



2012 Academic Challenge

MATHEMATICS TEST - STATE FINAL

This Test Consists of 40 Questions

Mathematics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader

Linda Wiggins, Illinois State University – Author

Matthew Childers, Illinois State University – Author

Sandra Cox, Kaskaskia College – Reviewer

Mary Weaver, WYSE – Coordinator of Test Production

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

© 2012 Worldwide Youth in Science and Engineering

“WYSE”, “Worldwide Youth in Science and Engineering” and the “WYSE Design” are service marks of and this work is the Copyright © 2012 of the Board of Trustees of the University of Illinois at Urbana - Champaign.

All rights reserved.

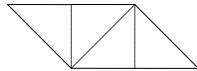
WYSE Academic Challenge
Mathematics Test (State Final) - 2012

1. Which of the following statements is NOT true?
- a) Every rhombus is a kite.
 - b) Every square is a rectangle.
 - c) Every rhombus is a parallelogram.
 - d) Some trapezoids may be isosceles.
 - e) Every trapezoid is a parallelogram.
2. In a food pantry, one shelf has a row of 6 different cans of soup and 4 different cans of fruit. Find the probability that 3 particular cans of soup will be together.
- a) $\frac{1}{15}$ b) $\frac{2}{13}$ c) $\frac{3}{13}$ d) $\frac{4}{13}$ e) $\frac{5}{13}$
3. A secant PR to a circle intersects the circle at both points P and Q. A tangent to that same circle, labeled RS, touches the circle at point S. If PQ has length 3 m and PR has length 12 m, what is the length of RS? Please round to the nearest meter.
- a) 3 m b) 4 m c) 5 m d) 6 m e) 7 m
4. In the given exponential equation $i = 0.6(1 - e^{-10t})$, solve for t .
- a) $t = \frac{-\ln(1 - \frac{5i}{3})}{10}$
- b) $t = \frac{-\ln(\frac{i}{0.6})}{10}$
- c) $t = \frac{-\ln(\frac{i}{0.6} + 1)}{10}$
- d) $t = \frac{-\ln(\frac{5i}{3} - 1)}{10}$
- e) $t = \frac{-\ln(\frac{i}{0.6} + 1)}{10}$
5. Determine $\lim_{x \rightarrow 0} \frac{2 \cos(3x) - 2}{x^2}$.
- a) -9 b) -6 c) -1 d) 0 e) Limit does not exist

6. What is the area of the region completely enclosed by the x-axis and the piecewise

$$\text{function } g(x) = \begin{cases} (x+1)^2 & \text{if } x \leq 0 \\ \left| \frac{1}{3}x \right| - 3 & \text{if } x > 0 \end{cases}.$$

- a) 13 b) 15 c) $19/3$ d) $79/6$ e) $83/6$
7. Simplify the expression $\sqrt{\frac{16 + 16 \cos 6x}{8}}$.
- a) $2 \sin 3x$ b) $2 \cos 3x$ c) $-2 \sin 3x$ d) $-2 \cos 3x$ e) $\frac{1}{2} \cos 3x$
8. A parallelogram is built using four right isosceles right triangles as shown below. If the parallelogram has a perimeter of 20 inches, what is its area? Round your answer to the nearest tenth of a square inch.



- a) 16.7 in^2 b) 17.2 in^2 c) 21.6 in^2 d) 22.2 in^2 e) 25.0 in^2
9. If we know only three of the six possible measures of a triangle (three angles and three sides), which of the following sets of known quantities is best tailored for utilization in Heron's Formula?
- a) Two sides and the angle between them.
 b) Two angles and the side between them.
 c) All three angles
 d) Two sides and an angle not between them.
 e) All three sides.
10. Given the function $f(x) = 8x^3 - 36x^2 + 54x - 27$, find the inverse function and call it $g(x)$.
- a) $g(x) = \frac{\sqrt[3]{x+3}}{2}$
 b) $g(x) = \frac{\sqrt[3]{x}}{2} + 1.5$
 c) $g(x) = \frac{\sqrt[3]{2x+3}}{2}$
 d) $g(x) = 2\sqrt[3]{x+3}$
 e) function does not have an inverse
11. What is the dot product of $\langle 3, 4, 5 \rangle$ and $\langle 3, 4, -7 \rangle$?

- a) $\langle -48, 36, 0 \rangle$ b) $\left\langle -\frac{4}{5}, \frac{3}{5}, 0 \right\rangle$ c) 60 d) -10 e) -42

12. Joan, Sarah and Phyllis went to lunch together and bought the same lunch. Each had a different amount of money on hand. Sarah had three times as much money as Phyllis and Phyllis had one-fourth as much money as Joan. Together they have \$200. After each person paid for their lunch, Joan noticed that she had $\frac{7}{8}$ of her money left. How much money did Phyllis have after paying for her lunch?
- a) \$ 2.50 b) \$ 5.00 c) \$ 12.50 d) \$ 62.50 e) \$ 82.50
13. A multiple choice test has forty questions, each with responses A through E. Assuming all possible answer keys are equally likely, what is the probability that a randomly chosen key has four or fewer of the answers as “C”? Round your answer to the nearest thousandth.
- a) 0.000 b) 0.047 c) 0.050 d) 0.076 e) 0.100
14. Six Congressmen are seated on the same side of a rectangular table for a hearing. If two of them MUST sit next to one another, in how many ways can the six be seated?
- a) 120 b) 240 c) 360 d) 480 e) 720
15. Given $f(x) = \frac{3x^2 - x - 2}{x - 1}$, find $f(1)$.
- a) 0 b) 1 c) 5 d) 8 e) undefined
16. Given the equation $\frac{A}{B} + 1 = \frac{B}{C}$, solve for B.
- a) $B = \pm\sqrt{AC+1}$
- b) $B = \pm\sqrt{AC+C}$
- c) $B = \frac{A \pm \sqrt{A^2 + 4AC}}{2}$
- d) $B = \frac{C \pm \sqrt{C^2 - 4AC}}{2}$
- e) $B = \frac{C \pm \sqrt{C^2 + 4AC}}{2}$
17. How many zeroes are there at the end of 638! ?
- a) 127 b) 152 c) 158 d) 319 e) 638
18. A professional free style ice skater goes into a spin making 0.80 as many revolutions in a second as the previous second. How many revolutions does the ice skater make in coming to a rest if the skater makes 1.50r in the first second?
- a) 7.5 r b) 6.0 r c) 4.5 r d) 2.0 r e) 1.5 r

19. Consider the stars “Zubeneschamali”, “Zubengelgenubi” and “al Minliar al Asad”. If we ignore the capitalization of some letters and the spaces between words, which one of the three stars has the most unique rearrangements?
- Zubeneschamali
 - Zubengelgenubi
 - al Minliar al Asad
 - Two of them are tied for the higher number of rearrangements.
 - All three have the same number of rearrangements.
20. John has a garden in the shape of an isosceles trapezoid. The equal sides of the trapezoid are 2 yards long. He wants to extend his garden so that the new shape of the garden is a right isosceles triangle with a garden perimeter of 10 yards. What should be the length of the equal sides of this new garden?
- $(8 - 2\sqrt{2})$ yds.
 - $8\sqrt{2}$ yds.
 - $(16 - 2\sqrt{2})$ yds.
 - $(8 - 5\sqrt{2})$ yds.
 - $(10 - 5\sqrt{2})$ yds.
21. How many vertices does a regular dodecahedron have?
- 10
 - 12
 - 20
 - 24
 - 30
22. For $3 + i$, what is its modulus' square divided by its conjugate?
- $3 + i$
 - $3 - i$
 - $\sqrt{13}$
 - 1
 - $\frac{4 + 3i}{5}$
23. A particular stretch of jogging trail is one half mile long. Art can jog the first quarter mile of the trail at 7 mph, but he can only jog the second quarter mile of the trail at 3 mph (it's much hillier). Ben can jog the first quarter mile at 6 mph, and can jog the second quarter mile at 4 mph. The two of them would like to finish the trail at the same time. Who should start first, and how much head start should they receive? Round your values for the head start to the nearest whole minute.
- Art should receive a 1 minute head start
 - Art should receive a 2 minute head start
 - Ben should receive a 1 minute head start
 - Ben should receive a 2 minute head start
 - They should basically start at the same time (result rounds to 0)
24. 14 men stand in a line that is laid out from east to west, with the first man furthest east and the last furthest west. They each stand 1 m behind the man ahead of them. The first notices that a spot across a pond is at a bearing of 23° west of south of him. The last notices that same spot is 51° east of south from him. How far is the spot from the first man? Round to the nearest tenth of a meter.
- 8.5 m
 - 8.9 m
 - 16.1 m
 - 19.9 m
 - 32.0 m

25. Reduce matrix $A = \begin{pmatrix} 2 & 4 & -6 & 0 \\ 4 & 8 & -4 & 4 \\ 6 & 12 & -8 & 6 \end{pmatrix}$ into echelon form.

a) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 4 & -6 & 0 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 6 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 3 \end{pmatrix}$

e) $\begin{pmatrix} 2 & 4 & -6 & 0 \\ 0 & 8 & -4 & 4 \\ 0 & 0 & -8 & 6 \end{pmatrix}$

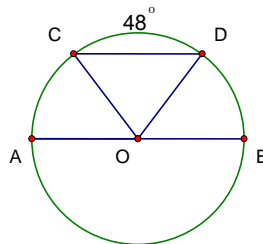
26. Given the equation $16(x - 3)^2 - n(y + 3)^2 = 16n$, find n so that the equation is a graph of a hyperbola with a foci $(8, -3)$.

- a) $n = 4$ b) $n = 9$ c) $n = 16$ d) $n = 25$ e) $n = 36$

27. The cosine of $5x$ is positive. In which quadrants can we find the terminal side of x , if the angle has been placed in standard form?

- a) I b) II c) III d) All four e) I and II

28. Using the diagram below, find the measure of arc BD given that $\overline{AB} \parallel \overline{CD}$. [The diagram below is not drawn to scale]



- a) 24° b) 42° c) 48° d) 66° e) 72°

29. Given the equation $\sum_{i=1}^x b^i = c$, solve for x .

a) $x = \frac{\ln(cb - c + 1)}{\ln b}$

b) $x = \frac{\ln(cb - c + 1)}{\ln b} - 1$

c) $x = \frac{\ln(cb - c + b)}{\ln b}$

d) $x = \frac{\ln(cb - c + b)}{\ln b} - 2$

e) $x = \frac{\ln(cb - c + b)}{\ln b} - 1$

30. Together, $x = 1 + \tan^2 t$ and $y = \sin t$ describe a curve. Which of the following is a solution for x in terms of y ?

a) $\frac{1}{y^2}$

b) $\frac{1}{1 - y^2}$

c) $1 + \frac{y^2}{1 + y^2}$

d) $\frac{1 - y^2}{1 + y^2}$

e) $\frac{2y^2}{1 + y^2}$

31. Anne and Beverly are 6 miles apart and are tracking their cat, Flash. At a certain instant, Flash is 5 miles from Anne with the distance increasing at a rate of 28 mph. At the same instant, the cat is also 5 miles from Beverly with the distance increasing at 4 mph. How fast is the cat moving?

a) $20\sqrt{2}$ mph

b) 20 mph

c) $10\sqrt{2}$ mph

d) 10 mph

e) 5 mph

32. Substance A has a half-life of 20 days. Substance B has a half-life of 30 days. If we start with three times as much substance A as substance B, how many days from now will it take for us to have three times as much substance B as substance A? Round your answer to the nearest whole day.

a) 60 days

b) 95 days

c) 155 days

d) 190 days

e) Never happens

33. If a hobbyist's rabbit population after t months can be modeled by the function $p(t) = \frac{225t}{3t + 25}$, what is the eventual trajectory of the population?

a) It will crash.

b) It will eventually settle at 9.

c) It will oscillate forever and never settle at a single value.

d) It will eventually settle at 75.

e) It will grow without restraint.

34. Find the rectangular equation of the polar equation $r = \frac{8}{4 - 2\cos\theta}$.

- a) $16x^2 + y^2 - 2x + 8 = 0$
- b) $4x^2 + 4y^2 - 2x - 8 = 0$
- c) $4x^2 + 3y^2 - 2x + 8 = 0$
- d) $3x^2 - 4y^2 - 2x + 8 = 0$
- e) $3x^2 + 4y^2 - 8x - 16 = 0$

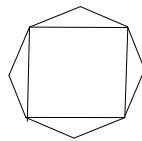
35. A square pyramid is built such that each side of the base is four times longer than the height of the pyramid. If it has a volume of 40,000 cubic feet, what is its height? Round to the nearest tenth of a foot.

- a) 13.6 ft.
- b) 19.6 ft.
- c) 21.5 ft.
- d) 32.0 ft.
- e) 34.2 ft.

36. Find $\int 4\csc^6 x \cot x \, dx$

- a) $(\frac{2}{3})\csc^5 x + c$
- b) $\frac{-2\csc^6 x}{3} + c$
- c) $\frac{-4\csc^7 x}{7} + c$
- d) $\frac{-2}{3}\csc^7 x + c$
- e) $\frac{-4\csc^7 x}{6} + c$

37. A square is inscribed inside a regular octagon as shown below. If the area of the square is 1 square inch, determine the area of the octagon. Round your answer to the nearest hundredth of a square inch.



- a) 1.33 in^2
- b) 1.41 in^2
- c) 1.44 in^2
- d) 1.50 in^2
- e) 1.67 in^2

38. The third derivative of a function, $f(x)$, with respect to x , is $f'''(x) = 9e^{3x}$. Which of the following could be $f(x)$?

a) $\frac{3}{2}x^3e^{3x}$

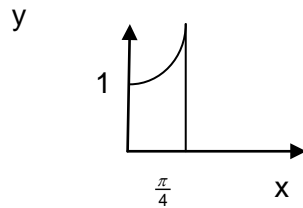
b) $\frac{1}{3}e^{3x} + \frac{1}{100}x^3$

c) $\frac{3}{40}e^{6x} + 6x^2 + 4x + 8$

d) $9e^{3x} + 8$

e) $\frac{1}{3}e^{3x} + 6x^2 + 4x + 9$

39. Linda received a trial size lipstick tube and wishes to find the volume of the actual lipstick in the tube. The volume of the lipstick is generated by revolving about the x axis the region bounded by $y = \sec x$, $y = 0$, $x = 0$, and $x = \frac{\pi}{4}$. See diagram below:



- a) $\frac{\pi^3}{3}$ cu. unit b) π^3 cu. unit c) $\frac{\pi^2}{2}$ cu. unit d) π cu. unit e) $\frac{\pi}{2}$ cu. unit

40. At a recent amateur stock car race, four racers, Art, Brett, Chris, and Dale, competed. The race was a total length of 150 miles long, and each racer made at least two pit stops. The following facts are true:

- i. Everyone's average pit stop time was a whole number.
- ii. Art drove at a speed of 100 mph and took a total of 4 pit stops.
- iii. Brett drove 10 mph faster than Chris, and averaged 5 minutes per pit stop.
- iv. One of the drivers drove at a speed of 90 mph, and took 12 minutes total in pit stops.
- v. Dale drove 80 mph, but took the fewest pit stops and averaged 3 minutes per stop.
- vi. The drivers took a total of 14 pit stops for a total pit stop time of 67 minutes.
- vii. No one took the same number of stops and no one had the same average stop time

Who won the race? (i.e., had the lowest total combined driving and pit stop time?)

- a) Art
- b) Brett
- c) Chris
- d) Dale
- e) Not enough information given