2012 Academic Challenge
State Physics Exam Solutions

1. **Correct response: A**

$$1.00\frac{\text{gram}\cdot\text{mm}}{\text{min}^2}\times\frac{1\text{kg}}{1000\,\text{gram}}\times\frac{1\text{m}}{1000\,\text{mm}}\times\left(\frac{1\text{min}}{60\,\text{s}}\right)^2=2.78\times10^{-10}\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}=2.78\times10^{-10}\,\text{N}$$

2. **Correct response: B**

$$a=\left|\frac{v_f-v_i}{t}\right|=\left|\frac{20.0\,\text{m/s}-0.00\,\text{m/s}}{5.00\,\text{s}}\right|=4.00\,\text{m/s}^2$$

3. **Correct response: D**

$$x_f-x_i=\tfrac{1}{2}\left(v_f+v_i\right)t=\tfrac{1}{2}(20.0\,\text{m/s}+0.00\,\text{m/s})(5.00\,\text{s})=50.0\,\text{m}$$

4. **Correct response: B**

The magnitude of the average velocity is the magnitude of the displacement divided by the time interval and the average speed is the path length divided by the time interval. The path length is always greater than or equal to the magnitude of the displacement, therefore the average speed is always greater than or equal to the magnitude of the average velocity.

5. **Correct response: C**

$$y-y_0=v_{0y}t-\tfrac{1}{2}gt^2 \quad \text{and} \quad \text{initial horizontal velocity} \quad \Rightarrow \quad v_{0y}=0 \quad \Rightarrow$$

$$t=\sqrt{\frac{2\left(y-y_0\right)}{g}}=\sqrt{\frac{2(0.800\,\text{m})}{\left(9.80\,\text{m/s}^2\right)}}=0.404\,\text{s}$$

6. **Correct response: A**

$$v_y^2-v_{y0}^2=2a\left(y-y_0\right) \quad \Rightarrow \quad v_y=\sqrt{v_{y0}+2a_y\left(y-y_0\right)}=\sqrt{0+2\left(9.80\,\text{m/s}^2\right)(0.800\,\text{m})}=3.96\,\text{m/s}$$

$$\tan\theta=\frac{v_y}{v_x} \quad \Rightarrow \quad v_x=\frac{v_y}{\tan\theta}=\frac{(3.96\,\text{m/s})}{\tan(80.0^\circ)}=0.698\,\text{m/s}$$

7. **Correct response: E**

Using upward as the positive y-direction,

$$\sum\vec{F}=ma \quad \Rightarrow \quad F_{shroud\,lines\,y}+F_{gravity\,y}=ma \quad \Rightarrow \quad F_{shroud\,lines\,y}=ma-F_{gravity\,y}=ma+mg$$

$$=(60.0\,\text{kg})\left(4.00\,\text{m/s}^2\right)+(60.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)=828.\text{N}$$

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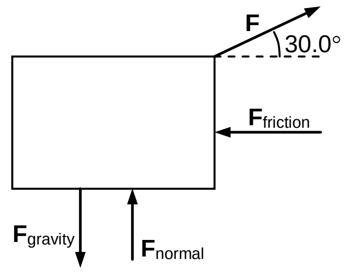
8. **Correct response: C**



$$\mathbf{F}_{gravity} + \mathbf{F}_{normal} + \mathbf{F}_{friction} + \mathbf{F} = 0 \quad \Rightarrow \quad F_{gravity\,y} + F_{normal\,y} + F_{friction\,y} + F_y = 0 \quad \Rightarrow$$
$$F_{normal\,y} = -F_{gravity\,y} - F_{friction\,y} - F_y = -(-40.0\,\text{kg})(9.80\,\text{m/s}^2) - 0 - (70.0\,\text{N})\sin 30.0° = 357.\text{N}$$
$$F_{gravity\,x} + F_{normal\,x} + F_{friction\,x} + F_x = 0 \quad \Rightarrow \quad 0 + 0 - \mu F_{normal} + F\cos 30.0° = 0 \quad \Rightarrow$$
$$\mu = \frac{F\cos 30.0°}{F_{normal}} = \frac{(70.0\,\text{N})\cos 30.0°}{357.\text{N}} = 0.170$$

9. **Correct response: B**

Considering only the force components parallel to the inclined plane:

$$F_T - mg\sin 25.0° = 0 \quad \Rightarrow \quad m = \frac{F_T}{g\sin 25.0°} = \frac{200.\text{N}}{(9.80\,\text{m/s}^2)(\sin 25.0°)} = 48.3\,\text{kg}$$

10. **Correct response: E**

$$\omega = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad k = m\omega^2 = m\left(\frac{2\pi}{T}\right)^2 = (3.00\,\text{kg})\left(\frac{2\pi}{2.00\,\text{s}}\right)^2 = 29.6\,\text{N/m}$$

11. **Correct response: A**

$$I_1\omega_1 = I_2\omega_2 \quad \Rightarrow \quad I_2 = \frac{I_1\omega_1}{\omega_2} = \frac{(30.0\,\text{kg}\cdot\text{m}^2)(4.00\,\text{rad/s})}{(6.00\,\text{rad/s})} = 20.0\,\text{kg}\cdot\text{m}^2$$
$$W = \Delta K = \tfrac{1}{2}I_2\omega_2^2 - \tfrac{1}{2}I_1\omega_1^2 = \tfrac{1}{2}(20.0\,\text{kg}\cdot\text{m}^2)(6.00\,\text{rad/s})^2 - \tfrac{1}{2}(30.0\,\text{kg}\cdot\text{m}^2)(4.00\,\text{rad/s})^2 = 120.\text{J}$$

12. **Correct response: D**

Constant angular velocity implies only a non-zero centripetal component of acceleration,

$$a_c = \omega^2 r = (2.00\,\text{rev/s} \times 2\pi\,\text{rad/rev})^2(0.0400\,\text{m}) = 6.32\,\text{m/s}^2$$

13. **Correct response: D**

$$d = \sum v\Delta t = (4.00\,\text{m/s})(1.00\,\text{s}) + \tfrac{1}{2}(4.00\,\text{m/s} + 8.00\,\text{m/s})(1.00\,\text{s}) + \tfrac{1}{2}(8.00\,\text{m/s})(1.00\,\text{s}) = 14.0\,\text{m}$$

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14. **Correct response: B**

$$\frac{Gm_{star}m_{planet}}{R^2} = m_{planet}a = m_{planet}\omega^2 R = m_{planet}\frac{4\pi^2}{T^2}R \quad \Rightarrow$$

$$m_{star} = \frac{4\pi^2}{GT^2}R^3 = \frac{4\pi^2\left(8.00\times10^{11}\text{ m}\right)^3}{\left(6.673\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(30.0\text{ yr}\right)^2\left(3.156\times10^7\text{ s/yr}\right)^2} = 3.38\times10^{29}\text{ kg}$$

15. **Correct response: E**

$$\frac{F}{A} = Y\frac{\Delta L}{L} \quad \Rightarrow \quad Y = \frac{FL}{A\Delta L} = \frac{FL^2}{V\Delta L} = \frac{\left(40.0\text{N}\right)\left(2.00\text{m}\right)^2}{\left(3.00\times10^{-6}\text{ m}^3\right)\left(2.00050\text{ m} - 2.00000\text{ m}\right)} = 1.07\times10^{11}\frac{\text{N}}{\text{m}^2}$$

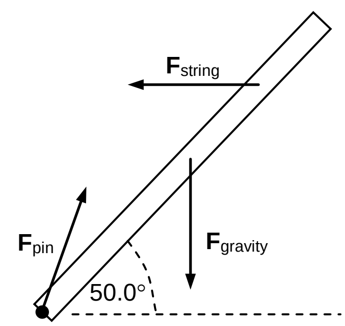
16. **Correct response: B**

Let the mass of each 1.00 m by 1.00 m piece outlined by grid lines be *M*. By symmetry, the center of mass of each of these pieces is at its geometric center.

$$x_{cm} = \frac{M\left(0.500\text{ m} + 0.500\text{ m} + 0.500\text{ m} + 1.500\text{ m} + 2.500\text{ m} + 2.500\text{ m}\right)}{6M} = 1.33\text{ m}$$

$$y_{cm} = \frac{M\left(-0.500\text{ m} + -0.500\text{ m} + -0.500\text{ m} + 0.500\text{ m} + 0.500\text{ m} + 1.500\text{ m}\right)}{6M} = 0.167\text{ m}$$

17. **Correct response: B**



The total torque about the pin must be zero:

$$\sum RF\sin\theta = 0 \quad \Rightarrow \quad R_{pin}F_{pin}\sin\theta_{pin} + R_{string}F_{string}\sin\theta_{string}CCW + R_{gravity}F_{gravity}\sin\theta_{gravity}CW = 0$$

$$\Rightarrow \quad F_{string} = \frac{-R_{pin}F_{pin}\sin\theta_{pin} - R_{gravity}F_{gravity}\sin\theta_{gravity}CW}{R_{string}F_{string}\sin\theta_{string}CCW} = \frac{0 + \left(1.00\text{ m}\right)\sin 40.0°\left(100.\text{ N}\right)CCW}{\left(1.50\text{ m}\right)\sin 50.0°\,CCW} = 55.9\text{ N}$$

18. **Correct response: D**

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = y_0 + v_0\sin\theta\cdot t + \tfrac{1}{2}a_y t^2 \quad \Rightarrow \quad t = \frac{-v_0\sin\theta \pm \sqrt{\left(v_0\sin\theta\right)^2 - 2a_y\left(y_0 - y\right)}}{a_y}$$

$$t = \frac{-\left(40.0\text{ m/s}\right)\left(\sin 50.0°\right) \pm \sqrt{\left[\left(40.0\text{ m/s}\right)\left(\sin 50.0°\right)\right]^2 - 2\left(-9.80\text{ m/s}^2\right)\left(30.0\text{ m} - 0.00\text{ m}\right)}}{-9.80\text{ m/s}^2} = 7.11\text{ s}$$

19. **Correct response: C**

$$W = \Delta K + \Delta U \quad \Rightarrow \quad \Delta U = W - \Delta K = 0 - 200.\text{ J} = -200.\text{ J}$$

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20. **Correct response: E**

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad v_{2f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2} \quad \Rightarrow$$

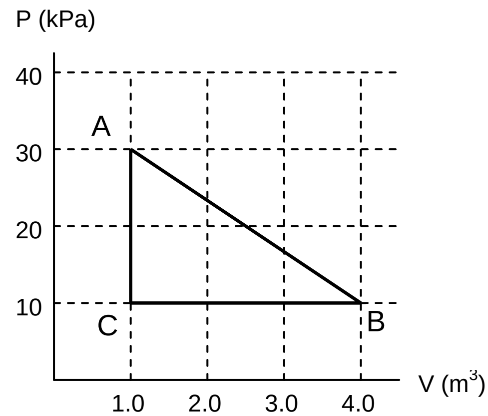
$$v_{2f} = \frac{(2.00\,\text{kg})(3.00\,\text{m/s}) + (4.00\,\text{kg})(3.00\,\text{m/s}) - (2.00\,\text{kg})(-1.00\,\text{m/s})}{(4.00\,\text{kg})} = 5.00\,\text{m/s}$$

21. **Correct response: D**

$$I_1 = I_{cm} + md_1^2 \quad and \quad I_2 = I_{cm} + md_2^2$$

$$\Rightarrow \quad I_2 = I_1 - md_1^2 + md_2^2 = 2.00\,\text{kg}\cdot\text{m}^2 - (4.00\,\text{kg})(0.400\,\text{m})^2 + (4.00\,\text{kg})(0.300\,\text{m})^2 = 1.72\,\text{kg}\cdot\text{m}^2$$

22. **Correct response: C**



$$W = W_{AB} + W_{BC} + W_{CA} = -\tfrac{1}{2}(30\,\text{kPa} + 10\,\text{kPa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) - (10\,\text{kPa})(1.0\,\text{m}^3 - 4.0\,\text{m}^3) - 0 = -30.\text{kJ}$$

23. **Correct response: A**

$$PV = nRT \quad \Rightarrow \quad \frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} \quad \Rightarrow \quad P_2 = \frac{P_1 V_1}{V_2} = \frac{P(3.00\,\text{m}^3)}{(1.00\,\text{m}^3)} = (3.00)P$$

24. **Correct response: A**

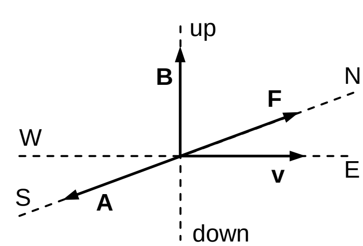
Direct application of Pascal's Principle.

25. **Correct response: C**

$$W_{\infty \to d} = -\Delta U = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{d} - \frac{1}{\infty}\right) = \frac{q^2}{4\pi\varepsilon_0 d} = W \quad \text{and} \quad W_{d \to d/2} = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{d/2} - \frac{1}{d}\right) = \frac{q^2}{4\pi\varepsilon_0 d} = W$$

26. **Correct response: A**

Applying the right-hand rule, vector **A** lies in the direction of **v** × **B**, which is south. The charge is negative, so the force, **F**, is in the opposite direction of **v** × **B**, or north.



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27. **Correct response: C**

$$EMF = -\frac{\Delta\phi}{\Delta t} = -\frac{\Delta(BA\cos\theta)}{\Delta t} = -\frac{(2.00\,\text{T} - 6.00\,\text{T})\pi(0.300\,\text{m})^2\cos 60.0°}{8.00\times10^{-3}\,\text{s}} = 70.7\,\text{V}$$

28. **Correct response: E**

The equivalent resistance of the three resistors connected to the battery is

$$R_{eq} = 400.\Omega + \frac{(300.\Omega)(600.\Omega)}{300.\Omega + 600.\Omega} = 600.\Omega$$

which implies the current from the battery is $I = V / R_{eq} = 6.00\,\text{V}/600.\Omega = 0.0100\,\text{A}$.
Applying Kirchhoff's Voltage Loop Rule to the outer loop if the circuit,

$$6.00\,\text{V} - V_{R_1} - V_{R_2} = 0 \quad\Rightarrow\quad V_{R_2} = 6.00\,\text{V} - V_{R_1} = 6.00\,\text{V} - IR_1 = 6.00\,\text{V} - (0.0100\,\text{A})(400.\Omega) = 2.00\,\text{V}$$

$$P_{R_2} = \frac{V_{R_2}^2}{R_2} = \frac{(2.00\,\text{V})^2}{300.\Omega} = 13.3\,\text{mW}$$

29. **Correct response: A**

$$V_0 = I_0 Z = I_0\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = (0.0500\,\text{A})\sqrt{(5.00\Omega)^2 + \left(\frac{1}{(800.\text{s}^{-1})(60.0\times10^{-6}\,\text{F})}\right)^2} = 1.07\,\text{V}$$

30. **Correct response: A**

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad\Rightarrow\quad f = \frac{ss'}{s+s'} = \frac{(20.0\,\text{cm})(-30.0\,\text{cm})}{(20.0\,\text{cm}) + (-30.0\,\text{cm})} = 60.0\,\text{cm}$$

31. **Correct response: B**

$$m = \frac{h'}{h} = -\frac{s'}{s} = -\frac{(-30.0\,\text{cm})}{20.0\,\text{cm}} = +1.50$$

32. **Correct response: E**

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \quad\Rightarrow\quad L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{200.\text{m}}{\sqrt{1 - \frac{(0.900c)^2}{c^2}}} = 459\,\text{m}$$

33. **Correct response: E**

$$E = h\nu = \frac{h}{T} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{10.0\,\text{s}} = 6.63\times10^{-35}\,\text{J}$$

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34. **Correct response: D**

$$N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = \left(1.00\times10^{18}\right)\left(\frac{1}{2}\right)^{2.00\text{hr}/4.00\text{hr}} = 0.707\times10^{18}$$

35. **Correct response: E**

An alpha decay removes 2 protons and 2 neutrons from the nucleus. Therefore, the atomic number is reduced by two and the atomic mass number is reduced by four.