WYSE MATH SECTIONAL 2013

1. Ans A: Since there are two proportional rows (6 -2 5 2 -4) and (-18 6 -15 -6 12) the determinant is 0.

2. Ans E: Think of $f(x)=\frac{u}{v}$, where $u=10x^2$ and $v=3x+1$. The derivative of $\frac{u}{v}$, by the product rule, is $\frac{u'v-v'u}{v^2}=\frac{(20x)(3x+1)-(3)(10x^2)}{(3x+1)^2}=\frac{30x^2+20x}{(3x+1)^2}$.

3. Ans B: Using implicit differentiation we find $3x^2+3y^2\frac{dy}{dx}=3y+3x\frac{dy}{dx}$. Solving for $\frac{dy}{dx}$ we find $3y^2\frac{dy}{dx}-3x\frac{dy}{dx}=3y-3x^2 \Rightarrow \frac{dy}{dx}\left(3y^2-3x\right)=3y-3x^2$.
$\frac{dy}{dx}=\frac{y-x^2}{y^2-x} \Rightarrow \frac{\frac{3}{2}-\left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2-\frac{3}{2}}=-1$. Then the equation of the line with slope -1 and passing through the point $\left(\frac{3}{2},\frac{3}{2}\right)$ is given by $y-\frac{3}{2}=-1\left(x-\frac{3}{2}\right)$. In general form we have $x+y-3=0$.

4. Ans C: As $x=\sin t$ and $y=\cos^2 t=1-\sin^2 t$, $y=1-x^2$. Thus, y is quadratic in terms of x and we have a parabola.

5. Ans A: The old total was 50 * 10 = 500. The new total is 560. The new mean is 560/11.

6. Ans C: Using the relationship between polar and rectangular ($r=\sqrt{x^2+y^2}$, $x=r\cos\theta$ and $y=r\sin\theta$) we substitute r and $\sin\theta$ with the rectangular equivalent. We have $x^2+y^2=\frac{4}{1+3\left(\frac{y^2}{x^2+y^2}\right)} \Rightarrow x^2+y^2=\frac{4\left(x^2+y^2\right)}{x^2+4y^2} \Rightarrow x^2+4y^2=4$.

7. Ans B: For Vega, $0.03=-2.5\log\frac{B_{Vega}}{B_0}$. So $-0.012=\log\frac{B_{Vega}}{B_0}$. Thus, $10^{-0.012}=\frac{B_{Vega}}{B_0}$ and $B_{Vega}=10^{-0.012}B_0$. For Mars, $1.84=-2.5\log\frac{B_{Mars}}{B_0}$. So $-0.736=\log\frac{B_{Mars}}{B_0}$. Thus, $10^{-0.736}=\frac{B_{Mars}}{B_0}$ and $B_{Mars}=10^{-0.736}B_0$. So the ratio between the two is given as follows: $\frac{B_{Vega}}{B_{Mars}}=\frac{10^{-0.012}B_0}{10^{-0.736}B_0}=10^{0.724}\approx 5$ times.

8. Ans D: Each side is 2.5. Height is $\sqrt{2.5^2 - 1.25^2} \approx 2.16506$. Area is 2.5*2.16506 = 5.413.

9. Ans D: $\frac{36}{.08} = 450$

10. Ans D: For a sinusoidal function of the form $y = a + b\sin kt$, let p be the period and f be the frequency. Then $f = \frac{1}{p} = \frac{1}{2\pi/k} = \frac{k}{2\pi} = \frac{7\pi}{2\pi} = 3.5$. The maximum height of the wave is 46, and the minimum is 0, so the wave's height is 46 feet.

11. Ans E: Dividing each equation by 7, we have the following augmented matrix:

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \left(a^2 - 14\right) & a + 2 \end{bmatrix}$$

Performing the operations of $-3R_1 + R_2 \Rightarrow R_2$ and $-4R_1 + R_3 \Rightarrow R_3$, we have

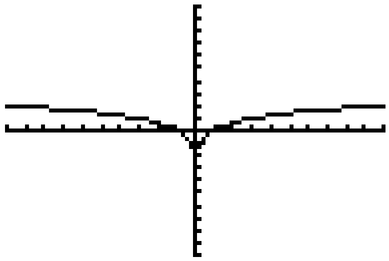
$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2 - 2 & a - 14 \end{bmatrix}$$

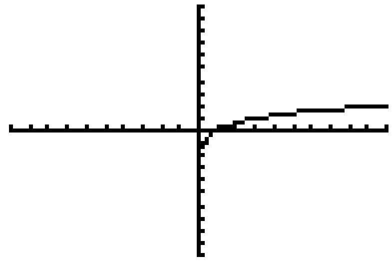
If $a^2 - 2 = 14$ and $a - 14 = -10$ to create identical rows, then $a^2 - 16 = 0$ when $a = 4$ or $a = -4$. $a - 14 = -10$ when $a = 4$. Then $a = 4$ will cause the second and third row to be identical (same line) so infinitely many solutions will occur if $a = 4$.

12. Ans B: The area of a sector of a circle is given by $s = r\theta$, where r is the radius of the circle and θ is the radian measure of the central angle. We can use the relationship between degrees and radians ($\frac{\theta_d}{180^\circ} = \frac{\theta_r}{\pi}$) to see that $\theta = \frac{216\pi}{180} = \frac{6\pi}{5}$. So, if we fill in the known quantities in our area equation, we get $\frac{216}{5}\pi = \frac{6}{5}\pi \cdot r$. If we multiply both sides by $\frac{5}{6\pi}$, we get $r = 36''$.

13. Ans A: Start by multiplying both sides by A + 1 to get $A^2 + A - B = A + 1$. Rearrange the equation to get $A^2 - B - 1 = 0$. Use quadratic formula to get $A = \frac{-(0) \pm \sqrt{(0)^2 - 4(-B - 1)}}{2}$. This can simplify down to $A = \frac{\pm\sqrt{4B + 4}}{2} = \pm\sqrt{B + 1}$

14. Ans A: The graph of $y = \log x^2$ is while the graph of $y = 2\log x$ is





The graphs are the same on the interval $(0, \infty)$.

15. Ans B: Choose a denomination of which you will roll four (there are six ways to do that) and another one which you will roll just once (there are five ways to do that). Then choose the four dice which will show the same number (there are five ways to do that). If you multiply all of those numbers together, you get 150.

16. Ans E: $A^B = B \Rightarrow \left(A^B\right)^{\frac{1}{B}} = B^{\frac{1}{B}} \Rightarrow A = B^{\frac{1}{B}}$. Using logarithms is possible, but unnecessary.

17. Ans B: Since $2x^2$ and $-y^2$ have different signs, we can rewrite the equation in standard form of a hyperbola. $2\left(x^2 + 6x + 9\right) - y^2 = -14 + 18 \Rightarrow 2\left(x+3\right)^2 - y^2 = 4$. Dividing each side by 4 we have $\frac{(x+3)^2}{2} - \frac{y^2}{4} = 1$. The center is $(-3, 0)$ and $c = \sqrt{2+4} = \sqrt{6}$. The focus points are $\left(-3 - \sqrt{6}, 0\right)$ and $\left(-3 + \sqrt{6}, 0\right)$.

18. Ans D: We can find the number of phone numbers with two 633s by making three blanks and finding the number of ways to fill two of them with 633 and the third one with one of the ten digits. There are C(3, 2) = 3 ways to fill in the 633s and then ten ways to fill in the digit slot. There are thus 30 ways to have two 633s.

To overcount the number of one 633 phone numbers, make five blanks and fill in one of them with 633 (there are five ways to do that) and the others with four digits (10,000 ways to do that), so we have at most 50,000 of these. But we have counted the dual 633 numbers twice, so there are only 49,970 numbers with 633 in them.

19. Ans C: By the half angle formula, we have $2\left(\frac{1+\cos x}{2}\right) - \cos x \Rightarrow 1 + \cos x - \cos x = 1$.

20. Ans C: The number of degrees in an n-sided polygon is $(n-2)180^\circ$, so the average number of degrees in its angles would be $\frac{(n-2)180^\circ}{n}$. Though this is always less than 180 degrees, we can see that its limit would be 180 degrees, which means that is the least upper bound.

21. Ans B: An irregular tetragon is simply another name for a quadrilateral.

22. Ans D: There are 3 vowels in the word. The vowels can be arranged 3! ways (6 ways). The possibilities regarding the four consonants and the one cluster of vowels is 5! ways (120 ways). By the counting principals we have 6*(120) = 720. There are 720 different arrangements with the 3 vowels together. The total arrangements possible for the 7 letters is 7! (5040 ways). The probability is $\frac{720}{5040} \Rightarrow \frac{1}{7}$.

23. Ans C: Teams can score 0 points by losing all of their games. Teams can score six points by winning two matches or winning one match and tying three of them. They cannot score 14 points as if they lose a match, the best they can do is twelve points—so the number of wins plus the number of ties needs to be five and three times the number of wins plus the number of ties would need to be fourteen. But if x + y = 5 and 3x + y = 14, this means that x = 4.5 and y = 0.5.

24. Ans A: The car takes 120/40 = 3 hours for the first part. The car takes 100/50 = 2 hours for the second part. The average speed is (40 * 3 + 50 * 2) / (3 + 2) = 44 mph

25. Ans E: Since the ends are triangles, the two end areas are calculated by using the area of a triangle formula. The rest of the partitions are estimated by using the trapezoidal rule. We have

$$\frac{50(72.6)}{2}+\frac{40(60.1)}{2}+50\left(\frac{72.6}{2}+81+74.4+61.2+75.5+95.8+\frac{80.2}{2}\right)+35\left(\frac{60.1+80.2}{2}\right)$$

The total area is 28687 square feet. Dividing this total by 2500 we get 11.47. Then there are only 11 campsites.

26. Ans B: The arithmetic mean of the numbers is $\frac{4+3+8+2+11+x+y}{7}=8$. So $4+3+8+2+11+x+y=56$. Thus $x+y=28$. Then the average of the two numbers is half of that 28 (which is 14).

27. Ans A: Using law of cosines we find $a=\sqrt{(187)^2+(206)^2-2(187)(206)\cos(96.29)}=292.9948884$. Rounding to the nearest inch we have 293.

28. Ans B: For a + bi, the tangent of the argument is the ratio of b to a, so we need to take the arctangent of that ratio, and then round that. (We do not need to adjust that as the arctangent of a positive number is the smallest positive number which satisfies the equation in question.) In this case, the smallest argument would be $\theta=\tan^{-1}\frac{4}{3}\approx 1$.

29. Ans E: Turn each rate value into parts of a trail per minute. Alan is 1/50 or 0.02 trail per minute, Bob is 1/40 or 0.025 trail per minute, and Carl is 1/10 or 0.1 trail per minute. In the five minute head start, Alan is done with 5 * 0.02, or 0.1 of the trail. Bob will catch up to Alan when 0.1 + 0.02t = 0.025t. Solve this to get t = 20 minutes more. At this point, both Alan and Bob have traveled (5 +20) * 0.02 = 20 * 0.025 = 0.5, or half of the trail. To see if Carl catches Alan, solve 0.5 + 0.02t = 0.1t, or t = 6.25 minutes. This is less than it would take Carl to finish the trail, so yes, he catches Alan. To see if Carl catches Bob, solve 0.5 + 0.025t = 0.1t, or t = 6.67 minutes, so yes, he catches Bob as well.

30. Ans B: Using the formula for the volume of a cone and solving for the height *h* we find $12\pi = \frac{1}{3}h\pi(3)^2 \Rightarrow 12\pi = 3h\pi$. Then *h* = 4. The slant height represents the hypotenuse of the right triangle that has legs 3inches and 4 inches long. Using Pythagorean theorem we can solve for the slant height *s*. $s = \sqrt{3^2 + 4^2} \Rightarrow \sqrt{25} = 5$.

31. Ans C: By filling in the relevant details, we have $15 = \sqrt{\frac{2000}{16\sin\theta}}$. So $225 = \frac{2000}{16\sin\theta}$. Multiplying both sides by the denominator, we have $3600\sin\theta = 2000$. Then $\sin\theta = \frac{5}{9}$ and $\theta = \sin^{-1}\frac{5}{9} \approx 34^\circ$.

32. Ans D: You need one roll, then a different one, then another different one, and then a third different one. This is 1 * 5/6 * 4/6 * 3/6 = 0.2778, or about 28%.

33. Ans C: Dividing each side by – 6 , we have $x^3 - 4x^2 + x + 6 < 0$. Factor the left we have $(x+1)(x-2)(x-3) < 0$. The critical values are – 1 , 2 , and 3. Creating a sign graph we have the following:

Interval	$(x+1)$	$(x-2)$	$(x-3)$	Sign of *f*(*x*)
$x < -1$	-	-	-	-
$-1 < x < 2$	+	-	-	+
$2 < x < 3$	+	+	-	-
$x > 3$	+	+	+	+

We want $f(x) < 0$ which occurs when $x < -1$ or $2 < x < 3$.

34. Ans D: The sine of the angle formed by the balloon and the ground is initially .8. By the Pythagorean Identity for sine and cosine, the cosine of that angle is .6. By the fact that the sine of double an angle is two times the sine of that angle times the cosine of the angle, the sine of the doubled angle is 2(.6)(.8) = .96. The hypotenuse of the newly formed triangle is 320 feet, and the side opposite the new angle is what we want to find. So, $.96 = \frac{h}{320}$. Thus h = 307.2 ft.

35. Ans D: Using $D = RT$, we find the set up $50t = 150(24 - t)$. We find that *t* = 18. The distance is given by 50(18) = 900.

36. Ans D: If all letters were distinguishable, there would be 23! arrangements. Since there are, however, 2 Hs, 2 Is, 2 Ns, 6Es, 3Rs, 2Ms, along with the singleton letters C, S, A, D, T and O, we have $\frac{23!}{2!2!2!6!3!2!} \approx 3.740 \times 10^{17}$ arrangements instead.

37. Ans B: Given the similarity of the triangular components of the pentagons, we end up with the ratio $\frac{AB}{DE} = \frac{AB + DE}{AB}$. If DE = 1, we get $AB^2 = AB + 1$. Solve to get $AB = \frac{1+\sqrt{5}}{2} \approx 1.618$. Some students may recognize this value as the Golden Ratio.

38. Ans E: $fp + fq = pq \Rightarrow fp - pq = -fq \Rightarrow p(f - q) = -fq \Rightarrow p = -\frac{fq}{f - q}$ or $p = \frac{fq}{q - f}$.

39. Ans E: For the first month, the price paid is 110% of the price at the end of the prior year. A 10% discount from that would be .9 times 1.1 = .99 = 99% of the prior cost. Thus, over the first 11 months, a consumer would pay 110% for one month and 99% for the next ten, for a total of 1100% of the prior monthly rate. (This is equivalent to what would be paid over the second plan.) In the final month, with the first plan, one would pay 99% of the prior year's rate and with the second plan, one would still be paying the prior year's rate—this puts the first plan ever so slightly ahead after a year.

40. Ans A: If read properly, the answer falls out fairly reasonably. Based on I and II, we find that the prices range from \$5 up to \$75. By I and III we find that Bret and Denise bought a total of five items. Since VI and VII say there are distinct people buying the least and most, one of the two women must buy one item, and the other four. Since Bret is paying \$75, she must buy one item, the most expensive one. From there it is possible to determine more information, but it's not necessary. With the given information, it's not possible to completely determine uniquely who bought exactly which items