



2013 Academic Challenge

MATHEMATICS TEST - SECTIONAL

This Test Consists of 40 Questions

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

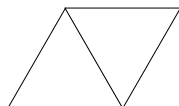
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WYSE Academic Challenge
Mathematics Test (Sectional) – 2013

1. Evaluate $\det(A)$ where $A = \begin{bmatrix} 6 & -2 & 5 & 2 & -4 \\ 3 & -1 & 4 & -5 & 9 \\ 5 & 8 & 1 & 4 & 2 \\ -18 & 6 & -15 & -6 & 12 \\ -4 & 11 & -5 & 7 & 19 \end{bmatrix}$
- a) 0 b) 8 c) 20 d) 28 e) 30
2. What is the derivative, with respect to x , of $f(x) = \frac{10x^2}{3x+1}$?
- a) $\frac{20x}{3}$
- b) $\frac{10}{3}$
- c) $20x - 3$
- d) $\frac{10x^2}{(3x+1)^2}$
- e) $\frac{30x^2 + 20x}{(3x+1)^2}$
3. Find the equation of the tangent line at the point $(\frac{3}{2}, \frac{3}{2})$ for the graph of the equation $x^3 + y^3 = 3xy$
- a) $4x - 5y + 4 = 0$
- b) $x + y - 3 = 0$
- c) $5x - 4y + 2 = 0$
- d) $x - y - 1 = 0$
- e) $3x - 2y = 0$
4. When graphed in the xy -coordinate plane, what is the shape of the graph of $x = \sin t$, $y = \cos^2 t$?
- a) Circle
- b) Single line
- c) Parabola or parabolae
- d) Hyperbola
- e) Dual lines

5. A set of ten numbers has a mean of 50. Determine what will happen to the mean if we include an eleventh data value of 60. Round your answer to the nearest tenth.
- New mean is 50.9
 - New mean is 51.0
 - New mean is 55.0
 - New mean is 55.5
 - Can't be determined without actual data values
6. Find the rectangular equation for the polar equation $r^2 = \frac{4}{1 + 3\sin^2 \theta}$
- $x^2 + y^2 = 0.81$
 - $x^2 + y^2 = 4y$
 - $x^2 + 4y^2 = 4$
 - $x^2 - y^2 = 4y$
 - $x^2 - y^2 = 0.81$
7. The decibel intensity level of a sound is given by $M = -2.5 \log \frac{B}{B_0}$, where B is the true brightness and B_0 is merely a constant. Vega has a magnitude of 0.03. The minimum magnitude of Mars is 1.84. How many times brighter than Mars is Vega? Round to the nearest whole number.
- 2 times
 - 5 times
 - 10 times
 - 53 times
 - 100 times
8. A rhombus is formed with two equilateral triangles as shown below. If the rhombus has perimeter of 10 inches, determine the area of the rhombus. Round your answer to the nearest tenth of a square inch.



- 3.1 in^2
 - 4.4 in^2
 - 5.2 in^2
 - 5.4 in^2
 - 6.3 in^2
9. Willy Wonka's chocolate waterfall mixes his chocolate. This chocolate is continuously collected in his river-like holding tank such that during a given hour only 8% as much chocolate liquid is not collected in the tank (as some is used to make candy) as the previous hour. If 36 gallons are collected in the first hour, what must be the minimum capacity of the river-like tank? Round to the nearest gallon.
- 39 gal
 - 64 gal
 - 200 gal
 - 450 gal
 - 500 gal
10. A wave's height is the difference between its crest and trough. If the heights of waves 100 feet out from shore after t minutes is given by $h(t) = 23 + 23 \sin 7\pi t$, what is the sum of the number of waves per minute and the wave height in feet? Please round to the nearest integer.
- 23
 - 27
 - 30
 - 50
 - 53

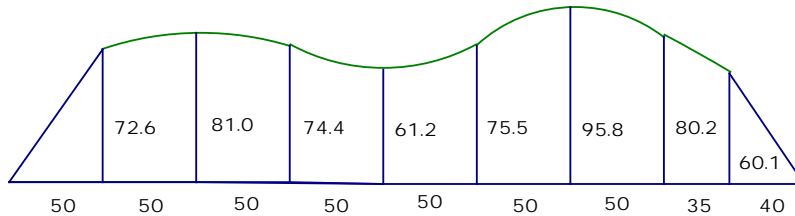
11. Find a value for a that will produce infinitely many solutions for the system below:

$$\begin{aligned} 7x + 14y - 21z &= 28 \\ 21x - 7y + 35z &= 14 \\ 28x + 7y + (7a^2 - 98)z &= 7a + 14 \end{aligned}$$

- a) -4 b) -3 c) 2 d) 3 e) 4
12. What is the radius of a circle which has an arc whose length is $\frac{216}{5}\pi$ inches and whose central angle is 216° ? Round to the nearest inch.
- a) 8" b) 36" c) 18" d) 24" e) 14"
13. Given the equation $A - \frac{B}{A+1} = 1$, solve for A.
- a) $\pm\sqrt{B+1}$
 b) $\frac{\pm\sqrt{4B+1}}{2}$
 c) $\frac{B \pm \sqrt{B^2 + 4}}{2}$
 d) $\frac{B \pm \sqrt{B^2 - 4}}{2}$
 e) No real solution
14. Determine the statement below that best describes the relationship of the graphs of $y = \log x^2$ and $y = 2\log x$.
- a) The graphs are identical on the interval $(0, \infty)$.
 b) The graphs are identical on the interval $(-\infty, \infty)$.
 c) One graph is a parabola while the other is a line.
 d) Both graphs are even functions.
 e) Both graphs are odd functions.
15. In Yahtzee, we roll five standard dice. How many ways are there to get a four of a kind? Please assume that the dice are distinguishable.
- a) 30 b) 150 c) 750 d) 1,296 e) 7,776
16. Given the equation $A^B = B$, where $B > 1$, solve for A.
- a) $A = e^{-B}$ b) $A = e^B$ c) $A = B^{-B}$ d) $A = B^B$ e) $A = B^{\frac{1}{B}}$

17. Given the equation $2x^2 + 12x - y^2 = -14$, find a focus point.
- a) $(-3 - \sqrt{2}, 0)$
 - b) $(-3 - \sqrt{6}, 0)$
 - c) $(-3 + \sqrt{2}, 0)$
 - d) $(-3, -2)$
 - e) $(-3, 2)$
18. If all phone numbers with the consecutive digits 633 are outlawed, under this law, how many 7 digit numbers are no longer available?
- a) 30
 - b) 7,290
 - c) 10,000
 - d) 49,970
 - e) 50,000
19. Simplify $2\cos^2\left(\frac{x}{2}\right) - \cos(x)$.
- a) $\sin x$
 - b) $\cos x$
 - c) 1
 - d) $1 - \sin^2 x$
 - e) $\cos(2x) - 1$
20. What is the least upper bound on the average size of an angle in a polygon?
- a) 150 degrees
 - b) 179 degrees
 - c) 180 degrees
 - d) 270 degrees
 - e) 360 degrees
21. How many vertices does an irregular tetragon have?
- a) 3
 - b) 4
 - c) 6
 - d) 8
 - e) Impossible to determine
22. The letters of the word 'ALFREDO' are arranged in all possible ways. If an arrangement is picked at random, what is the probability that all of the vowels are together?
- a) $\frac{6}{21}$
 - b) $\frac{4}{21}$
 - c) $\frac{2}{7}$
 - d) $\frac{1}{7}$
 - e) $\frac{1}{21}$
23. A soccer tournament has a round robin portion in which teams play five matches. Teams get zero points for a loss, one point for a tie and three points for a win. What is the smallest non-negative integer number of points that a team cannot possibly score?
- a) 0
 - b) 6
 - c) 14
 - d) 15
 - e) 16
24. A car travels for 120 miles at 40 mph. The car then drives another 100 miles at 50 mph. What was the average speed of the car for the 220 mile trip?
- a) 44 mph
 - b) $44.\overline{4}$ mph
 - c) $44.\overline{54}$ mph
 - d) 45 mph
 - e) Not enough information given

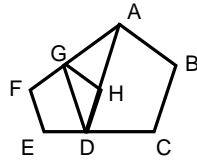
25. Mark bought a plot of land shown in the diagram below. Using the foot measures given in the diagram, Mark approximates the area using the trapezoidal rule. He plans to section the land into 2500 square feet campsites. Approximately how many campsites will Mark have on this land? [Diagram is not drawn to scale.]



- a) 15 b) 14 c) 13 d) 12 e) 11
26. The mean of 7 numbers is 8. The first five numbers are 4, 3, 8, 2 and 11, respectively. What is the average of the last two numbers? Round to the nearest whole number.
- a) 0 b) 14 c) 28 d) 32 e) 63
27. The area of a triangle is 19,100 square inches. Two sides of the triangle measure 206 inches and 187 inches. The included angle has a degree measure of 96.29. What is the measure of the remaining side of the triangle? Round to the nearest inch.
- a) 293 in. b) 278 in. c) 223 in. d) 186 in. e) 168 in.
28. What is the smallest nonnegative argument of $3 + 4i$? If necessary, round to the nearest integer.
- a) 0 b) 1 c) 3 d) 5 e) 25
29. Alan can walk a stretch of trail in 50 minutes. Bob can jog the same stretch of trail in 40 minutes, and Carl can bike the trail in 10 minutes. Suppose Alan starts walking and gets a five minute head start. Bob then starts jogging. When Bob catches up to Alan, Carl starts biking. Will Carl catch up to either Alan or Bob before they are finished with the trail? If so, who all does he catch, and how many minutes of biking does it take him? Round your answer to the nearest minute.
- a) Carl doesn't catch up to either Alan or Bob
 b) Carl catches up to Alan after biking 8 minutes, but Bob finishes the trail first
 c) Carl catches up to Alan after biking 7 minutes, and catches Bob after 8 minutes
 d) Carl catches up to Alan after biking 6 minutes, and catches Bob after 8 minutes
 e) Carl catches up to Alan after biking 6 minutes, and catches Bob after 7 minutes
30. Find the slant height of a cone that has a radius of 3 inches and a volume of 12π cubic inches .
- a) 4 in. b) 5 in. c) 6 in. d) 8 in. e) 9 in.

31. The time in seconds that it takes for a luge to go down a hill of length d feet inclined at an angle θ (in degrees) is given by $t = \sqrt{\frac{d}{16 \sin \theta}}$. Please find the angle of inclination of a 2000 foot hill which it takes 15 seconds to slide down. Round to the nearest degree.
- 15°
 - 24°
 - 34°
 - 40°
 - 44°
32. A person is playing a dice game where they toss four dice at once. As long as all of the dice have a different number facing up, the player wins. What is the probability of winning? Round to the nearest whole percent.
- 9%
 - 16%
 - 25%
 - 28%
 - 33%
33. Solve the inequality $-6x^3 + 24x^2 - 6x - 36 > 0$.
- $x < -1$
 - $x < 3$
 - $x < -1$ or $2 < x < 3$
 - $x > 3$ or $-1 < x < 2$
 - $x < -1$ or $x > 2$
34. A 320-foot rope tethers a balloon to the ground. Let the balloon have an initial height of 256 feet off the ground. If the angle of elevation to the balloon is doubled, how high is it then above the ground? Round to the nearest foot.
- 90 feet
 - 270 feet
 - 288 feet
 - 307 feet
 - 315 feet
35. At a water amusement park in Canada, a log flume lift takes a rider up the log flume incline at a rate of 50m/min. The rider then log flumes down the same incline at a rate of 150m/min. If the round trip takes 24 minutes, how long is the log flume ride?
- 300m
 - 450m
 - 600m
 - 900m
 - 1050m
36. In how many distinguishable ways can one rearrange the letters of "Chinese Remainder Theorem"? Assume that neither spaces nor capital letters are either necessary or acceptable. Round to four significant figures and put your answer in scientific notation
- 1.352×10^6
 - 5.379×10^{13}
 - 5.195×10^{14}
 - 3.740×10^{17}
 - 2.585×10^{22}

37. Pentagons ABCDG and DEFGH are both regular. If the length of DE is 1 inch, what is the length of AB? In other words, if each side of the smaller pentagon has a length of one inch, what is the length of each side of the larger pentagon? Round your answer to the nearest hundredth of an inch.



- a) 1.50 in b) 1.62 in c) 1.67 in d) 1.75 in e) 1.80 in
38. Given the equation $f = \frac{pq}{p+q}$, solve for p .
- a) $p = \frac{f}{q-f}$ b) $p = \frac{q}{f-q}$ c) $p = \frac{-fq}{q-f}$ d) $p = \frac{fq}{f-q}$ e) $p = \frac{fq}{q-f}$
39. A cable company offers people a plan in which they may take a 10% increase in rate for the first month, a 10% decrease from that the next month, and no change in the cost for the remainder of the first year. (After that, who knows?) The other plan is to keep what one paid for the previous month for the entire year. The question at hand is in which month the first plan is a better deal (if it ever is!)
- a) The first plan is always a worse deal.
 b) Both plans are equal.
 c) After six months, the first plan is better.
 d) After eleven months, the first plan is better.
 e) At the end of the year, the first plan is better.
40. Six women, Amy, Bret, Carol, Denise, Erin, and Fran, went to a garage sale, and each woman bought at least one item. Each of the following statements are also true:
- I. They bought a total of 15 items worth a total value of \$600.
 - II. All prices were different, and all were divisible by \$5
 - III. Amy and Erin each bought two items, Carol and Fran each bought three items
 - IV. Amy and Bret each spent \$75, Carol and Denise each spent \$140
 - VI. The person who spent the most money, the person who bought the most items, and the person who bought the most expensive item are all different, distinct people
 - VII. The person who spent the least money, the person who bought the fewest items, and the person who bought the least expensive item are also all different, distinct people

Who bought the most expensive item?

- a) Bret b) Carol c) Denise d) Erin e) Fran