WYSE MATH REGIONAL 2013 SOLUTIONS

1. Ans E: $AB - EA = CD - BC \Rightarrow A(B - E) = CD - BC \Rightarrow A = \frac{CD - BC}{B - E}$

2. Ans C: Let W represent the event that she finishes the hot wing eating contest and let H represent the event that she finishes the 20 lb hamburger eating contest. Since $P(W \cup H)$= 0.8 and $P(W \cap H)$= 0.4 and $P(W^c)$ = 0.3, then $P(W)$= 0.7. Then $P(W \cup H)$ = $P(W)$+ $P(H)$ - $P(W \cap H)$. Substituting in known values we have 0.8 = 0.7 + $P(H)$ - 0.4. So $P(H)$ must equal 0.5.

3. Ans C: For a rational function which has the same degree in both its numerator and denominator, there is a horizontal asymptote and it is the ratio of the leading coefficients of the numerator and denominator.

4. Ans B: Let's call the integers x, x+2, and x+4. Since x + (x+2) = 3(x+4) – 15, then x = 5. The three numbers are 5, 7, and 9, so the sum must be 21.

5. Ans D: To find the radius for the polar equivalent form we calculate $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2} = 5$. This gives us the radius of 5 units. To find θ we solve the equation $\frac{5}{2} = 5\cos\theta$. Here θ must equal $\frac{\pi}{3}$. Then one polar coordinate to match the Cartesian coordinate is $\left(5, \frac{\pi}{3}\right)$. Since (b) has a radius denoted as -5 we are looking for the polar opposite. The radian measure $\frac{4\pi}{3}$ places us in the third quadrant, but once we take the polar opposite, we are back in the first quadrant which is equivalent to solution (a). In (c) we are moving clockwise so we are in the third quadrant, but again with the negative radius measure we take the polar opposite which produces the same polar as (a). Finally, looking at (e) we see that we rotate clockwise $\frac{5\pi}{3}$ which places us into the first quadrant with the radius of 5 units. This is the same as answer (a). As we look at (d) we notice that the radians $\frac{2\pi}{3}$ places us in the second quadrant with the negative radius this moves us to the polar opposite which is the 4th quadrant, not the first. Therefore (d) is incorrect.

6. Ans C: There are C(13,1) = 13 ways to pick the denomination that shows up thrice. There are C(12, 2) = 66 ways to pick the two denominations that each show up once. Then there are C(4, 3) = 4 ways to pick the three cards of the first denomination and C(4,1) = 4 ways each to find the two other cards of the other two denominations. Thus there are C(13,1)*C(12,2)*C(4,3)*C(4,1)*C(4,1) = 54,912 ways to draw a three of a kind.

7. Ans A: Let a_n denote the nth term of the sequence and S_n represent the sum of the first n terms. The value of the 16th term is given by $5 + 15(\frac{2}{5}) = 11$. The sum of the first 16th terms is given by $\frac{16(5+11)}{2} = 128$. The value of the 31st term is given by $5 + 30\left(\frac{2}{5}\right) = 17$. The sum of the first 31st terms is given by $\frac{31(5+17)}{2} = 341$. The difference of the two sums is $341 - 128 = 213$.

8. Ans A: The above information can be rewritten as x + 4 = 3 cos t and y – 8 = 3 sin t. So we have $(x+4)^2+(y-8)^2=(3\cos t)^2+(3\sin t)^2=9\cos^2 t+9\sin^2 t=9\left(\cos^2 t+\sin^2 t\right)=9$. Thus the center is (-4, 8) and the radius is 3.

9. Ans C: Stan's rate is 4 mph, Bill's is 6 mph. Stan gets a 1/5 hour head start. If we let x be the number of hours that Bill jogs, then 4(x + 1/5) + 6x = 3, and x = 0.32 hours, which ends up being 19 minutes after 1:12. The two meet a little after 1:31 PM.

10. Ans B: Let R represent the amount of money in Ron's pocket. Then the equation that can be formed is $125=2.50R+R+.60(2.50R)\Rightarrow 125=5R\Rightarrow 25=R$. In this equation, 2.5R represents algebraically the amount of money in Doug's pocket. Replacing R with 25 we find that Doug has \$62.50 .

11. Ans B: In its initial form, this is $\frac{1}{2+3i}$. To rationalize, multiply top and bottom by the conjugate of the denominator, 2 – 3i. $\frac{2-3i}{(2-3i)(2+3i)}=\frac{2-3i}{4-9i^2}=\frac{2-3i}{4+9}=\frac{2-3i}{13}$.

12. Ans E: This population can be modeled by the function $H=4000\cdot 0.5^{\left(\frac{t}{2}\right)}$. If we let H = 10, we find that the solution for t is 17.3. For those who want to use the half-life formula $0.5=e^{kt}$, we find that if we solve $0.5=e^{k3}$ for k, we get k = $0.5\ln 0.5\approx -0.34657$. Plugging in 10 for H in $H=4000\cdot e^{-0.34657t}$ gives the same result of 17.3.

13. Ans C: Since the given equation is the graph of a parabola, the eccentricity is 1.

14. Ans B: Since we have a population that is increasing exponentially, we know that $Q(t)=Q_o e^{kt}$. We can also replace some values based on the fact that the initial quantity is 10 and the quantity after 38 minutes $\left(\frac{19}{30}\text{ hours}\right)$ have elapsed is 20. So an equation would look something like this: $20=10e^{\frac{19}{30}k}$. This tells us that $2=e^{\frac{19}{30}k}$. So $k=\frac{30\ln 2}{19}$. If we wish to solve for how long it takes to get to 1,000,000, we would then solve the following equation: $1{,}000{,}000=10e^{\frac{30\ln 2}{19}t}$. So $100{,}000=e^{\frac{30\ln 2}{19}t}$ and thus $t=\frac{19\ln 100000}{30\ln 2}$.

15. Ans E: The 2 by 2 matrix when multiplied with a 2 by 3 will result in a 2 row 3 column matrix. Then the 2 by 3 resulting matrix when multiplied with a 3 row 2 column, the final resulting matrix will have 2 rows and 2 columns making this a square matrix.

16. Ans D: Since all of the circles are contained within the largest one, the area of the union is the same as the area of the largest circle.

17. Ans E: If the two angles were at opposite ends of the same side, then we could conclude the quadrilateral must be a trapezoid, since this would create at least one pair of parallel lines. However, since that condition was not stated, we cannot conclude anything in particular about the quadrilateral beyond what was given.

18. Ans B: For this word problem, the system of equation may be written as
$$x + y = 18000$$
$$.055x + .030y = 700$$
The home determinant is $\begin{vmatrix} 1 & 1 \\ .055 & .03 \end{vmatrix}$. Replacing the second column (these values represent the values relating to the lower interest investment) of the home determinant with the values of 18000 and 700 and placing this determinant over the home determinant, we now have a proper set up for finding the value of the lower investment, $\dfrac{\begin{vmatrix} 1 & 18000 \\ .055 & 700 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ .055 & .030 \end{vmatrix}}$.

19. Ans E: First of all, the cotangent is positive in quadrant III, so we can immediately eliminate b and c. Remember the Pythagorean identity involving cotangent and cosecant is $\cot^2 x + 1 = \csc^2 x$ and thus $\cot^2 x = \csc^2 x - 1$. So $\cot x = \sqrt{\csc^2 x - 1}$ and thus $\cot x = \sqrt{\dfrac{1}{\sin^2 x} - 1}$.

20. Ans A: The tetrahedron can be thought of as a pyramid, so its volume is 1/3 base*height. The area of the base is $\frac{1}{2} \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$. The height is a little trickier. Consider a triangle that has three corners of top of the tetrahedron, one of the bottom vertices, and the centroid of the bottom face. The centroid is found by splitting the base up into six right triangles. With this, you will find that the triangle has a hypotenuse of 1, and legs that are $\frac{\sqrt{3}}{3}$, and the height. The height of the tetrahedron must be $\frac{\sqrt{6}}{3}$. Use the volume formula to get a final volume of $\frac{\sqrt{2}}{12}$ cubic inches, which is about 0.12 cu in.

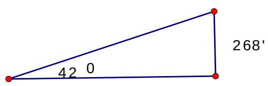
21. Ans C: Letting the bottom length represent the hypotenuse side of a right triangle with the base of the triangle 5 ft. (8.5 – 3.5), the height of the triangle measuring 78 ft. We can find the hypotenuse side by using Pythagorean theorem.
$\sqrt{78^2 + 5^2} = \sqrt{6109} = 78.16009212$. Since the top of the pool measures 78 ft. in length, the bottom is 0.16009212. Rounding to the tenths place we have 0.2.

22. Ans E: The function above can be rewritten as $y = \dfrac{(x-3)(x+1)}{(x-2)(x-3)(x+1)}$, so there are holes where x = -1, 3. If we plug those into the roughly equivalent $y = \dfrac{1}{x-2}$, we get holes at $\left(-1, -\dfrac{1}{3}\right)$ and $(3,1)$.

23. Ans B: Using Heron Formula to find the area of the triangle we find s=0.5(30+50+60)=70 and $A = \sqrt{70(70-30)(70-50)(70-60)} = 748.3314774$. For the volume we need to multiply this area by 20 (the thickness). This gives us 14966.62955. Rounding to the nearest whole millimeter we have 14967cubic mm.

24. Ans E: The cosine of any right angle is 0.

25. Ans C: the median is (8+9)/2 = 8.5, the mean is (2+5+7+8+8+9+9+10+10+10)/10=7.8. This means the difference between the two is 8.5-7.8=0.7

26. Ans A: 

The length of the string is found by calculating $\frac{268}{\sin(42°)}$. We find 400.5197154. Rounding to the nearest feet we have 401 feet.

27. Ans A: The easiest way to handle this problem is to use the total number of committees and subtract from that the number of committees which are either all male or all female. There are C(25, 4) committees in total. C(13, 4) of those are all male. C(12, 4) of those are all female. This leaves 11440 possibilities.

28. Ans E: The other two angles of the triangle must be 20 degrees. Let x be the length of the third side and use law of sines to get $\frac{\sin 20}{5} = \frac{\sin 140}{x}$. $x = 5 \cdot \frac{\sin 140}{\sin 20}$, so $x \approx 9.4$.

29. Ans D: Divide each side of the equation by 2 we have $2\log x - 1 = \log(1-2x)$. Then continuing to solve we have $\log x^2 - \log(1-2x) = 1 \Rightarrow \log\frac{x^2}{1-2x} = 1$. Rewrite in exponential form we have $\frac{x^2}{1-2x} = 10^1 \Rightarrow x^2 = 10 - 20x \Rightarrow x^2 + 20x - 10 = 0$. Using the quadratic formula we find $x = -10 \pm \sqrt{110}$. However, since we cannot have a negative value in the first term of the original equation there is only one solution; $x = -10 + \sqrt{110}$.

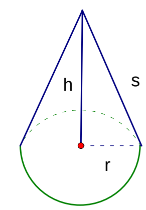
30. Ans C: This can be solved using backsolving. Otherwise, let x be the number of hours spent driving at 33 mph. Then the man drove 12 – x hours at 43 mph. Since distance is equivalent to rate times time, the total distance driven will be $33x + 43(12 - x)$ miles, which we set equal to 416. The result then follows as 10 hours spent driving at 33 mph.

31. Ans C: Since $x^2 - 6xy + 9y^2$ factors as $(x-3y)^2$, we have $(x-3y)^2$-100. We can now completely factor by using the difference of squares; $(x-3y-10)(x-3y+10)$.

32. Ans E: In order to answer this problem, we would need to know something about either the angles of the kite (and then we can use the sine formula for the areas of triangles) or the lengths of the diagonals of the kite. The extension of Heron's formula known as Brahmagupta's formula does not apply as this quadrilateral is not guaranteed to be cyclic.

33. Ans B: There are multiple ways to solve this. One method is as follows: Connect the centers of the three circles to create an equilateral triangle with sides of 2 inches each. The area of the triangle would be $\sqrt{3}$ square inches. Take a 60 degree circular wedge out of each corner, each wedge having an area of $\frac{1}{6} * \pi * 1^2$ square inches. This makes the overall area of the region $\sqrt{3} - 3 * \frac{1}{6}\pi \approx 0.16$ square inches.

34. Ans A: Using the diagram below, let *h* represent the height, *r* represent the radius and *s* represent the slant height:



The Volume is given by $V = \frac{1}{3}\pi r^2 h$. Substituting in the known values into this equation we can solve for *h*. $1540 = \frac{1}{3}\pi 11.9^2 h$. Here we find that *h* is 10.38480103. Now that we know *h* and *r*, we can use Pythagorean Theorem to find the hypotenuse, the slant height *s*. $s = \sqrt{10.38480103^2 + 11.9^2} = 15.79411575$. The slant height is 15.8 cm.

35. Ans A: By the Pythagorean theorem, each diagonal of the rectangle is 10 inches. Since each angle which is inscribed in the circle is 90 degrees, each diagonal is a diameter of the circle. Thus, the radius of the circle is 5 inches, and the area of the circle is 25π square inches.

36. Ans C: There are ten possible combinations of who Dana could room with. The best approach is process of elimination. The possibilities listed by first letter of the name are 1-DAB, 2-DAC, 3-DAE, 4-DAF, 5-DBC, 6-DBE, 7-DBF, 8-DCE, 9-DCF, and 10-DEF. The ones that would conflict with statement I are 2, 6, 7, and 10. The ones that conflict with II are 2, 4, 6, and 9. That only leaves 1, 3, 5, and 8 possible. 1 would put Chris, Emily and Flo together, which conflicts with IV. 3 has no problems yet, 5 would put Flo, Anne, and Emily together, which conflicts with III, and 8 puts Anne, Beth, and Flo together, which conflicts with III. The only legitimate one is 3.

37. Ans D: The perimeter is 20 yards, so we just add all sides together and set the sum equal to 20 and solve for *x*. $\sqrt{x-1} + \sqrt{5x-1} + 10 = 20$. Isolating the first term on the left side of the equation we have $\sqrt{x-1} = 10 - \sqrt{5x-1}$. Squaring each side we now have $x - 1 = 99 - 20\sqrt{5x-1} + 5x$. Isolating $\sqrt{5x-1}$ on the right we have $\frac{4x+100}{20} = \sqrt{5x-1}$. Squaring each side and setting the equation equal to 0 we now have $x^2 - 75x + 650 = 0$ $\Rightarrow (x-10)(x-65) = 0$. Then *x* = 10 or *x* = 65. But 65 does not work in the original equation. The only solution is 10.

38. Ans D: This scenario has the equation $\sec x = 7\cos x$ and thus $\frac{1}{\cos x} = 7\cos x$. By multiplying both sides by the cosine, we have $7\cos^2 x = 1$. Then the square of the cosine of x is equivalent to one seventh. So $\cos x = \pm\sqrt{\frac{1}{7}}$. This means that the smallest possible angle is the arccosine of the positive square root (the smallest positive first-quadrant angle which meets the above criterion), and the second smallest possible angle is the arccosine of the negative square root (the smallest positive second-quadrant angle which meets the above criterion). This means that the answer is D.

39. Ans E: Substituting x^{-1} with U, the equation becomes $U^2 - 3U - 40 = 0$. Then factoring we have $(U-8)(U+5) = 0 \Rightarrow U = 8$ or $U = -5$. Replacing U with the true value x^{-1} and solving for x we find $x = -\frac{1}{5}$; $x = \frac{1}{8}$.

40. Ans D: Let x be the number of questions that one man can create in one day and let y be the number of questions that one woman can create in one day. Then 16x + 14y = 178 and 20x + 22y = 254. If we multiply that first equation by 5 and the second one by -4, we get 80x + 70y = 890 while -80x – 88y = -1,016. Adding those together, -18y = -226, so women can create 7 questions per day. So 16x + 98 = 178, which means that 16x = 80 and men can create 5 questions per day. Thus twenty men and seven women can create 149 questions per day, and seven times that per week.