



2013 Academic Challenge

MATHEMATICS TEST - REGIONAL

This Test Consists of 40 Questions

Mathematics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader
Linda Wiggins, Illinois State University – Author
Matthew Childers, Illinois State University – Author
Sandra Cox, Kaskaskia College – Reviewer
Mary Weaver, WYSE – Coordinator of Test Production

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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WYSE Academic Challenge
Mathematics Test (Regional) – 2013

- Given the equation $AB + BC = CD + EA$, solve for A.
 - $A = CD + E + B + BC$
 - $A = CD + E - B - BC$
 - $A = \frac{CD + E}{B + BC}, B + BC \neq 0$
 - $A = \frac{CD + BC}{B + E}, B + E \neq 0$
 - $A = \frac{CD - BC}{B - E}, B - E \neq 0$
- Veronica is registered for two eating contests, Devil's Pit 12 Hot Wings and 20 lb everything on it Hamburger. The probability that she will finish at least one of the eating contests is 0.8, that she will finish both eating contests is 0.4 and that she will fail the hot wing eating contest is 0.3. Find the probability that she will finish the 20 lb everything on it Hamburger contest.
 - 0
 - 0.4
 - 0.5
 - 0.6
 - 0.7
- Given the function $f(x) = \frac{7x^2 - 11x + 6}{3x^2 - 4}$, what is $\lim_{x \rightarrow -\infty} f(x)$?
 - 2
 - 0
 - $\frac{7}{3}$
 - 7
 - $-\infty$
- Suppose we have three consecutive odd integers, and the sum of the first two is fifteen less than three times the third. What is the sum of the three numbers?
 - 15
 - 21
 - 33
 - 39
 - no solution
- Which polar coordinate below does not represent the Cartesian coordinate $(\frac{5}{2}, \frac{5\sqrt{3}}{2})$?
 - $(5, \frac{\pi}{3})$
 - $(-5, \frac{4\pi}{3})$
 - $(-5, -\frac{2\pi}{3})$
 - $(-5, \frac{2\pi}{3})$
 - $(5, -\frac{5\pi}{3})$
- When drawing five cards simultaneously from a standard 52 card deck, how many ways are there to draw three of a kind (three cards of one denomination, one card of each of two different denominations)?
 - 858
 - 13,728
 - 54,912
 - 462,384
 - 43,490,304
- Given the sequence 5, $\frac{27}{5}$, $\frac{29}{5}$, $\frac{31}{5}$, ..., find the difference of the sum of the first 31st terms and the sum of the first 16th terms.
 - 213
 - 210
 - 196
 - 180
 - 173

8. What are the center and radius of a circle parameterized by the following equations:
 $x = 3 \cos t - 4$, $y = 3 \sin t + 8$?
- a) (-4, 8), 3 b) (4, -8), 9 c) (3, 3), 12 d) (3, 4), 8 e) (-1, 11), 24
9. Stan can walk a mile in 15 minutes. Bill can jog a mile in 10 minutes. Stan starts walking from the north end of road at 1:00 PM and heads south. Twelve minutes later (1:12 PM), Bill starts jogging from the south end of the same road and heads north. If the road is four miles long, when do they meet? Round your answer to the nearest whole minute.
- a) 1:27 PM b) 1:28 PM c) 1:31 PM d) 1:36 PM e) 1:52 PM
10. Doug, Ron and Sally put all of their pocket money together and find that they have an overall total of \$125. If it is known that Doug has 150% more cash than Ron, and Sally has 60% as much cash as Doug, how much money did Doug have in his pocket?
- a) \$80 b) \$62.50 c) \$37.50 d) \$25 e) \$12.50
11. Which of the following is the reciprocal of $2 + 3i$?
- a) $\frac{2+3i}{13}$ b) $\frac{2-3i}{13}$ c) $2 - 3i$ d) $\frac{1}{2} + \frac{1}{3i}$ e) $\frac{1}{5i}$
12. After a recent thunderstorm, a city had 4000 homes without power. Assuming the electric company can exponentially reduce the number of homes without power by half every two hours, how long should it take for the company to reduce the number of homes without power down to 10? Round your answer to the nearest tenth of an hour.
- a) 12.0 hr b) 16.8 hr c) 17.0 hr d) 17.1 hr e) 17.3 hr
13. Given the equation $3x^2 + 6x - y + 5 = 0$, Find the eccentricity e .
- a) 0 b) $\frac{3}{4}$ c) 1 d) $\frac{4}{3}$ e) $\frac{5}{3}$
14. A certain species of bacteria has the property that, under ideal laboratory conditions, the population doubles every 38 minutes. If 10 are placed in a petri dish with a nutrient broth medium, how many hours will a scientist need to wait until she has 1,000,000 bacteria available to her? Please round to the nearest hour.
- a) 5 hrs b) 11 hrs c) 148 hrs d) 161 hrs e) 192 hrs
15. If a 2 by 2 matrix is multiplied by a 2 by 3 matrix and this resulting matrix is then multiplied by a 3 by 2 matrix, describe the final resulting matrix.
- a) The matrix is a 3 by 3 matrix
 b) The matrix is a 2 by 3 matrix
 c) The matrix is a 3 by 2 matrix
 d) The matrix cannot be determined
 e) The matrix is a 2 by 2 square matrix

16. Picture three concentric circles—one with radius 3, one with radius 4, and one with radius 5. What is the total area enclosed by the union of these circles?
- a) 9π b) 10π c) 16π d) 25π e) Insufficient information.
17. If we know two of the angles of a quadrilateral are supplementary, what conclusion must be true about the quadrilateral?
- a) the quadrilateral must be a kite
 b) the quadrilateral must be a rhombus
 c) the quadrilateral must be a trapezoid
 d) the quadrilateral must be a parallelogram
 e) none of the given conclusions are guaranteed to be true
18. Two investments totaling \$18,000.00 yield an annual income of \$700.00. If the first investment has an interest rate of 5.5% and the second 3%, select the set up below that will give the value of the lower investment rate.

a)
$$\frac{\begin{vmatrix} 1 & 18000 \\ .055 & 700 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ .03 & .055 \end{vmatrix}}$$

b)
$$\frac{\begin{vmatrix} 1 & 18000 \\ .055 & 700 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ .055 & .030 \end{vmatrix}}$$

c)
$$\frac{\begin{vmatrix} 1 & 1 \\ .055 & .030 \end{vmatrix}}{\begin{vmatrix} 1 & 18000 \\ .055 & 700 \end{vmatrix}}$$

d)
$$\frac{\begin{vmatrix} 18000 & 1 \\ 700 & .030 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ .055 & .030 \end{vmatrix}}$$

e)
$$\frac{\begin{vmatrix} 18000 & 700 \\ .055 & .030 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ .055 & .030 \end{vmatrix}}$$

19. Which of the following is an alternative formula for the cotangent of an angle x in the third quadrant?

a) $\frac{\sin x}{\cos x}$

b) $-\sqrt{1 - \frac{1}{\sin^2 x}}$

c) $-\sqrt{\tan^2 x + 1}$

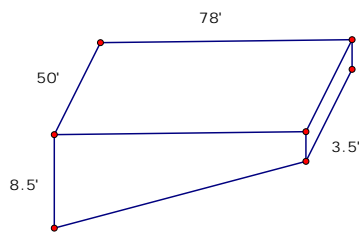
d) $\frac{1 - \sin^2 x}{\cos^2 x}$

e) $\sqrt{\frac{1}{\sin^2 x} - 1}$

20. A tetrahedron is a three dimensional figure with equilateral triangles on all four faces. If each triangular face has edges 1 inch long, what is the volume of the tetrahedron? Round your answer to two decimal places.

- a) 0.12 cu in b) 0.18 cu in c) 0.25 cu in d) 0.33 cu in e) 0.36 cu in

21. A swimming pool is 50 ft. wide, 78 ft. long, 3.5 ft deep at one end and 8.5 ft. at the other end. If the slope of the bottom of the pool is constant, how much longer is the bottom of the pool compared to the top? Round to the nearest tenth. See diagram below. [Note: diagram is not drawn to scale]



- a) 5.0 ft b) 2.2 ft c) 0.2 ft d) 0.1 ft e) 0 ft

22. Are there holes in the graph of $y = \frac{x^2 - 2x - 3}{x^3 - 4x^2 + x + 6}$? If yes, which one of the following given points is a hole?

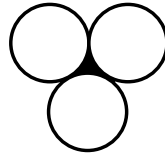
- a) No holes b) $\left(0, -\frac{1}{2}\right)$ c) $\left(-1, \frac{1}{2}\right)$ d) (2,0) e) (3,1)

23. Find the volume of a triangular piece of milk chocolate that is 20 mm thick with sides that are 30 mm, 50 mm and 60 mm. Round to the nearest whole number.

- a) 14960 mm³ b) 14967 mm³ c) 15000 mm³ d) 18000 mm³ e) 18697 mm³

24. What is the cosine of the largest angle of a right triangle whose long leg is 40 inches and whose hypotenuse is 41 inches?
- a) $\frac{41}{40}$ b) $\frac{40}{11}$ c) $\frac{7}{41}$ d) $\frac{40}{41}$ e) 0
25. For the data set {2, 5, 7, 8, 8, 9, 9, 10, 10, 10}, determine the difference between the median and the mean.
- a) 0 b) 0.5 c) 0.7 d) 1.5 e) 2.2
26. Find the length of a kite string to the nearest feet when the string makes an angle of 42 degrees with the ground when the kite is 268 feet high.
- a) 401 ft b) 361 ft c) 300 ft d) 298 ft e) 280 ft
27. There are thirteen men and twelve women vying to be on a four person committee. How many ways are there to get a committee which includes representatives of both genders?
- a) 11440 b) 12650 c) 274560 d) 303600 e) 1081575
28. An obtuse isosceles triangle has one angle that is 140 degrees, and two sides that are both 5 inches. Determine the length of the third side. Round your answer to the nearest tenth of an inch
- a) 5.4 in b) 7.1 in c) 7.5 in d) 8.2 in e) 9.4 in
29. Solve for x in the equation $4\log x - 2 = 2\log(1 - 2x)$
- a) $x = -20 - \sqrt{220}$
 b) $x = -20 + \sqrt{220}$
 c) $x = -10 \pm \sqrt{110}$
 d) $x = -10 + \sqrt{110}$
 e) no solution
30. A man drives part of a trip at 33 mph and part of that same trip at 43 mph. The trip takes twelve hours, and the man drove 416 miles in total. How long did he drive at the slower speed? (Please round your answer to the nearest hour if necessary.)
- a) 8 hrs. b) 9 hrs. c) 10 hrs. d) 11 hrs. e) All 12 hrs.
31. Completely factor $x^2 - 6xy + 9y^2 - 100$.
- a) $x(x - 6y)(3y - 10)(3y + 10)$
 b) $(x - 10)(x + 10) + 3y(3y - 2x)$
 c) $(x - 3y - 10)(x - 3y + 10)$
 d) $(x - 3y)(x + 3y) - 2(3xy - 50)$
 e) Does not factor

32. Given a kite with pairs of sides of 3 m and 4 m, rounded to the nearest square meter, what is the area of that kite?
- a) 6 m b) 8 m c) 10 m d) 12 m e) Multiple possible answers
33. Three circles, each with a one inch radius, are placed so each of them is tangent to the other as shown below. What is the area of the central shaded region? Round your answer to the nearest hundredth of a square inch.



- a) 0.10 sq in b) 0.16 sq in c) 0.25 sq in d) 0.31 sq in e) 0.43 sq in
34. Calculate the slant height s of a right circular cone for which the radius r is 11.9 cm and the volume of the cone is 1540 cm^3 . Round the slant height to the tenth of a centimeter.
- a) 15.8 cm b) 13.7 cm c) 10.4 cm d) 10 cm e) 9.7 cm
35. What is the area of the circle which is circumscribed about a rectangle with sides 6 inches and 8 inches? Please round the answer to the nearest square inch.
- a) 79 sq in b) 113 sq in c) 154 sq in d) 201 sq in e) 314 sq in
36. Six girls, Anne, Beth, Chris, Dana, Emily, and Flo, are being assigned hotel rooms for a class trip. They need to be assigned to two hotel rooms, three girls per room. In order to make sure everyone is happy, they consider the following:
- I. Anne refuses to room with Chris
 - II. Emily refuses to room with Beth
 - III. Flo will room with Anne, but only if Dana rooms with them
 - IV. Chris will room with Emily, but only if Dana rooms with them

Who will Dana end up rooming with?

- a) Anne and Beth
 b) Anne and Flo
 c) Anne and Emily
 d) Beth and Chris
 e) Chris and Emily
37. A garden has a 20 meter plastic border in the shape of a triangle with sides $\sqrt{x-1}$, $\sqrt{5x-1}$ and 10. Find x .
- a) 65 m b) 37 m c) 13 m d) 10 m e) 2 m

38. To the nearest hundredth, what is the measure of the second-smallest positive angle where the secant of the angle is seven times the cosine of that same angle?
- a) 0.14 b) 1.18 c) 1.43 d) 1.96 e) 4.32
39. Solve for x when $x^{-2} - 3x^{-1} - 40 = 0$.
- a) $x = 5$
b) $x = 8$
c) $x = -\frac{1}{8}$
d) $x = \frac{1}{5}$
e) $x = -\frac{1}{5}; x = \frac{1}{8}$
40. Eight men and seven women can make 178 test questions in two days. Ten men and eleven women can create 254 questions in that same time period. If there are twenty men and seven women, how many test questions can they create in one full week (seven days)?
- a) 149 b) 2 c) 298 d) 1,043 e) 1,575