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1. Ans E: Think of $f(x) = uv$, where $u = 10x$ and $v = e^{2x}$. The derivative of uv, by the product rule, is $u'v + v'u = 10e^{2x} + 10x \cdot 2e^{2x} = 10e^{2x} + 20xe^{2x}$.

2. Ans A: Using Cramer's Rule we have $x = \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ Where $\begin{matrix} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{matrix}$. We see that $a_1 = 1, a_2 = -4, b_1 = 10, b_2 = 2, c_1 = 1, c_2 = -22$. The system is $\begin{matrix} x + 10y = 1 \\ -4x + 2y = -22 \end{matrix}$

3. Ans E: As $x = \sin t$ and $y^2 = \sin^2 t$, $y^2 = x^2$. Thus, $y = x$ and $y = -x$ both satisfy this parametric relationship. That means that its graph would have dual lines.

4. Ans B: Interchange variables and solve for y. We have $x = 2^{y+3} - 7 \Rightarrow x + 7 = 2^{y+3}$ $\Rightarrow \log_2(x+7) = y + 3$. $y = \log_2(x+7) - 3$.

5. Ans A: We can split this figure left to right into an isosceles right triangle, a rectangle, and a second isosceles right triangle. The two right triangles will each have area of 0.5. The rectangle will have area of $\sqrt{2}$. Add these up to get $1 + \sqrt{2}$, which rounds to 2.4.

6. Ans E: For the rock concert, $120 = 10\log\dfrac{I_{rock}}{10^{-12}}$. So $12 = \log\dfrac{I_{rock}}{10^{-12}}$. Thus, $10^{12} = \dfrac{I_{rock}}{10^{-12}}$ and $I_{rock} = 1\ W/m^2$. For the whisper, $30 = 10\log\dfrac{I_{whisper}}{10^{-12}}$. So $3 = \log\dfrac{I_{whisper}}{10^{-12}}$. Thus, $10^3 = \dfrac{I_{whisper}}{10^{-12}}$ and $I_{whisper} = 10^{-9}\ W/m^2$. So the ratio between the two is given as follows: $\dfrac{I_{rock}}{I_{whisper}} = \dfrac{1}{10^{-9}} = 10^9 = 1{,}000{,}000{,}000$.

7. Ans C: Taking any point on the hyperbola, say (4,0), and the foci (-5,0) and (5,0), we find the difference of the distance from (4,0) to (-5,0) and the distance from (4,0) to (5,0). We have $|4 - -5| - |4 - 5| \Rightarrow 9 - 1 = 8$.

8. Ans C: The old standard deviation was 10, so the old variance was 100, and the sum of the squared distances from the mean was 100*4 = 400. The new mean is still 100. All of the old distances and squared distances will remain the same, and the new number adds no distance. The new sum of the squared distances is 400, but the new variance will be 80. This means the standard deviation will be $\sqrt{80}$, roughly 8.94, rounded to 9.

9. Ans D: For a sinusoidal function of the form $f(t) = a\sin(kt)$, let p be the period and f be the frequency. $f = \dfrac{1}{p} = \dfrac{1}{\pi 2\pi/k} = \dfrac{k}{2\pi 2} = \dfrac{110}{\quad} = 55$. t is in seconds, so frequency is in Hz.

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10. Ans D: $\frac{1.5}{1-0.80} = 7.5$

11. Ans B: The area of a sector of a circle is given by $A = \frac{1}{2}r^2\theta$, where r is the radius of the circle and θ is the radian measure of the central angle. We can use the relationship between degrees and radians ($\frac{\theta_d}{180^\circ} = \frac{\theta_r}{\pi}$) to see that $\theta = \frac{108\pi}{180} = \frac{3\pi}{5}$. So, if we fill in the known quantities in our area equation, we get $\frac{216}{5}\pi = \frac{1}{2}\cdot\frac{3}{5}\pi\cdot r^2$. If we multiply both sides by $\frac{10}{3\pi}$, we get $144 = r^2$. So $r = 12"$.

12. Ans E: Let t represent taxi time, p the plane time and n the connection time. The system of equations is
$$\begin{aligned} 55t + 400p + 40c + 0n &= 1500 \\ -220t + 400p - 160c + 0n &= 0 \\ 1t + 0p - 1c - 1n &= 0 \\ 1t + 1p + 1c + 1n &= 11 \end{aligned}$$
Placing this into matrix form and solving we find the following:
$$\begin{matrix} 55 & 400 & 40 & 0 & 1500 \\ -220 & 400 & -160 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 11 \end{matrix} \Rightarrow \begin{matrix} 11 & 80 & 8 & 0 & 300 \\ 0 & 1 & 0 & 0 & 3 \\ 2 & 1 & 0 & 0 & 11 \\ 1 & 1 & 1 & 1 & 11 \end{matrix}$$
At this point, working backwards, we find that p = 3 , 2 t +1 p = 11 so t = 4. Using the top equation we can find the value of c. Finally, since c = 2, the value of n can be found using the last equation, so n = 2.

13. Ans B: The equilateral triangle can be split into six identical right triangles, each a 30-60-90 triangle. The shorter leg has a length equal to the smaller circle's radius, and the hypotenuse has a length equal to the larger circle's radius. Since the hypotenuse is twice as long as the shorter leg, the radius of the larger circle is twice the radius of the smaller, making its area four times that of the smaller circle.

14. Ans D: There are three types of meals listed here, main course-dessert-salad-appetizer, main course-dessert-salad-soup and main course-dessert-soup-appetizer. Since all three types are disjoint, we may find the number of each type and then add those together to get the total number of meal options. For each meal type, multiply together the number of options for each dish type (i.e. the number of main course-dessert-salad-appetizer meals is found by taking $6\times8\times14\times12 = 8064$, the number of main course-dessert-salad-soup meals is $6\times8\times14\times5 = 3360$ and the number of main course-dessert-soup-appetizer meals is $6\times8\times5\times12 = 2880$). Adding up those products, we receive a slate of 14,304 potential meals.

15. Ans A: Making a chart of values we find

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	1	-0.4	-1	-0.4	1	2.4	3	2.4	1

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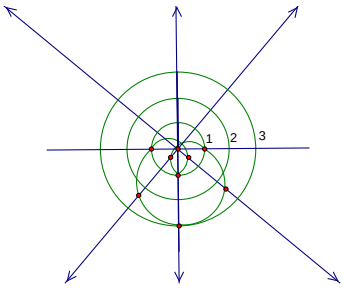


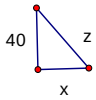
Therefore the graph is a limacon with radius 3.

16. Ans E: Start by multiplying both sides by AB to get $A^2 - B^2 = AB$. Rearrange the equation to get $A^2 - AB - B^2 = 0$. Use quadratic formula to get $A = \frac{-(-B) \pm \sqrt{(-B)^2 - 4(-B^2)}}{2}$. This can simplify down to $A = \frac{B \pm \sqrt{5B^2}}{2} = \frac{B \pm B\sqrt{5}}{2}$

17. Ans B: In creating the list of PINs, our first step might be to choose the two digits that need to be zeroes. There are C(5, 2) = 10 ways to do that. The next steps would be to fill in each of the remaining three spaces. There are only 9 options at each. By the multiplication principle, we can multiply 10 by the cube of 9 in order to get 7,290 possible 5-digit PINs with exactly two zeroes. You may take solace in the fact that she is highly unlikely to guess correctly before the machine, as a security measure, eats your card.

18. Ans B: $x^2 + 40^2 = z^2$. When z = 50, by Pythagorean theorem, x = 30. $\frac{dz}{dt} = 6$.

Plugging these values into $2x\frac{dx}{dt} = 2z\frac{dz}{dt}$ we find $\frac{dx}{dt} = 10$.



19. Ans C: The area of a regular polygon is given by the equation $A = \frac{1}{2}ap$, where a is the length of the apothem (a segment emanating from the center of the polygon which happens to serve as both an altitude and a bisector to one of the sides) and p is the perimeter. Since an 11-gon has 11 sides, the perimeter of this particular figure would be 176 inches (11 times 16). The easiest way to find the area of a non-triangular, non-square, non-hexagonal regular polygon is likely found by forming an isosceles triangle between the center thereof and two adjacent vertices, the vertex angle of which measures $\frac{360^\circ}{11}$. Thus, the other two angles of the triangle are. $\frac{180^\circ - \frac{360^\circ}{11}}{2} = \frac{810^\circ}{11}$. So, if one draws an altitude from the vertex angle to the other side, they have formed the apothem a as the long leg of a $\frac{810^\circ}{11} - \frac{180^\circ}{11} - 90^\circ$ right triangle, the short leg of which is 8

(half of the side length of the original figure). By the Law of Sines, we can create the following relationship: $\frac{a}{\sin\left(\frac{810^\circ}{11}\right)} = \frac{8}{\sin\left(\frac{180^\circ}{11}\right)}$. So $a = \frac{8\sin\left(\frac{810^\circ}{11}\right)}{\sin\left(\frac{180^\circ}{11}\right)}$. Thus, by our equation for the area of a regular polygon, $A = \frac{1}{2} \cdot \frac{8\sin\left(\frac{810^\circ}{11}\right)}{\sin\left(\frac{180^\circ}{11}\right)} \cdot 176 \approx 2{,}398$. Whew.

(There is an alternative solution in which the same triangle is used, but you note the size of the halved angle, and use a tangent approach in order to find the apothem.)

20. Ans C: Using Law of Cosines we have $c^2 = 60.5^2 + 90^2 - 2(60.5)(90)\cos 45^\circ$. Therefore c = 63.72 . The desired distance is approximately 63 ft. 9 in.

21. Ans D: This can be modeled using the function $P = 1000 * 0.5^{t/3}$. Set P = 10 and solve for t to get $10 = 1000 * 0.5^{t/3} \Rightarrow 0.01 = 0.5^{t/3} \Rightarrow \ln 0.01 = \ln 0.5^{t/3} \Rightarrow \ln 0.01 = \frac{t}{3} \cdot \ln 0.5 \Rightarrow t = 3 \cdot \frac{\ln 0.01}{\ln 0.5} \approx 19.9$. We can also model this using a $P = Ae^{kt}$ function, but the results would be the same.

22. Ans C: If we multiply .85 (15% off), .65, .92, .85, .8 and 1.06 (6% sales tax) together, we get, to the nearest hundredth, .37, which would correspond to a 63% discount.

23. Ans D: The price range within two standard deviations for each store is given below: Kohl's - \$18 to \$50 , Sears - \$22 to \$42 , Macy's - \$19.75 to \$43.75 , Dress Barn - \$26 to \$34 and JC Penney - \$23 to \$39. Therefore, Kohl's will have the lowest price.

24. Ans A: If all sides have length x, the surface area must be $\frac{\sqrt{3}}{4}x^2 + \frac{\sqrt{3}}{4}x^2 + x^2 + x^2 + x^2$. Solve $\left(\frac{\sqrt{3}}{2} + 3\right)x^2 = 100$ for x to get $x \approx 5.085898$. The volume ends up being $\left(\frac{\sqrt{3}}{4}x^2\right) \cdot x$, so insert the value of x to get a volume of approximately 56.96 cubic inches.

25. Ans A: The geometric mean of the numbers is $\sqrt[7]{4 \cdot 3 \cdot 8 \cdot 2 \cdot 11 \cdot x \cdot x} = 8$. So $2112x^2 = 2097152$. Thus $x^2 = \frac{32768}{33}$. So the two possibilities for x are $x = \pm\sqrt{\frac{32768}{33}}$. The sum of those two potential x-values is 0.

26. Ans E: The side of the square is $\sqrt{144} = 12$. So the perimeter of one-fourth circle is given by $\frac{2\pi(12)}{4} = 6\pi$. Since each side of the square is 12 feet long, the perimeter of the fish pond will be (6π + 2(12)) ft.

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27. Ans C: Every complex number c can be thought of as an ordered pair (r,θ), where r is the unique distance between that point in the complex Cartesian plane and the origin and θ is an angle between the line segment between that number's corresponding point in the plane and the origin and the positive x-axis. De Moivre's theorem says that c^n can be thought of as the ordered pair $\left(r^n, n\theta\right)$. An angle that the segment from the origin to $3+i$ makes with the positive x-axis can be given by $\theta = \tan^{-1}\frac{1}{3}$. So, by de Moivre's theorem, an angle that the segment from the origin to $(3+i)^{50}$ makes with the positive x-axis can be given by $\varphi = 50\theta = 50\tan^{-1}\frac{1}{3} \approx 921.747^\circ$, the terminal edge of which would be in qIII.

28. Ans A: $2\pi r = 7.5$. So r = 1.193662073. Since $A = \pi r^2$ then A = 4.476.

29. Ans B: Car B would catch car A when they have traveled the same distance. If x is the number of hours after noon, then solving 60x = 70(x – 0.5) would give us the correct time. Solving gives us x = 3.5, which means at 3:30 PM. The distance for both would be 210 miles, so car B does catch car A.

30. Ans C: First, it must be noted that 35 yards is 105 feet. Then, if we form a right triangle whose legs are from my vantage point to the point on the building directly across from me and from that point to the top of the building, we can note that we have a right triangle with a 15 degree angle, whose adjacent side is 105 feet and whose opposite side is unknown. So $\tan 15^\circ = \frac{a}{105}$ and the distance from the point directly across from me to the top of the building, a, is $105\tan 15^\circ$. If we subsequently form a right triangle whose legs are from my vantage point to the point on the building directly across from me and from that point to the bottom of the building, we can note that we have a right triangle with a 35 degree angle, whose adjacent side is 105 feet and whose opposite side is unknown. So $\tan 35^\circ = \frac{b}{105}$ and the distance from the point directly across from me to the bottom of the building, b, is $105\tan 35^\circ$. Thus the height of the building is $a + b = 105\tan 15^\circ + 105\tan 35^\circ \approx 102'$.

31. Ans B: From the given, we know that $m\angle CAB = 30$ since $m\angle EAC + m\angle CAB = 180^\circ$. This makes $m\angle DCB = 30^\circ$. $\angle ABC$ is the alternate interior angle of $\angle DCB$. Therefore the $m\angle ABC = 30^\circ$. The final vertex angle in the triangle, $\angle ACB$, must be 120° for the vertex angle sum of the triangle to add to 180°. Therefore, the triangle must be an obtuse isosceles triangle.

32. Ans E: Turn each rate value into tank per minute. Faucet 1's rate is 1/50 = 0.02 tank per minute, faucet 2's rate is 1/40 = 0.025 tank per minute, and the drain's rate is 1/20 = 0.05 tank per minute. At 12:10, the tank is 0.02*10 = 0.2, or 20% full. At 12:20, the tank is 0.2 + 10*(0.02+0.25) = 0.65, or 65% full. Next, we notice that 0.02 + 0.025 – 0.05 = -0.005. In other words, the tank will lose half a percent each minute. Since the tank is 65% full, it will take another 130 minutes to empty, or until 2:30 PM.

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33. Ans D: From the denominator we know that x cannot be 0 or $-\frac{3}{2}$ as both values make the denominator 0. Multiplying the inequality by the LCD $4x(2x+3)$ and setting the right side to 0 we have $-6x^2-9x+6<0$. Dividing by –3 produces the inequality $2x^2+3x-2>0$. The critical values found here are $\frac{1}{2}$ and – 2. These values must be excluded since the inequality does not contain values equal to 0. Plugging in values around the excluded values, we find that only numbers chosen in $(-\infty,-2)\cup(-\frac{3}{2},0)\cup(\frac{1}{2},\infty)$ will work.

34. Ans D: Let $u=43+9\cos(8+9t)$. The cosine of a linear function can take on all values between -1 and 1. This means that u can take on all values between 34 and 52. Since this covers a potential range of 18 units (much larger than the period of a sine function, which is only 2π), sin u can vary between -1 and 1, and $f(t)$ can thus vary between 116 and 128. So the difference between the peak and trough is twice the amplitude of the sinusoidal function described therein, \$12M.

35. Ans A: There are 4 quarts in a gallon, and thus 4 pecks in a bushel. So a bushel and a peck would be equivalent to five pecks. He must then pick a total of 45 pecks of pickled peppers, 44 more than he already has. Peter Piper is switching to the metric system at the next opportunity.

36. Ans D: $\left(-\frac{5}{2}\right)^2 \Rightarrow \frac{25}{4} \Rightarrow 6.25$

37. Ans B: On each roll, there is a 5/6 chance that the person does not roll a double. To get this result three times, we take 5/6 * 5/6 * 5/6 = 0.5787

38. Ans A: Let x be the probability that Clyde makes his shot (as a decimal). Then Bonnie has a 1.5x probability of sinking her shot. The probability that Clyde misses is then 1 – x and the probability that Bonnie misses is 1 – 1.5x. We know that the probability that they both miss, expressed as a decimal, is .46875 (as it's the complement of the event where at least one makes the shot). So we have that $(1-x)(1-1.5x)=.46875$ and, if we expand the left-hand side, we get $1.5x^2-2.5x+1=.46875$. Bringing everything over to one side for quadratic formula purposes, the equation becomes $1.5x^2-2.5x+.53125=0$. If we go through the quadratic formula and simplification, we get:

$$x=\frac{2.5\pm\sqrt{(-2.5)^2-4(1.5)(.53125)}}{2(1.5)}=\frac{2.5\pm\sqrt{6.25-3.1875}}{3}=\frac{2.5\pm\sqrt{3.0625}}{3}=\frac{2.5\pm1.75}{3}$$

If we simplify at the end, we get either x = .25 or x = 1.4166666…. Since the latter one of those solutions is not a possible probability, Clyde has a .25 = 25% chance of making his shot.

39. Ans E: $(-x^2)-2(x-5-(7-(2x-x^2)-3x))-12(2-x)$

$\Rightarrow -x^2-2x+10+2(7-2x+x^2-3x)-24+12x$

$\Rightarrow -x^2-2x+10+14-4x+2x^2-6x-24+12x \Rightarrow x^2$.

40. Ans C: Based on the given beams and total, we have the following four equations: $2A + B = C + 2D$, $2D + A = B$, $A + B + C + D = 20$, and $D = C + 2$. This can be solved slowly with substitution, or rather quickly with matrix row reduction. Either way, we end up with A = 1, B = 11, C = 3, and D = 5.