$

5. Polygon ABCDEF is a hexagonal shape as shown below. All sides have length 1 inch. Angles A and D are right angles, but all the other angles are 135 degree angles. What is the area of this shape? Round your answer to the nearest tenth of a square inch.



- a)  $2.4 \text{ in}^2$       b)  $2.5 \text{ in}^2$       c)  $2.6 \text{ in}^2$       d)  $2.8 \text{ in}^2$       e)  $3.0 \text{ in}^2$

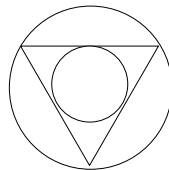
6. The decibel intensity level of a sound is given by  $B = 10 \log \frac{I}{I_0}$ , where  $I$  is the intensity of a sound, given in  $W / m^2$  and  $I_0$  is the intensity of a reference sound ( $I_0 = 10^{-12} W / m^2$ ). If a rock concert has a decibel intensity level of 120 dB, how many times more intense is it than a whisper (decibel intensity: 30 dB)? Round to the nearest whole number.

- a) 2 times  
 b) 9 times  
 c) 600,000 times  
 d) 9,000,000 times  
 e) 1,000,000,000 times

7. Given the equation  $\frac{x^2}{16} - \frac{y^2}{9} = 1$ , find the constant value distance used to verify that the locus of points form a hyperbola.

- a) 5 units      b) 6 units      c) 8 units      d) 9 units      e) 10 units

8. A set of four numbers has a population mean of 100 and a population standard deviation of 10. Determine what will happen to the population standard deviation if we include a fifth data value of 100. All answers have been rounded to the nearest whole number.
- S.D. decreases to 6
  - S.D. decreases to 8
  - S.D. decreases to 9
  - S.D. remains 10
  - Can't be determined without actual data values
9. A musical note has its variation from normal air pressure represented by the function  $f(t) = 0.3 \sin 110\pi t$ , where  $t$  is in seconds. What is the frequency of that note?
- $\frac{1}{55}$  Hz
  - 0.3 Hz
  - $\frac{10}{3}$  Hz
  - 55 Hz
  - $\frac{1100}{3}$  Hz
10. In coming to rest, a coin travels on a cone funnel making 0.80 revolutions in a second as in the previous second. How many revolutions does the coin make in coming to a rest if it makes 1.50 revolutions in the first second after the coin is released onto the funnel?
- 3 rev
  - 4.5 rev
  - 6 rev
  - 7.5 rev
  - 9 rev
11. What is the radius of a circle which has a sector whose area is  $\frac{216}{5}\pi$  square inches and whose central angle is  $108^\circ$ ? Round to the nearest inch.
- 4"
  - 12"
  - 6"
  - 24"
  - 14"
12. George went on a 1500 mile trip that took a total of 11 hours. He took a taxi to the airport, then a plane, and finally a rental car to reach the final destination. The taxi ride took as long as the final car ride and the time for connections. The taxi averaged 55 mph, the plane averaged 400 mph and the rental car averaged 40 mph. The plane traveled 4 times as far as the taxi and car combined. How long did the connections take?
- 6 hours
  - 5 hours
  - 4 hours
  - 3 hours
  - 2 hours
13. A circle is inscribed inside an equilateral triangle, and the triangle is in turn inscribed inside a larger circle, as shown below. If the smaller circle has an area of 1 square unit, determine the area of the larger circle. Round your answer to the nearest tenth of a square unit.



- $3.0 \text{ in}^2$
- $4.0 \text{ in}^2$
- $4.5 \text{ in}^2$
- $5.0 \text{ in}^2$
- $5.2 \text{ in}^2$

14. A restaurant has a menu with eight desserts, six main courses, fourteen salads, 12 appetizers and five soups. How many possible meals can be made by combining a main course, a dessert, and two of the other three types of dishes?
- a) 45                      b) 434                      c) 1,488                      d) 14,304                      e) 40,320
15. Identify the basic curve and the maximum radius given by the function  $r = 1 - 2 \sin \theta$ .
- a) a limaçon with maximum radius of 3 units  
 b) a cardioid with maximum radius of 3 units  
 c) a four-leaf rose with maximum radius of 3 units  
 d) a lemniscate with maximum radius 1 unit  
 e) a semiellipse with maximum radius 1 unit
16. Given the equation  $\frac{A}{B} - \frac{B}{A} = 1$ , solve for A.
- a)  $\pm\sqrt{1+B^2}$                       b)  $\pm\sqrt{B+B^2}$                       c)  $\frac{B \pm B\sqrt{3}}{2}$                       d)  $\frac{-B \pm B\sqrt{5}}{2}$                       e)  $\frac{B \pm B\sqrt{5}}{2}$
17. A crook overhears you tell a friend that you have a 5-digit PIN on your ATM card. She further hears that you have exactly two zeroes in the PIN. Once she inevitably steals your ATM card, what is the maximum number of PINs that she must systematically check before she finds yours?
- a) 1,000                      b) 7,290                      c) 10,000                      d) 14,580                      e) 59,049
18. A fisherman at sea has cast his line into the still water and has hooked a sea creature that is located 40 feet directly below him. The hooked sea creature moves horizontally in the water while the fishing line is being reeled out at a rate of 6 ft/s. When 50 feet of line is already out, what rate is the sea creature moving?
- a) 1.5 ft/s                      b) 10 ft/s                      c) 15 ft/s                      d) 18 ft/s                      e) 20 ft/s
19. What is the area of a regular 11-gon whose sides are each 16"? Round your answers to the nearest whole number.
- a) 915 sq. in.  
 b) 1,970 sq. in.  
 c) 2,398 sq. in.  
 d) 2,866 sq. in.  
 e) 8,082 sq. in.
20. A baseball diamond (square) has 90 feet between bases. The distance from the center of the front edge of the pitcher's mound to the home plate vertex point of the baseball diamond is 60ft. 6 in. Find the distance from the center of the front edge of the pitcher's mound to the first plate vertex point of the baseball diamond. State the answer as a denominate number given in both feet and inches. Round to the nearest inch.
- a) 60 ft. 6 in.                      b) 63 ft.                      c) 63 ft. 9 in.                      d) 66 ft.                      e) 66 ft. 8 in.

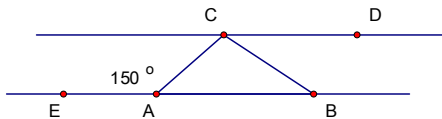
21. A crowd started with 1000 people. The crowd began to disperse so that every three minutes, the number of people remaining was half. In other words, 500 at three minutes, 250 at six, etc. Determine how long it should take for the number of remaining people to reach 10. Round your answer to the nearest minute.
- a) 9 min            b) 18 min            c) 19 min            d) 20 min            e) 99 min
22. A store has five sales on dresses in five consecutive weeks. One is a 15% off sale, the next, a 35% off sale, and the subsequent three are 8% off, 15% off again and finally 20% off. Through some expert flirtation, Jamie manages to get all five discounts applied concurrently, one after another. If she must pay 6% sales tax, what percent discount (to the nearest whole number) did she get after tax versus the original, untaxed price.
- a) 57%            b) 59%            c) 63%            d) 87%            e) 92%
23. Kathy has recorded the price in dollars of dress slacks at Kohl's, Sears, JC Penney, Dress Barn and Macy's over a 20 week period. She has compiled the following information regarding the mean prices and standard deviations:

Store	Mean Price	Standard Deviation
Kohl's	\$34	8
Macy's	\$31.75	6
Sears	\$32	5
JC Penney	\$31	4
Dress Barn	\$30	2

Since Kathy is a bargain hunter, which store should she plan to shop?

- a) Dress Barn    b) JC Penney    c) Macy's    d) Kohl's    e) Sears
24. A regular triangular prism has two equilateral triangles on each end, with three squares connecting them. If this prism has a surface area of 100 square inches, determine its volume. Round your answer to the nearest cubic inch.
- a)  $57 \text{ in}^3$             b)  $68 \text{ in}^3$             c)  $84 \text{ in}^3$             d)  $89 \text{ in}^3$             e)  $132 \text{ in}^3$
25. The geometric mean of 7 numbers is 8. The first five numbers are 4, 3, 8, 2 and 11, respectively. The last two numbers are exactly the same. Find the sum of the possible value or values for the last two numbers. Round to the nearest whole number.
- a) 0            b) 14            c) 28            d) 32            e) 63
26. A man has a square garden area totaling 144 sq. ft. He plans to use part of this area for an elaborate fish pond with a border made of stone all around the pond. The rest of this garden area is in the shape of a quarter circle with a radius equal to the length of the side of the square. How many feet of material will be needed for the border around the pond?
- a) 24 ft.  
 b)  $36\pi$  ft.  
 c)  $(36\pi + 24)$  ft.  
 d)  $6\pi$  ft.  
 e)  $(6\pi + 24)$  ft.

27. If drawn in a complex Cartesian plane (i.e. x-axis represents the real part and y-axis represents the complex part), in which quadrant would one find  $(3+i)^{50}$ ?
- a) I                      b) II                      c) III                      d) IV                      e) On an axis
28. Kenobi's circular dog bed has a sleeping area enclosed with a circular foam tubing that is 7.5 ft. long. What is the sleeping area of this dog bed rounded to the nearest tenth of a square foot?
- a) 4.5 sq.ft.  
b) 4.0 sq.ft.  
c) 3.8 sq.ft.  
d) 3.5 sq.ft.  
e) 3.1 sq.ft.
29. At noon, car A leaves city A at 60 mph and heads for city B, which is 300 miles away. At 12:30, car B also leaves city A and heads for city B, but traveling at 70 mph. Determine whether or not car B catches up to car A before it reaches city B. If it does, determine what time car B catches up to car A. Round your answer to the nearest whole minute.
- a) Yes, car B catches up to car A at 3:00 PM  
b) Yes, car B catches up to car A at 3:30 PM  
c) Yes, car B catches up to car A at 3:50 PM  
d) Yes, car B catches up to car A at 4:30 PM  
e) No, car B does not catch up to car A
30. I stand on a hill, 35 yards away from a building. From my vantage point, the angle of elevation to the top of the building is 15 degrees, while the angle of depression to the bottom of the building is 35 degrees. How tall is the building, in feet? Round to the nearest foot.
- a) 34 ft.  
b) 74 ft.  
c) 102 ft.  
d) 130 ft.  
e) 541 ft
31. Given  $AB \parallel CD$ ,  $m\angle EAC = 150^\circ$ , and  $m\angle CAB = m\angle DCB$ . What type of triangle is  $\triangle ABC$ ? See diagram below. Diagram is not drawn to scale.



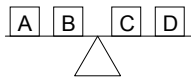
- a) A right triangle  
b) Obtuse isosceles triangle  
c) Equilateral triangle  
d) Acute scalene triangle  
e) Obtuse scalene triangle

32. A tank has two faucets that fill it, and one drain that empties it. Faucet 1 can fill the tank in 50 minutes, faucet 2 can fill the tank in 40 minutes, and the drain can empty the tank in 20 minutes. At noon, faucet 1 is turned on and the tank begins to fill. At 12:10, faucet 2 is turned on and helps fill the tank faster. At 12:20, the drain is opened and starts emptying the tank while the faucets are still running. Will the tank eventually be empty or full, and at what time will this happen? Round your answer to the nearest minute.
- Full at 12:30 PM
  - Full at 2:10 PM
  - Empty at 1:30 PM
  - Empty at 2:10 PM
  - Empty at 2:30 PM
33. Solve the inequality  $\frac{1}{2x} - \frac{3}{4} < \frac{1}{2x+3}$ .
- $(-\infty, -2) \cup (\frac{1}{2}, \infty)$
  - $(-\infty, -2) \cup (-\frac{3}{2}, \infty)$
  - $(-\infty, -2] \cup [-\frac{3}{2}, \infty)$
  - $(-\infty, -2) \cup (-\frac{3}{2}, 0) \cup (\frac{1}{2}, \infty)$
  - $(-\infty, -2] \cup [-\frac{3}{2}, 0) \cup (\frac{3}{2}, \infty)$
34. The price of a type of jetliner, in millions of dollars, varies according to the function  $f(t) = 122 + 6 \sin(43 + 9 \cos(8 + 9t))$ , where  $t$  is the number of months since the particular jetliner was introduced. Wayne buys one at the peak value, while Garth buys one at the trough valuation. What is the difference between the amount that each paid? Round to the nearest million dollars.
- \$2 million
  - \$6 million
  - \$9 million
  - \$12 million
  - \$18 million
35. Peter Piper picked a peck (8 dry quarts) of pickled peppers. He wanted to show his wife that he loved her “a bushel [8 dry gallons] and a peck” for each time it came up in the eponymous Frank Loesser song as sung by Doris Day (9 times) (and nothing says love like pickled peppers!) How many more times must he pick a peck of pickled peppers in order to display his affection?
- 44
  - 11
  - 8
  - 36
  - He’s already done.
36. Given the equation  $x^2 - 5x = 17$ , when solving for  $x$  by completing the square what value must be added to both sides of the equation so that the left side may be written as a binomial squared?
- 25
  - 11.56
  - 9
  - 6.25
  - 3.4

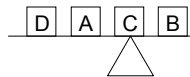


37. A person is playing a dice game where they roll a pair of fair dice three separate times. They win if they don't roll doubles on any of the three rolls. What is the probability of winning? Round to the nearest whole percent.
- a) 42%                      b) 58%                      c) 63%                      d) 67%                      e) 83%
38. Bonnie and Clyde Turner have taken up the game of basketball and have decided to specialize in shooting three-pointers. Clyde is a poor shot from beyond the arc. Bonnie is 50% better than Clyde when it comes to sinking treys. Together, if they each take a shot (and we assume independence of success), we know there's a 53.125% chance that at least one will hit from downtown. What is the chance that Clyde can sink one? Round to the nearest percent.
- a) 25%                      b) 20%                      c) 38%                      d) 10%                      e) 34%
39. Simplify  $(-x^2) - 2(x - 5 - (7 - (2x - x^2) - 3x)) - 12(2 - x)$
- a)  $x^2 + 12x$   
 b)  $11x^2 - 50x + 60$   
 c)  $13x^2 - 50x + 60$   
 d)  $3x^2$   
 e)  $x^2$
40. Two balance bars have been set up so that they each balance while having four weights placed on them as shown below:

Balance beam 1



Balance beam 2



Note that both beams balance even though the weights are in different positions, due to differing values of torque. You are told that the total weight of the four objects is 20 pounds, and that weight D is 2 pounds heavier than weight C. If that is the case, how much must weight C weigh?

- a) 2.0 lb                      b) 2.5 lb                      c) 3.0 lb                      d) 5.0 lb                      e) 5.5 lb