2013 Academic Challenge
Sectional Physics Exam Solutions

1. **Correct response: C**

$$v^2 = v_0^2 + 2a(x - x_0) \quad \Rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(12.0\,\text{m/s})^2 - 0^2}{2(20.0\,\text{m})} = 3.60\,\text{m/s}^2$$

2. **Correct response: B**

$$x_1 = x_0 + \tfrac{1}{2}(v_1 + v_0)t_1 \quad \Rightarrow \quad t_1 = \frac{2(x_1 - x_0)}{(v_1 + v_0)} = \frac{2(20.0\,\text{m})}{(12.0\,\text{m/s} + 0\,\text{m/s})} = 3.33\,\text{s}$$

$$x_2 = x_1 + v_1 t_2 \quad \Rightarrow \quad t_2 = \frac{(x_2 - x_1)}{v_1} = \frac{(100.0\,\text{m} - 20.0\,\text{m})}{(12.0\,\text{m/s})} = 6.67\,\text{s}$$

$$t = t_1 + t_2 = 3.33\,\text{s} + 6.67\,\text{s} = 10.0\,\text{s}$$

3. **Correct response: D**

Using a coordinate system with positive x direction toward the east and positive y direction toward the north,

$$\mathbf{v}_{car\,to\,truck} = \mathbf{v}_{car\,to\,earth} + \mathbf{v}_{earth\,to\,truck} = \mathbf{v}_{car\,to\,earth} - \mathbf{v}_{truck\,to\,earth}$$

$$v_{car\,to\,truck} = \sqrt{{v_{car\,to\,truck}}_x^2 + {v_{car\,to\,truck}}_y^2} = \sqrt{(v_{car\,to\,earth\,x} - v_{truck\,to\,earth\,x})^2 + (v_{car\,to\,earth\,y} - v_{truck\,to\,earth\,y})^2}$$

$$= \sqrt{(0.0\,\text{m/s} - 15.0\,\text{m/s})^2 + (-20.0\,\text{m/s} - 0.0\,\text{m/s})^2} = 25.0\,\text{m/s}$$

4. **Correct response: A**

Using a coordinate system with positive x direction toward the east and positive y direction toward the north,

$$\mathbf{p}_{total\ final} = \mathbf{p}_{total\,initial} \quad \Rightarrow \quad (m_{car} + m_{truck})\mathbf{v}_{cm\ final} = m_{car}\mathbf{v}_{car\,initial} + m_{truck}\mathbf{v}_{truck\,initial}$$

$$\Rightarrow \quad \mathbf{v}_{cm\ final} = \frac{m_{car}\mathbf{v}_{car\,initial} + m_{truck}\mathbf{v}_{truck\,initial}}{(m_{car} + m_{truck})}$$

Finding the x component of the final velocity:

$$v_{cm\ final\ x} = \frac{m_{car}v_{car\,initial\ x} + m_{truck}v_{truck\,initial\ x}}{(m_{car} + m_{truck})} = \frac{(1500.\,\text{kg})(0.0\,\text{m/s}) + (2000.\,\text{kg})(15.0\ \text{m/s})}{(1500.\,\text{kg}) + (2000.\,\text{kg})} = 8.57\ \text{m/s}$$

Finding the y component of the final velocity:

$$v_{cm\ final\ y} = \frac{m_{car}v_{car\,initial\ y} + m_{truck}v_{truck\,initial\ y}}{(m_{car} + m_{truck})} = \frac{(1500.\,\text{kg})(-20.0\,\text{m/s}) + (2000.\,\text{kg})(0.0\ \text{m/s})}{(1500.\,\text{kg}) + (2000.\,\text{kg})} = -8.57\ \text{m/s}$$

$$v_{cm\ final} = \sqrt{(v_{cm\ final\ x})^2 + (v_{cm\ final\ y})^2} = \sqrt{(8.57\ \text{m/s})^2 + (-8.57\ \text{m/s})^2} = 12.1\,\text{m/s}$$

$\theta = \tan^{-1}\dfrac{v_{cm\ final\ y}}{v_{cm\ final\ x}} = \tan^{-1}\dfrac{-8.57\ \text{m/s}}{8.57\ \text{m/s}} = -45.0°$, which is 45.0° south of east

5. **Correct response: B**

$$\Delta U = \tfrac{1}{2}CV_f^2 - \tfrac{1}{2}CV_i^2 \quad \Rightarrow \quad C = \frac{2\Delta U}{V_f^2 - V_i^2} = \frac{2(30.0\,\text{J})}{(8.00\,\text{V})^2 - (4.00\,\text{V})^2} = 1.25\,\text{F}$$

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6. **Correct response: E**

Newton's 2nd Law for 4.00 kg mass in the horizontal direction:

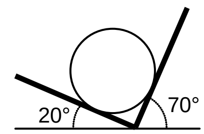
$$F_{tension} = m_{4.00\,\text{kg}} a_{4.00\,\text{kg}} = (4.00\,\text{kg})(3.00\,\text{m/s}^2) = 12.0\,\text{N}$$

Newton's 2nd Law for mass *m* in the vertical direction:

$$F_{tension} - mg = ma_m \quad \Rightarrow \quad m = \frac{F_{tension}}{g + a_m} = \frac{12.0\,\text{N}}{(9.8\,\text{m/s}^2) + (-3.00\,\text{m/s}^2)} = 1.76\,\text{kg}$$

7. **Correct response: D**

As the surfaces of the V are frictionless, the forces of those surfaces are in a direction normal to the surface. Using toward the right as the positive x direction and upward as the positive y-direction,



$$\sum \vec{F} = m\vec{a} = 0 \quad \Rightarrow \quad F_{20x} + F_{70x} = 0 \quad \Rightarrow \quad F_{20}\cos 70° - F_{70}\cos 20° = 0 \quad \Rightarrow \quad F_{70} = \frac{\cos 70.0°}{\cos 20.0°} F_{20}$$

and $F_{20y} + F_{70y} - mg = 0 \quad \Rightarrow$

$$\Rightarrow \quad F_{20}\sin 70° + F_{70}\sin 20° = F_{20}\sin 70° + \frac{\cos 70.0°}{\cos 20.0°} F_{20}\sin 20°0 - mg = 0$$

$$\Rightarrow \quad F_{20} = \frac{mg}{\sin 70.0° + \dfrac{\cos 70.0°}{\cos 20.0°}\sin 20.0°} = \frac{(2.00\,\text{kg})(9.80\,\text{m/s}^2)}{\sin 70° + \dfrac{\cos 70.0°}{\cos 20.0°}\sin 20°} = 18.4\,\text{N}$$

8. **Correct response: D**

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (50.0\,\text{rev/s})(2\pi\,\text{rad/rev})}{-2.00\,\text{rad/s}^2} = 157.\text{s}$$

9. **Correct response: A**

$$\tau = I\alpha \quad \Rightarrow \quad I = \frac{\tau}{\alpha} = \frac{0.400\,\text{N}\cdot\text{m}}{2.00\,\text{rad/s}^2} = 0.200\,\text{kg}\cdot\text{m}^2$$

10. **Correct response: D**

This is a statement of the parallel axis theorem.

11. **Correct response: E**

Using upward as the positive y-direction,

$$\sum \vec{F} = ma \quad \Rightarrow \quad F_{shroud\ lines\ y} + F_{gravity\ y} = ma \quad \Rightarrow \quad F_{shroud\ lines\ y} = ma - F_{gravity\ y} = ma + mg$$
$$= (70.0\text{kg})(2.00\,\text{m/s}^2) + (70.0\text{kg})(9.80\,\text{m/s}^2) = 826.\text{N}$$

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12. **Correct response: A**

$$\sum F_y = F_{grav\,y} + F_{normal} = -mg\cos\theta + F_{normal} = ma_y = 0 \quad \Rightarrow \quad F_{normal} = mg\cos\theta$$

On the verge of slipping:

$$\sum F_x = F_{grav\,x} + F_{friction\,x,\max} = mg\sin\theta_{\max} - \mu_s F_{normal} = mg\sin\theta_{\max} - \mu_s mg\cos\theta_{\max} = ma_x = 0$$

$$\Rightarrow \quad \frac{\sin\theta_{\max}}{\cos\theta_{\max}} = \mu_s \quad \Rightarrow \quad \theta_{\max} = \tan^{-1}\mu_s = \tan^{-1}(0.500) = 26.6^\circ$$

13. **Correct response: E**

$$\Delta K_1 + \Delta U_1 = \Delta K_2 + \Delta U_2 \quad \Rightarrow \quad \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \quad \Rightarrow$$

$$k = \frac{m\left(v_2^2 - v_1^2\right)}{\left(x_1^2 - x_2^2\right)} = \frac{(3.00\,\text{kg})\left((1.00\,\text{m/s})^2 - (1.50\,\text{m/s})^2\right)}{(0)^2 - (0.800\,\text{m})^2} = 5.86\,\text{N/m}$$

14. **Correct response: A**

This is a statement of the law of conservation of linear momentum.

15. **Correct response: C**

The heat required, Q, is the heat to raise the temperature to the melting temperature, Q_1, plus the heat to change the solid to a liquid, Q_2:

$$Q = Q_1 + Q_2 = mC\Delta T + mL_f$$

$$= (2.00\,\text{kg})(800\,\text{J/kg}\cdot{}^\circ\text{C})(40.0\,{}^\circ\text{C} - [-20.0\,{}^\circ\text{C}]) + (2.00\,\text{kg})(4.00\times10^4\,\text{J/kg}) = 1.76\times10^5\,\text{J}$$

16. **Correct response: E**

The electric field magnitude E at a distance d from a point charge q is

$$E = \frac{kq}{d^2}.$$

17. **Correct response: B**

$$A = A_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\ln(A/A_0) = \ln\left[\left(\frac{1}{2}\right)^{t/t_{1/2}}\right] = \left(\frac{t}{t_{1/2}}\right)\ln\left(\frac{1}{2}\right)$$

$$t_{1/2} = \frac{t\cdot\ln(1/2)}{\ln(A/A_0)} = \frac{(3.00\,\text{hr})\ln(1/2)}{\ln\left(3.00\times10^4/2.00\times10^6\right)} \Rightarrow \quad t_{1/2} = 0.495\,\text{hr}$$

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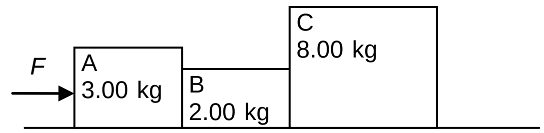
18. **Correct response: D**



$$F_{AB} = -F_{BA}; \quad F_{BC} = -F_{CB} \quad \text{(Newton's 3rd Law)}$$

$$a_{whole} = a_A = a_B = a_C \quad \text{(All the blocks move together)}$$

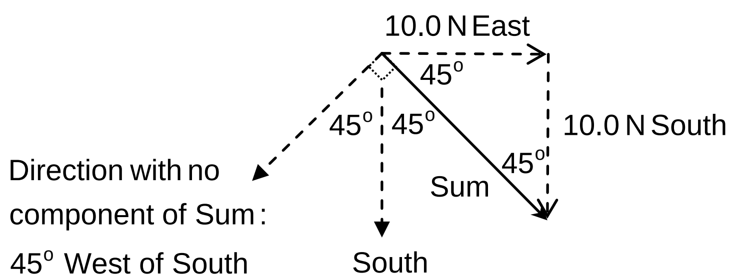
$$a = \frac{F}{13.0\text{ kg}} = \frac{F + F_{AB}}{3.00\text{ kg}} = \frac{F_{BA} + F_{BC}}{2.00\text{ kg}} = \frac{F_{CB}}{8.00\text{ kg}} \quad \text{(Newton's 2nd Law)}$$

$$\frac{F_{BA} - F_{CB}}{2.00\text{ kg}} = \frac{F_{CB}}{8.00\text{ kg}}$$

$$\frac{F_{BA}}{2.00\text{ kg}} = \frac{F_{CB}}{2.00\text{ kg}} + \frac{F_{CB}}{8.00\text{ kg}} = F_{CB}\left(\frac{1}{2.00\text{ kg}} + \frac{1}{8.00\text{ kg}}\right) = F_{CB}\left(\frac{10.0}{16.0\text{ kg}}\right)$$

$$\frac{F_{BA}}{2.00\text{ kg}} = F_{CB}\left(\frac{10.0}{16.0\text{ kg}}\right) \quad \Rightarrow \frac{F_{CB}}{F_{BA}} = \frac{16.0}{20.0} = 0.800 \quad \Rightarrow \frac{|F_{BC}|}{|F_{BA}|} = 0.800$$

19. **Correct response: B**



20. **Correct response: C**

$$F_{\text{on ball}} = m_{\text{ball}} a_{\text{ball}}$$

$$**\frac{GM_{\text{planet}} m_{\text{ball}}}{R^2} = m_{\text{ball}} a_{\text{ball}} = ** \, m_{\text{ball}}\left(\frac{v^2}{R}\right) = m_{\text{ball}}\left(\frac{(2\pi R/T)^2}{R}\right) = \frac{(4\pi^2 R) m_{\text{ball}}}{T^2}$$

$$\frac{GM_{\text{planet}}}{R^2} = \frac{(4\pi^2 R)}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_{\text{planet}}}}$$

Also, using $**$, $R = \dfrac{GM_{\text{planet}}}{v^2}$, so

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_{\text{planet}}}} = \sqrt{\frac{4\pi^2 (GM_{\text{planet}})^2}{v^6}}$$

$\therefore$ T depends upon G, R, M_{planet}, and the speed of bowling ball, but not the mass of the bowling ball.

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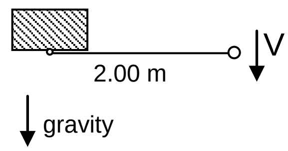
21. **Correct response: B**



$$|a_{tot}| = \sqrt{a_{centripetal}^2 + a_{vertical}^2}$$

$$2g = \sqrt{\left(\frac{v^2}{R}\right)^2 + g^2}$$

$$4g^2 = \frac{v^4}{R^2} + g^2$$

$$3g^2 = \frac{v^4}{R^2}$$

$$v = \left(3g^2R^2\right)^{1/4} = \left(3\times 9.8^2 \times 2^2\right)^{1/4}\text{ m/s} = 5.83\text{ m/s}$$

22. **Correct response: E**

$$GPE_i + KE_i = GPE_f + KE_f$$

$$0 + \frac{1}{2}(m)(7.00\text{ m/s})^2 = (m)g(-1.50\text{ m}) + \frac{1}{2}(m)v_f^2$$

$$v_f = 8.85\text{ m/s}$$

23. **Correct response: E**

$$Energy_{wave} \propto Amplitude^2$$

$$E \propto A^2$$

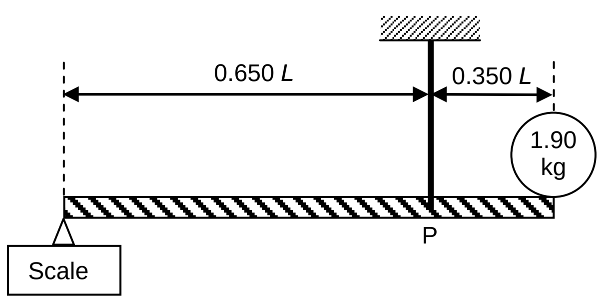
$$E_{new} = E_{original} - Heat$$

$$kA_{new}^2 = kA_{original}^2 - Heat$$

$$A_{new} = \sqrt{A_{original}^2 - Heat/k}$$

$$A_{new} < A_{original}$$

24. **Correct response: A**



$$\sum Torques_P = 0$$

$$(1.90\text{ kg})(9.80\text{ m/s}^2)(0.350\ L)CW + (0.125\ W)(0.650L)CW + (W)(0.500 - 0.350)(L)CCW = 0$$

$$(1.90\text{ kg})(9.80\text{ m/s}^2)(0.350)CW + (0.125\ W)(0.650)CW - (W)(0.500 - 0.350)CW = 0$$

$$(1.90\text{ kg})(9.80\text{ m/s}^2)(0.350) + (0.125\ W)(0.650) - (W)(0.500 - 0.350) = 0$$

$$W = 94.8\text{ N}$$

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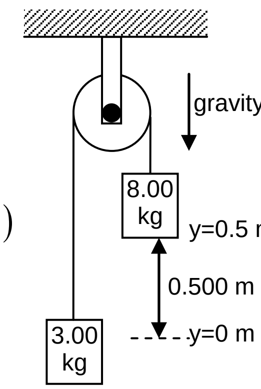
25. **Correct response: A**

$$x(t) = \left(20.000 \text{ m/s}^2\right)t^2 + \left(30.000 \text{ m/s}^3\right)t^3$$
$$x(0.100 \text{ s}) = \left(20.000 \text{ m/s}^2\right)(0.100 \text{ s})^2 + \left(30.000 \text{ m/s}^3\right)(0.100 \text{ s})^3 = 0.230 \text{ m}$$
$$x(0.00 \text{ s}) = \left(20.000 \text{ m/s}^2\right)(0.00 \text{ s})^2 + \left(30.000 \text{ m/s}^3\right)(0.000 \text{ s})^3 = 0.000 \text{ m}$$
$$v_{\text{ave}} = \frac{x(0.100 \text{ s}) - x(0.00 \text{ s})}{0.100 \text{ s} - 0.00 \text{ s}} = \frac{0.230 \text{ m} - 0.000 \text{ m}}{0.100 \text{ s} - 0.00 \text{ s}} = \frac{0.230 \text{ m}}{0.100 \text{ s}} = 2.30 \text{ m/s}$$

26. **Correct response: E**

$$f = \mu_K N = \mu_K mg$$
$$\mu_K = \frac{f}{mg} = \frac{355 \text{ N}}{(58.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)} = 0.625 \frac{\text{N}}{\text{N}} = 0.625$$

27. **Correct response: D**



$$Work_{\text{on Earth-block system}} = \Delta Energy_{\text{of Earth-block system}}$$
$$F_{friction} d \cdot \cos(\theta) = \left(KE_{8f} + KE_{3f} + GPE_{8f} + GPE_{3f}\right) - \left(KE_{8i} + KE_{3i} + GPE_{8i} + GPE_{3i}\right)$$

$$(12.5 \text{ N})(0.500 \text{ m}) \cdot \cos\left(180^o\right) =$$
$$\left(\frac{1}{2}(8.00 \text{ kg})v_f^2 + \frac{1}{2}(3.00 \text{ kg})v_f^2 + 0 + (3.00 \text{ kg})\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(0.500 \text{ m})\right)$$
$$-\left(\frac{1}{2}(8.00 \text{ kg})0^2 + \frac{1}{2}(3.00 \text{ kg})0^2 + (8.00 \text{ kg})\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(0.500 \text{ m}) + (3.00 \text{ kg})\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(0 \text{ m})\right)$$

$$-6.25 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = (5.50 \text{ kg})v_f^2 + 14.7 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} - 39.2 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$
$$v_f = 1.82 \text{ m/s}$$

28. **Correct response: C**

$$L = I\omega \quad \Rightarrow \omega = \frac{L}{I}$$
$$KE = \frac{1}{2} I\omega^2 = \frac{1}{2} I \left(\frac{L}{I}\right)^2 = \frac{L^2}{2I}$$
$$I = \frac{L^2}{2 \cdot KE} = \frac{\left(4.81 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}\right)^2}{2 \cdot \left(1.53 \times 10^5 \text{ J}\right)} = 75.6 \text{ kg} \cdot \text{m}^2$$

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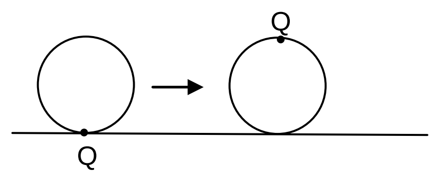
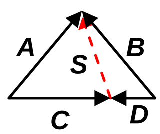
29. **Correct response: C**



$$d=\sqrt{d_x^2+d_y^2}=\sqrt{(\pi R)^2+(2R)^2}=\sqrt{(45.2\pi)^2+(2\cdot 45.2)^2}\text{ cm}=168\text{ cm}$$

30. **Correct response: D**

$$\mathbf{-C+A=S=-D+B}$$
$$\mathbf{A+-C=S=-D+B}$$



31. **Correct response: B**

$$v=f\lambda$$
$$72.1\text{m/s}=(110.\text{ Hz})\lambda$$
$$\lambda=\frac{72.1\text{m/s}}{110.\text{ /s}}$$
$$L=\frac{\lambda}{2}\qquad\text{(for the fundamental mode)}$$
$$L=\frac{1}{2}\left(\frac{72.1\text{m}}{110.}\right)=0.328\text{ m}$$

32. **Correct response: A**

$$B.F.=W_{\text{displaced fluid}}=\rho_{\text{fluid}}V_{\text{cannonball}}g$$
$$234\text{ N}=(1.00\times 10^3\text{ kg/m}^3)(V_{\text{cannonball}})(9.80\text{ m/s}^2)$$
$$234\text{ kg}\cdot\text{m/s}^2=(1.00\times 10^3\text{ kg/m}^3)(V_{\text{cannonball}})(9.80\text{ m/s}^2)$$
$$V_{\text{cannonball}}=0.0239\text{ m}^3$$

33. **Correct response: A**

$$P=\frac{\Delta Q}{\Delta t}=mc_w\frac{\Delta T}{\Delta t}$$
$$150.\frac{\text{J}}{\text{s}}=(185\text{ g})\left(4.19\frac{\text{J}}{\text{g}\cdot\text{C}^{\text{o}}}\right)\left(\frac{100\,^{\text{o}}\text{C}-23.6\,^{\text{o}}\text{C}}{\Delta t}\right)$$
$$\Delta t=395\text{ s}$$

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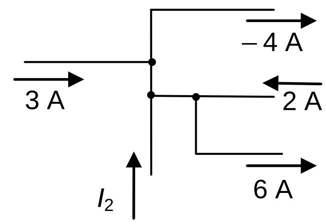
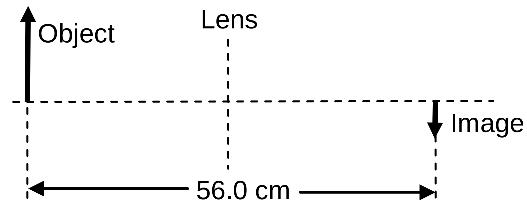
34. **Correct response: D**



A denotes amps.

$$\sum I_{\text{(inward reference=positive)}} = 0$$
$$+(3\text{ A})+(I_2)-(6\text{ A})+(2\text{ A})-(-4\text{ A})=0$$
$$I_2 = -3\text{ A}$$

35. **Correct response: E**



$$\frac{d_o}{d_i} = -\frac{h_o}{h_i} = -\left(\frac{1}{-0.400}\right) = \frac{5}{2}$$
$$d_o = \frac{5}{2} d_i$$
$$d_i + d_o = 56.0\text{ cm}$$
$$d_i = 56.0\text{ cm} - d_o$$
$$d_o = \frac{5}{2} d_i = \frac{5}{2}(56.0\text{ cm} - d_o)$$
$$d_o(1+5/2) = \frac{5}{2}(56.0\text{ cm})$$
$$d_o = 40.0\text{ cm}$$