



2013 Academic Challenge

MATHEMATICS TEST - STATE FINAL

This Test Consists of 40 Questions

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

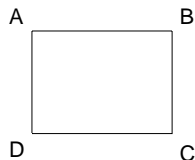
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WYSE Academic Challenge
Mathematics Test (State Final) – 2013

1. How many solid, non-reconstituted cubes with side length 2" can be cut from a brick with dimensions 7" by 11" by 3"?
a) 8 b) 15 c) 29 d) 30 e) 60
2. A committee consisting of two members is to be picked from the following: Mark, Mary, Luke, Lucy, and Bob. Find the probability that Lucy is on the committee.
a) 0.5 b) 0.4 c) 0.3 d) 0.2 e) 0.1
3. Two tangents hit a circle so that the intercepted minor arc has a measure of 110 degrees. What is the degree measure of the smaller angle created by the intersection of the tangents?
a) 27.5° b) 55° c) 70° d) 110° e) 125°
4. In the given logarithmic equation $t = \frac{-\ln\left(1 - \frac{5i}{3}\right)}{10}$, solve for i .
a) $i = 0.6(1 - e^{-10t})$
b) $i = 0.6e^{-10t} + 1$
c) $i = 0.60e^{10t} + 0.60$
d) $i = -0.60e^{10t} - 0.60$
e) $i = 0.6(1 + e^{10t})$
5. Determine the first derivative of the function $f(x) = \frac{\sin x}{x^2}$.
a) $f'(x) = \frac{\cos x}{2x}$
b) $f'(x) = \frac{2x - \cos x}{x^4}$
c) $f'(x) = \frac{\cos x - 2x}{x^4}$
d) $f'(x) = \frac{2 \sin x - x \cos x}{x^3}$
e) $f'(x) = \frac{x \cos x - 2 \sin x}{x^3}$
6. A farmer purchases 1000 linear feet of fencing and wishes to use it to create seven pens lined up next to one another. What is the maximum area for each pen? Round to the nearest square foot.
a) 1116 ft^2 b) 1276 ft^2 c) 2232 ft^2 d) 2551 ft^2 e) 3348 ft^2

7. Reduce the expression $4 \sin 3x \cos(3x - \pi) - 4 \cos 3x \sin(3x - \pi)$
- a) 1 b) 0 c) $\sin(3x - \pi)$ d) $\cos(3x - \pi)$ e) $\sin(6x + \pi)$
8. The golden rectangle shown below is created so that its sides satisfy the property $AB = \left(\frac{1 + \sqrt{5}}{2}\right)AD$. If the perimeter of the rectangle is 20 inches, what is its area? Round your answer to the nearest tenth of a square inch. Please note that the drawing isn't perfectly to scale.



- a) 23.6 in^2 b) 24.0 in^2 c) 25.0 in^2 d) 38.2 in^2 e) 40.5 in^2
9. If we know only three of the six possible measures of a triangle (three angles and three sides), which of the following sets of known quantities is the only one for which we can find two possible triangles?
- a) Two sides and the angle between them.
 b) Two angles and the side between them.
 c) All three angles
 d) Two sides and an angle not between them.
 e) All three sides.
10. Given the functions $f(x) = x - 3$ and $g(x) = x^2 + 4$ find $[(g \circ f) \circ f](x)$
- a) $x^2 - 6x + 10$
 b) $x^2 - 12x + 40$
 c) $x^2 - 6x - 10$
 d) $x^3 - 3x^2 - 12$
 e) $x^3 - 3x^2 + 4x - 12$
11. Which of the following is a normalized unit version of $\langle 8, 3 \rangle$?
- a) $\sqrt{73}$
 b) $\sqrt{55}$
 c) $\langle 8\sqrt{55}, 3\sqrt{55} \rangle$
 d) $\left\langle \frac{8}{\sqrt{73}}, \frac{3\sqrt{73}}{73} \right\rangle$
 e) 11

12. A man had a barrel full of pure lemon juice and a 3 gallon jug. One day he drew off a jug full of lemon juice from the barrel and filled up the barrel with water. Later when the lemon juice and water got thoroughly mixed he drew off another jug full and again filled up the barrel with water. The barrel then contained 50% lemon juice. What was the capacity of the barrel? Round to the nearest gallon.
- a) 12 gal b) 11 gal c) 10 gal d) 9 gal e) 8 gal
13. Each question on a multiple choice test has responses A through E. Assuming all possible answer keys are equally likely, what is the probability that a randomly chosen block of four consecutive questions all have the same letter answer? Round your answer to four decimal places.
- a) 0.0016 b) 0.0064 c) 0.0080 d) 0.0400 e) 0.2000
14. A famous songwriter plagiarizes songs at a rate of 5 per year and writes his own 11 times per year. If we have five of his average years of work in front of us, what is the probability that the first five songs we randomly select without replacement are all plagiarized? Round to four decimal places.
- a) .0022 b) .0030 c) .0095 d) .1447 e) .3125
15. Given $f(x) = \frac{6x^3 + 7x^2 - 7x - 6}{2x^2 + x - 3}$, find the range.
- a) $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$
 b) $\left(-\infty, \frac{-3}{2}\right) \cup \left(\frac{-3}{2}, 1\right) \cup (1, \infty)$
 c) $\left(-\infty, \frac{-5}{2}\right) \cup \left(\frac{-5}{2}, 5\right) \cup (5, \infty)$
 d) $\left(-\infty, \frac{-5}{2}\right) \cup \left(\frac{-5}{2}, \infty\right)$
 e) $(-\infty, \infty)$
16. Given the equation $\frac{A+B}{B+C} = B$, solve for B.
- a) $B = \frac{A}{C}$
 b) $B = \frac{A+1}{C+1}$
 c) $B = \frac{C \pm \sqrt{C^2 + 4A}}{2}$
 d) $B = \frac{1-C \pm \sqrt{C^2 + 4A - 1}}{2}$
 e) $B = \frac{1-C \pm \sqrt{C^2 + 4A - 2C + 1}}{2}$

17. A pair of a certain strain of guppies can, every month, make six pairs of guppies that survive for a month, at which point they are mature, which means they will make six pairs next month. How many pairs of guppies will there be after six months if we start with six mature pairs?
- a) 504 b) 798 c) 4788 d) 7566 e) 46656
18. A professional free style ice skater goes into a spin making .80 as many revolutions in a second as the previous second. If the ice skater makes 7.5 revolutions coming to a rest, how many revolutions does the skater make in the first second?
- a) 1.5 r b) 2.0 r c) 4.5 r d) 6.0 r e) 7.5 r
19. An IQ test has that its scores are normally distributed with a mean of 100 and a standard deviation of 16. What is the sum of the scores which represent the 5th and 95th percentiles? Please round to the nearest whole number.
- a) 147 b) 175 c) 200 d) 225 e) 253

20. Reduce matrix $A = \begin{pmatrix} 2 & 4 & -6 & 0 \\ 4 & 8 & -4 & 4 \\ 6 & 12 & -8 & 6 \end{pmatrix}$ into echelon form.

a) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 4 & -6 & 0 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

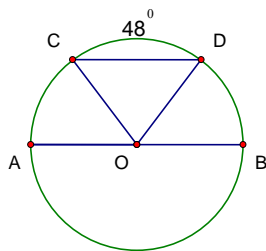
d) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 3 & 6 & -4 & 3 \end{pmatrix}$

e) $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 5 & 3 \end{pmatrix}$

21. How many faces does a regular square antiprism have?
- a) 5 b) 6 c) 8 d) 10 e) 12

22. If I randomly choose a number from 1 to 52, what is the probability that i raised to the power of that number is a real number?
- a) 0 b) $\frac{1}{4}$ c) $\frac{7}{26}$ d) $\frac{17}{52}$ e) $\frac{1}{2}$
23. A sandbox is to be constructed in the shape of an isosceles trapezoid where the equal sides of the trapezoid must be 2 meters long with a height of $\sqrt{3}$ meters. What should be the difference in length of the parallel sides of the trapezoid?
- a) 7 m b) 4 m c) $\sqrt{7}$ m d) $2\sqrt{3}$ m e) 2 m
24. Tom and Bill need to dig a hole that will be one thousand cubic feet. At noon, Tom starts digging the hole at 6 cubic feet per minute. At 12:30, Bill joins in digging the hole at a rate of 10 cubic feet per minute. When they are halfway done with the hole, they have to slow down to prevent the hole from collapsing. Tom reduces his speed to 4 cubic feet per minute, and Bill reduces his speed to 6 cubic feet per minute. At what time do they finish digging the hole? Round your time to the nearest minute.
- a) 1:17 PM b) 1:33 PM c) 1:40 PM d) 2:34 PM e) 2:50 PM
25. If I told you that the cosine of an angle was 0.7 and the sine of said angle is 0.6, which of the following could you tell me about the measure of the angle?
- a) Its measure could be $\cos^{-1} 0.7 + \frac{\pi}{3}$.
- b) Its measure could be $\sin^{-1} 0.6 + \frac{\pi}{2}$.
- c) Its measure could be $\cos^{-1} 0.7 + \frac{\pi}{2}$.
- d) Its measure could be $\tan^{-1} \frac{6}{7} + \frac{\pi}{2}$.
- e) The angle does not exist.

26. Using the diagram below, find the measure of angle BAD given that $\overline{AB} \parallel \overline{CD}$. [The diagram below is not drawn to scale]



- a) 72° b) 66° c) 48° d) 33° e) 24°

27. In which of the following quadrants can an angle be found if both its sine and cosecant are positive?

- a) I b) II c) III d) All four e) I and II

28. Given the equation $16(x-3)^2 + n(y+3)^2 = 16n$, find n so that the equation is a graph of an ellipse with a focus point at $(3, 0)$.

- a) $n = 4$ b) $n = 7$ c) $n = 16$ d) $n = 25$ e) $n = 36$

29. Given the equation $\frac{A}{1 + Be^{-kx}} = P$, solve for x .

a) $x = \frac{\ln\left(\frac{A-P}{B}\right)}{-k}$

b) $x = \frac{\ln\left(\frac{A-P}{BP}\right)}{-k}$

c) $x = \frac{\ln\left(\frac{A-P-1}{B}\right)}{-k}$

d) $x = \ln\left(\frac{P-A}{BPk}\right)$

e) $x = \ln\left(\frac{P+1-A}{Bk}\right)$

30. Together, $x = \frac{3}{1 + \tan^2 t}$ and $y = \sin t$ describe a curve. Which of the following is a solution for x in terms of y ?

a) $x = \frac{1}{3y^2}$

b) $x = 3 - 3y^2$

c) $x = 3 + \frac{3 + 3y^2}{y^2}$

d) $x = \frac{1 + y^2}{3 - 3y^2}$

e) $x = \frac{3 + 3y^2}{2y^2}$

31. Chuck is blowing up a spherical balloon whose radius is increasing by 1 inch every two seconds. How fast is the volume of the balloon increasing when the radius of the balloon is 3 inches?
- $18\pi \text{ in}^3/\text{sec}$
 - $12\pi \text{ in}^3/\text{sec}$
 - $4\pi \text{ in}^3/\text{sec}$
 - $6\pi \text{ in}^3/\text{sec}$
 - $3\pi \text{ in}^3/\text{sec}$
32. What is the measure of each vertex angle of a regular dodecagon?
- 135°
 - 140°
 - 144°
 - 150°
 - 157.5°
33. Solve $|3 - 5x| \leq |6x|$. Please put your answer in interval notation.
- \emptyset
 - $\left(-\infty, \frac{3}{11}\right]$
 - $(-\infty, -3] \cup \left[\frac{3}{11}, \infty\right)$
 - $\left[-3, \frac{3}{11}\right]$
 - $(-\infty, \infty)$
34. Convert $4x^2 + 4y^2 - x + y = 0$ to polar form.
- $r = \frac{\sin t + \cos t}{4}$
 - $r = -\frac{4}{\cos t + \sin t}$
 - $r = \frac{8}{4 + 2\cos t}$
 - $r = \frac{8}{4 - 2\cos t}$
 - $r = \frac{\cos t - \sin t}{4}$
35. A Crude Goldberg contraption includes a target, which, when hit with a golf ball, will turn a light bulb one quarter revolution. The bulb takes $66\frac{1}{8}$ revolutions to go from totally unscrewed to lit. The target is set so that John Daly only hits it one of every six times. If Daly stays consistent at this rate, what's the minimum number of sleeves of four balls he'll need to light the lamp?
- 382
 - 396
 - 397
 - 398
 - 1585

36. Find $\int 6x^2 + 3e^x dx$
- $2x^3 + e^{3x} + c$
 - $2x^3 + 3e^x + c$
 - $\frac{3}{2}x^3 + e^{3x} + c$
 - $\frac{2}{3}x^3 + e^{3x} + c$
 - $3x^3 + 3e^x + c$
37. Given the equation $\sum_{i=0}^x (i+1) = c$, solve for x.
- $x = -\frac{1}{2} \pm \sqrt{2c - \frac{7}{4}}$
 - $x = -\frac{1}{2} \pm \sqrt{2c + \frac{1}{4}}$
 - $x = -\frac{3}{2} \pm \sqrt{2c + \frac{9}{4}}$
 - $x = -\frac{3}{2} \pm \sqrt{2c + \frac{1}{4}}$
 - No possible real solution
38. The derivative of a function is $9e^x$. If that function has a y-intercept at the origin, what is that value of that function when $x = 3$? Round your answer to the nearest integer.
- 19
 - 91
 - 172
 - 181
 - 1811
39. Find the area of the region enclosed by $y = 2x - x^2$ and $y = x^2 - 4x + 4$.
- $\frac{25}{3}$
 - $\frac{11}{3}$
 - $\frac{5}{3}$
 - $\frac{1}{3}$
 - $-\frac{11}{3}$
40. Four biology students, Amy (f), Bill (m), Charles (m), and Dana (f), recently gave presentations on animals. The presentations were at 9:00, 9:10, 9:20, and 9:30. The following facts are also true:
- Dana went right after one boy, and right before the other.
 - The presentation on mammals was right after Bill's presentation
 - A boy presented on birds right before a girl presented on fish.
 - Amy went right after the presentation on reptiles
- Based on these facts, who presented third (at 9:20), and what was their topic?
- Bill, reptiles
 - Charles, reptiles
 - Charles, birds
 - Dana, fish
 - Dana, mammals