WYSE MATH STATE 2012 SOLUTIONS

1. Ans E: Trapezoids need only have one pair of parallel sides. Parallelograms are, by definition, forced to have two.

2. Ans A: All the cans can be arranged in ${}_{10}P_{10} = 10!$ ways. Grouping the 3 cans together and treating them as 1 can, we can arrange the cans now in ${}_{8}P_{8} = 8!$ ways. The 3 soup cans can be arranged by themselves ${}_{3}P_{3} = 3!$. The probability is then given by $\frac{8!3!}{10!} = \frac{1}{15}$.

3. Ans C: If given a secant segment and a tangent segment which both end at the same point outside of a circle and end at their points of intersection and tangency, respectively, the square of the length of the tangent segment is the product of the internal and external portions of the secant. So the square of RS is the product of PQ and QR. PQ has length 3 m and PR has length 12 m, so QR has length 9 m. Thus, the square of RS is 27 meters squared, the square root of which most closely rounds to 5 m.

4. Ans A: $i = 0.6\left(1 - e^{-10t}\right) \Rightarrow \frac{5}{3}i = 1 - e^{-10t} \Rightarrow e^{-10t} = 1 - \frac{5}{3}i \Rightarrow -10t = \ln\left(1 - \frac{5}{3}i\right) \Rightarrow \frac{-\ln\left(1 - \frac{5}{3}i\right)}{10}$

5. Ans A: If we plug in 0, we get the indeterminate form 0/0. Use L'Hospital's method to end up with $\lim_{x \to 0} \frac{-6\sin 3x}{2x}$. This is still 0/0, so L'Hospital's one more time to get $\lim_{x \to 0} \frac{-18\cos 3x}{2}$, which evaluates to -9.

6. Ans E: First, we should note that we can ignore the absolute value in g(x). Secondly, the left area is bounded below by the x-axis and above by the parabola, so we can integrate the quadratic function from -1 (the x-intercept on the left) to 0 (the place where the quadratic function no longer has precedence). The right area is bounded above by the x-axis and below by the absolute value function. So we can integrate the opposite of the absolute value function from 0 to 9 (the x-intercept on the right).

$$\text{Area} = \int_{-1}^{0} (x+1)^2\,dx + \int_{0}^{9}\left(-\frac{1}{3}x + 3\right)dx = \left[\frac{1}{3}(x+1)^3\right]_{-1}^{0} + \left[-\frac{1}{6}x^2 + 9x\right]_{0}^{9} = \frac{83}{6}$$

7. Ans B: $\sqrt{\frac{16 + 16\cos 6x}{8}} \Rightarrow \sqrt{\frac{16(1 + \cos 6x)}{4(2)}} \Rightarrow 2\left(\sqrt{\frac{1 + \cos 6x}{2}}\right) \Rightarrow 2\cos\frac{1}{2}(6x) \Rightarrow 2\cos 3x$

8. Ans B: Let x be the leg length of any one of the triangles. This would make the perimeter equal to $4x + (2\sqrt{2})x$. If the perimeter is 20, this makes x = $\frac{20}{4 + (2\sqrt{2})}$, which is approximately 2.9289 inches. The area of the parallelogram is $4 \cdot \frac{1}{2} x \cdot x$, which would be 17.2 in^2.

WYSE MATH STATE 2012 SOLUTIONS

9. Ans E: Since Heron's Formula depends directly on the knowledge of the measure of the three sides of a triangle, we would prefer to know all three of them first.

10. Ans B: Factor the function by using synthetic division or binomial theorem. We find that the function can be written as $f(x) = (2x-3)^3$. Interchanging the variables we have $x = (2y-3)^3$. Solving for y we find $2y - 3 = \sqrt[3]{x} \Rightarrow y = \frac{\sqrt[3]{x}+3}{2}$.

11. Ans D: The dot product is the sum of the products of corresponding entries in the vector, so the dot product is the product of 3 and 3 plus the product of 5 and 4 plus the product of 5 and -7, which is -10.

12. Ans C: $3p + p + 4p = 200 \Rightarrow p = 25$. Joan had 4 times as much so Joan has \$100. One-eighth of the \$100 represents the cost of the lunch. Lunch cost \$12.50. Phyllis (represented by p in the equation) only had \$25. Subtracting lunch means that Phyllis had \$12.50 left.

13. Ans D: Effectively we have a binomial distribution. We want 0 to 4 successes out of the 40. This is 5 distinct binomial probabilities that will need to be added, each one of the form $\mathrm{P(x)} = \mathrm{C(40, x)} \cdot 0.2^{\mathrm{x}} \cdot 0.8^{40-\mathrm{x}}$ where x is the number of successes. These five probabilities summed up are 0.00013 + 0.00133 + 0.00648 + 0.02052 + 0.04745 = 0.07596, which rounds to 0.076. Many calculators will have an option known as the binomial cumulative distribution function, which works in this case.

14. Ans B: It is easiest if we consider the two that must sit next to one another to be a bloc, and recognize that there are two possible arrangements of the bloc. Then there are essentially five places to seat the bloc, so there are 10 ways to seat them. The remaining four seats may be filled by the remaining four Congressmen in any order, so there are 24 = 4! ways to seat them. 24 times 10 is 240.

15. Ans E: Although the fraction reduces to $3x + 2$, we still cannot use x = 1 as the denominator will be 0. Therefore the correct response is undefined.

16. Ans E: $\mathrm{BC}\left(\frac{\mathrm{A}}{\mathrm{B}}+1\right) = \left(\frac{\mathrm{B}}{\mathrm{C}}\right)\mathrm{BC} \Rightarrow \mathrm{AC}+\mathrm{BC} = \mathrm{B}^2 \Rightarrow \mathrm{AC}+\mathrm{BC} = \mathrm{B}^2 \Rightarrow 0 = \mathrm{B}^2 - \mathrm{BC} - \mathrm{AC}$ use quadratic formula to give us our final answer of $\mathrm{B} = \dfrac{\mathrm{C} \pm \sqrt{\mathrm{C}^2 + 4\mathrm{AC}}}{2}$

17. Ans C: The only way we can have a certain number of zeroes at the end of a number is to have that many factors of 10, and thus at least that many factors of 2 and that many factors of 5. The limiting factor is the number of factors of 5. There are 127 numbers under 638 that are divisible by 5, 25 that are divisible by 25, 5 that are divisible by 125 and 1 that is divisible by 625. So there are a total of 127 + 25 + 5 + 1 factors of 5 available

18. Ans A: $s = \dfrac{1.5}{1-.8} \Rightarrow s = 7.5$

WYSE MATH STATE 2012 SOLUTIONS

19. Ans A: To count the number of unique rearrangements of strings of letters, we divide the factorial for number of letters by the factorials for the number of times each letter is repeated. For Zubeneschamali, that number is $\frac{14!}{2!2!}$, which is higher than $\frac{13!}{3!2!2!}$ for Zubenelgenubi and $\frac{15!}{5!3!2!}$ for al Minliar al Asad.

20. Ans E: One of the equal sides of the isosceles triangle is given by $2+x$. Using Pythagorean theorem we can find the algebraic expression for the base of the triangle, $\sqrt{2(2+x)^2} \Rightarrow \sqrt{2}(2+x)$. Then the equation to solve is $2(2+x)+\sqrt{2}(2+x)=10$. Solving for x we find $x=8-5\sqrt{2}$. The desired length is $2+8-5\sqrt{2} \Rightarrow 10-5\sqrt{2}$

21. Ans C: The dodecahedron has 12 faces, 20 vertices, and 30 edges.

22. Ans A: For every complex number, the square of its modulus is the number itself times its conjugate. Thus, if you divide the square of the modulus of a number by its conjugate, you are left with the number itself.

23. Ans A: Using the formula time = dist/rate, Art's total time is 0.25/7 + 0.25/3 = 5/42 hours, which is approximately 7.143 minutes. Ben's total time is 0.25/6 + 0.25/4 = 5/48 hours, which is 6.25 minutes. Art should receive a head start of exactly 5/336 hours, or approximately 0.893 minutes. Round this to one minute.

24. Ans A: We know that the first and last men are 13 meters apart. If we draw lines due south from each, and segments from those men to the aforementioned spot, we create the angles mentioned in the statement of the problem and a triangle which must thus have angles of 39 degrees off the vertex corresponding to the last man and 67 degrees off the vertex corresponding to the first man. Then, by the Law of Sines, if we denote the desired distance by x, $\frac{13}{\sin 74^\circ} = \frac{x}{\sin 39^\circ}$.

25. Ans C: Divide all rows by 2, then apply row operations $-2R_1 + R_2 \to R_2$. We now have $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 3 & 6 & -4 & 3 \end{pmatrix}$. Next, apply row operations $-3R_1 + R_3 \to R_3$. This gives us $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 5 & 3 \end{pmatrix}$. Finally, apply row operations $-5R_2 + 4R_3 \to R_3$. We have $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$.

26. Ans B: 8 – 3 = 5, Then $\sqrt{n+16} = 5$. Solving for n we find that n must be 9.

WYSE MATH STATE 2012 SOLUTIONS

27. Ans D: If we examine the following angles in standard form: 70 degrees, 140 degrees, 210 degrees and 280 degrees, they are in all four quadrants, but, when multiplied by 5, their measures are 350 degrees, 700 degrees, 1050 degrees and 1400 degrees, all of which are in quadrant IV when placed in standard position. Since the cosine is positive in quadrant IV, this means that we have angles in all four quadrants which, when quintupled, have positive cosines.

28. Ans D: First, since triangle COD has equal sides OC and OD, their subtending angles are congruent. So angle DCO and angle CDO are congruent. We know that angle COD is 48 degrees. This means that the remaining congruent angles of the triangle must be 66 degrees $\left(\frac{180-48}{2}\right)$. Since AB is parallel to CD, then the transversal OD forms congruent alternate interior angles. Angle CDO is congruent to angle BOD. So the central angle BOD is 66 degrees. This means arc BD is 66 degrees.

29. Ans E: The left side is the geometric series $b+b^2+b^3+\ldots+b^x$. A geometric series of the form $a+ar+ar^2+\ldots+ar^n$, has the sum $\dfrac{a(r^{n+1}-1)}{r-1}$. In our series, a = b, r = b, and n = x – 1, so it has a sum equal to $\dfrac{b(b^x-1)}{b-1}$. Now solve for x. $\dfrac{b(b^x-1)}{b-1}=c \Rightarrow$

$$b(b^x-1)=cb-c \Rightarrow \; b^x=c-\frac{c}{b}+1 \Rightarrow b^x=\frac{cb-c+b}{b} \Rightarrow \; x=\log_b\left(\frac{cb-c+b}{b}\right) \Rightarrow$$

$$x=\frac{\ln\left(\dfrac{cb-c+b}{b}\right)}{\ln b} \Rightarrow x=\frac{\ln\left(cb-c+b\right)-\ln b}{\ln b} \Rightarrow x=\frac{\ln\left(cb-c+b\right)}{\ln b}-1.$$

30. Ans B: We can rewrite the first equation as $x=\sec^2 t=\dfrac{1}{\cos^2 t}=\dfrac{1}{1-\sin^2 t}=\dfrac{1}{1-y^2}$.

31. Ans A: $x^2+y^2=u^2$ and $\left(6-x\right)^2+y^2=v^2$. Then $2x\dfrac{dx}{dt}+2y\dfrac{dy}{dt}=2u\dfrac{du}{dt}$ and $-2\left(6-x\right)\dfrac{dx}{dt}+2y\dfrac{dy}{dt}=2v\dfrac{dv}{dt}$ where $\dfrac{du}{dt}=28$, $\dfrac{dv}{dt}=4$; $u=v=5$. Using Pythagorean theorem we can solve for x as follow: $\left(6-x\right)^2+\left(\sqrt{25-x^2}\right)^2=25$. The equation simplifies to $36=12x$. Therefore $x=3$. When $x=3$, $y=4$. Plugging in these values we have the following:

$$3\frac{dx}{dt}+4\frac{dy}{dt}=140$$
$$-3\frac{dx}{dt}+4\frac{dy}{dt}=20$$

We deduce that $\dfrac{dx}{dt}=\dfrac{dy}{dt}=20$. So $\sqrt{20^2+20^2}=20\sqrt{2}$.

WYSE MATH STATE 2012 SOLUTIONS

32. Ans D: If we start with G grams of B, we start with 3G grams of A. The equations for the amount of each remaining after t days would be $A = 3G \cdot 0.5^{\left(\frac{t}{20}\right)}$ and $B = G \cdot 0.5^{\left(\frac{t}{30}\right)}$. We want to know when B = 3A. Solve the equation $G \cdot 0.5^{\left(\frac{t}{30}\right)} = 3 \cdot 3G \cdot 0.5^{\left(\frac{t}{20}\right)}$ which is also $0.5^{\left(\frac{t}{30}\right)} = 9 \cdot 0.5^{\left(\frac{t}{20}\right)}$. We get an answer of t = 190.196.

33. Ans D: Eventually, rational functions where the degree of the numerator is the same as that of the denominator tend toward their horizontal asymptotes. The y-value of the horizontal asymptotes in these cases is the ratio of the leading coefficients, in this case, 225 and 3. As the quotient of 225 and 3 is 75, y = 75 is the horizontal asymptote of this particular function.

34. Ans E: $r = \dfrac{8}{4 - 2\cos\theta} \Rightarrow 2r - r\cos\theta = 4 \Rightarrow 2r = 4 + r\cos\theta \Rightarrow 2\sqrt{x^2 + y^2} = 4 + x$
$4x^2 + 4y^2 - x^2 - 8x - 16 = 0 \Rightarrow 3x^2 + 4y^2 - 8x - 16 = 0$.

35. Ans B: The volume of a pyramid is one third of the product of its height and the area of its base. Let h be its height. Then each side of its base is 4h, and the area of the base is $16h^2$. Then its volume is $\dfrac{16}{3}h^3$. So $h^3 = 7500$.

36. Ans B: Let $u = \csc x$, then $du = -\csc x \cot x dx$. Then $\int 4\csc^6 x \cot x \, dx \Rightarrow$
$-4\int u^5 du \Rightarrow \frac{-4}{6}u^6 + c \Rightarrow \frac{-2}{3}\csc^6 x + c$.

37. Ans B: If the square has an area of 1 square unit, then each side has a length of 1 unit, and the distance from a vertex to the center would be $0.5\sqrt{2}$ units. The octagon can be split into eight isosceles triangles. Each triangle would have two side lengths equal to $0.5\sqrt{2}$ units. If you look at the triangle properly, you should notice that the $0.5\sqrt{2}$ side can act as a base, and the height of the triangle would be 0.5. This means each triangle has an area of $0.5 \cdot 0.5 \cdot 0.5 \cdot \sqrt{2}$. Since there are eight of these, the total area would end up being $8 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot \sqrt{2}$, which simplifies to $\sqrt{2} \approx 1.41$.

38. Ans E: The derivative of e to a power is the derivative of that power times e still raised to that power. So to get a valid possible f(x), we could divide $9e^{3x}$ by 27. (We could also add a polynomial of degree 2 or less, as the third derivative of that would completely disappear. This is why E is the valid answer.) (We could also just take the third derivatives of each of the five functions, and then note that, out of all of these, only E had the desired third derivative.)

39. Ans D: Using the disk method, we have
$$V = \pi\int_0^{\frac{\pi}{4}} \sec^2 x \, dx \Rightarrow V = \pi(\tan x)\Big|_0^{\frac{\pi}{4}} \Rightarrow V = \pi(1 - 0) \Rightarrow V = \pi.$$

40. Ans C: As we go through, we need to assign speed, number of stops, and time of stop. Art drove at 100 mph and so took 150/100*60 = 90 minutes in driving time. Dale drove at 80 mph and so took 150/80*60 = 118.5 minutes driving time. Since Brett can't average 5 minutes per stop and have a total stop time of 12 minutes, Chris must be the one driving at 90 mph and took 150/90*60 = 100 minutes driving time. This means Brett is driving at 100 mph and also took 90 minutes for driving. Since the number of stops totals 14 and no one is the same, we must have 2, 3, 4, and 5. Dale stopped 2 (the fewest) and Art stopped 4 times. Since Chris has the 12 minute stop total, he must have stopped 3 times, and Brett stopped 5 times. Dale spent 6 minutes in stops (2x3), Chris spent 12 minutes in stops (3x4), Brett spent 25 minutes in stops (5x5), meaning Art must have spent 67 – 6 – 12 – 25 = 24 minutes in stops. Since he stopped 4 times, he averaged 6 minutes per stop. Art took 90 + 24 = 114 minutes total, Brett took 90 + 25 = 115 minutes total, Chris took 100 + 12 = 112 minutes total, and Dale took 112.5 + 6 = 118.5 minutes total. Chris won the race.