WYSE MATH STATE 2013 SOLUTION SET

1. Ans B: We may as well cut the brick down to 6" by 10" by 2" because those extra inches cannot be used in making cubes. Then the new brick has volume 120 cubic inches. Since the cubes each have volume 8 cubic inches, 15 of them can fit in the smaller brick.

2. Ans B: There are 4 ways to pair Lucy with another member. There are 10 possible ways to pick a committee of two (${}_5C_2 = \frac{5!}{2(5-2)!} = 10$). The probability is then $\frac{4}{10}$.

3. Ans C: The major arc created by the tangents then must have a measure of 250 degrees by the computation 360 – 110 = 250. The difference between the two arc measures is 140 degrees, and half that difference is the measure of the included angle.

4. Ans A: $-10t = \ln\left(1-\frac{5i}{3}\right) \Rightarrow e^{-10t} = 1-\frac{5i}{3} \Rightarrow \frac{5i}{3} = 1-e^{-10t} \Rightarrow i = 0.6\left(1-e^{-10t}\right)$.

5. Ans E: Use the quotient rule to get $f'(x) = \frac{x^2(\cos x) - \sin x \cdot (2x)}{\left(x^2\right)^2} = \frac{x\cos x - 2\sin x}{x^3}$.

6. Ans C: Let x represent the total length of the seven pens when set next to one another and let y represent the width of each pen. There are eight width pieces and two length pieces, so 2x + 8y = 1000. Solving for x in terms of y, we get x = -4y + 500. So the area of the pens in terms of y is $A(y) = -4y^2 + 500y$. Because this is a quadratic function with negative leading coefficient, we know that the way to maximize the function is to set the derivative equal to 0, solve for y, and plug that back in to get the largest possible area. So -8y + 500 = 0, which means that 8y = 500, so to maximize the area, y must be 62.5 and x must thus be 250. Then the maximum area is 15,625 square feet and the area of each of the seven pens is one seventh of that, or approximately 2,232 square feet.

7. Ans B: $4\left(\sin 3x \cos(3x-\pi) - \cos 3x \sin(3x-\pi)\right)$. Letting $\alpha = 3x$ and $\beta = 3x - \pi$, we can see that the expression represents $4(\sin\alpha\cos\beta - \cos\alpha\sin\beta) \Rightarrow 4\sin(\alpha-\beta)$. Replace the true values for α and β. We have $4\sin\left(3x-(3x-\pi)\right) \Rightarrow 4\sin\pi \Rightarrow$ 4 * 0 = 0

8. Ans A: Abbreviate $\frac{1+\sqrt{5}}{2} = \Phi$, so $AB = \Phi AD$. Perimeter equals $2AB + 2AD = 20$, so $AD = \frac{10}{1+\Phi}$. Area equals $AB * AD = \Phi AD^2 = \Phi\left(\frac{10}{1+\Phi}\right)^2 = \left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{10}{1+\left(\frac{1+\sqrt{5}}{2}\right)}\right)^2 \approx 23.61$.

9. Ans D: The Side-Side-Angle case is sometimes known as the ambiguous case because there is a chance that we may find two different triangles with those sides and angle.

10. Ans B: $(g\circ f)(x)=(x-3)^2+4 \Rightarrow x^2-6x+13$. Then $[(g\circ f)\circ f](x)$ $=(x-3)^2-6(x-3)+13$. Which simplifies to $x^2-12x+40$.

11. Ans D: A vector is normalized when it is multiplied by the reciprocal of its magnitude. The magnitude in this case is $\sqrt{8^2+3^2}=\sqrt{73}$.

12. Ans C: First we need to find the percent of lemon juice after the 3 gallons of water is added to the barrel. Let B represent the number of barrels and P, the percent lemon juice. We have $100B-300=PB$. Then $P=\frac{100B-300}{B}$. For the second jug full of water mix we now have the following equation: $\left(\frac{100B-300}{B}\right)B-3\left(\frac{100B-300}{B}\right)=50B$. Solving for B we have $B^2-12B+18=0$. Using the quadratic formula, we find that $B=6+3\sqrt{2}$ or $B=6-3\sqrt{2}$. However $6-3\sqrt{2}=1.757359313$. This is less than the 3 gallon jug and does not make sense for the task at hand. The solution is $6+3\sqrt{2}=10.24264069$. Rounding to the nearest gallon we have 10 gallons.

13. Ans C: The first question can have any response, but we then want the second, third, and fourth to have the same response. The probability is (5*1*1*1)/(5*5*5*5)=0.008.

14. Ans A: There are 25 plagiarized songs and 80 total songs (as there are 16 total songs per year for five years). Thus the probability of pulling five plagiarized songs from the arrangement of music would be $\frac{25}{80}\frac{24}{79}\frac{23}{78}\frac{22}{77}\frac{21}{76}\approx .0022$

15. Ans C: The graph has a hole at both $(1,5)$ and $\left(\frac{-3}{2},\frac{-5}{2}\right)$.

16. Ans E: $(B+C)\left(\frac{A+B}{B+C}\right)=B(B+C) \Rightarrow A+B=B^2+BC \Rightarrow 0=B^2+BC-B-A$

$$\Rightarrow 0=B^2+(C-1)B-A\ ,\ B=\frac{-(C-1)\pm\sqrt{(C-1)^2-4\cdot 1\cdot(-A)}}{2\cdot 1}=\frac{1-C\pm\sqrt{C^2+4A-2C+1}}{2}$$

17. Ans D: After the first month, there will be 6 mature pairs and 36 immature pairs. After the second month, we will have 42 mature pairs and the 6 pairs that were already mature will have produced 36 pairs. The following table may make this easier:

Month	Mature Pairs	Immature Pairs (babies)
0	6	0
1	6	36
2	42	36
3	78	252
4	330	468
5	798	1980
6	2778	4788

There are thus 2778 + 4788 = 7566 pairs after six months.

18. Ans A: $7.5 = \frac{a}{1-.8} \Rightarrow a = 1.5$

19. Ans C: Since the 5th and 95th percentiles are equally far away from the mean (by virtue of the distribution of IQ scores being normal), their average is the mean, so their sum would be double the mean.

20. Ans C: Apply row operations $-2R_1 + R_2 \rightarrow R_2$. We now have $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 3 & 6 & -4 & 3 \end{pmatrix}$. Next, apply row operations $-3R_1 + R_3 \rightarrow R_3$. This gives us $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 5 & 3 \end{pmatrix}$. Finally, apply row operations $-5R_2 + 4R_3 \rightarrow R_3$. We have $\begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$. You may also logically deduce that (a) and (b) are impossible as they indicate that one row is a multiple of another by the row of zeros. (d) and (e) aren't in echelon form. This leaves just (c).

21. Ans D: A square antiprism is two squares connected by eight alternating equilateral triangles. This means the figure has a total of 10 faces.

22. Ans E: The sequence of powers of i goes as follows: i, -1, -i, 1, i, ..., which means that two out of every four powers of i is actually a real number. Since 52 is a perfect multiple of 4, that means that exactly half of those powers of i turns out to be a real number.

23. Ans E: Note that an isosceles trapezoid is made up of two equal right triangles and 1 rectangle. Taking out the middle piece, the rectangle, we see that the base of the right triangles together make up the difference in length of the two parallel sides. Using Pythagorean theorem we can solve for the base of the triangle. Here 2 represents the length of the hypotenuse and $\sqrt{3}$ represents the height. Then $2^2 - (\sqrt{3})^2 \Rightarrow 1$. The base of the right triangle must be 1 meter. Both triangles together gives the total difference of 2 meters.

24. Ans C: Using the formula amount = rate * time. By 12:30, Tom has dug 6*30 = 180 cubic feet. They have 820 cubic feet left. At the first speeds, solve 6t + 10t = 320. This takes them twenty more minutes, so its 12:50. At the second speed, solve 4t + 6t = 500. This takes another 50 minutes, so they are done at 1:40 PM.

25. Ans E: For every angle, the sum of the squares of the sine and cosine must be 1. In this case, the sum of the squares of the sine and cosine is 0.85, so this is a nonexistent angle.

26. Ans D: First, since triangle COD has equal sides OC and OD, their subtending angles are congruent. So angle DCO and angle CDO are congruent. We know that angle COD is 48 degrees. This means that the remaining congruent angles of the triangle must be 66 degrees $(\frac{180-48}{2})$. Since AB is parallel to CD, then the transversal OD forms congruent alternate interior angles. Angle CDO is congruent to angle BOD. So the central angle BOD is 66 degrees. This means arc BD is 66 degrees. Angle BAD is half the central angle BOD. Angle BAD must be 33 degrees.

27. Ans E: The sine and cosecant always have the same sign, so we only need to know where the sine is positive—quadrants I and II.

28. Ans B: Dividing the equation by 16n, we have the standard form of an ellipse. $\frac{(x-3)^2}{n}+\frac{(y+3)^2}{16}=1$. Here the center is (3, - 3). Since the y value changed in the given focus point, the major axis is vertical. $a^2=16$ and $b^2=n$, - 3 + c = 0. C must be 3. Then the formula $c^2=a^2-b^2$ gives us $9=16-n \Rightarrow n=7$.

29. Ans B: $A=P\left(1+Be^{-kx}\right) \Rightarrow A=P+BPe^{-kx} \Rightarrow A-P=BPe^{-kx} \Rightarrow \frac{A-P}{BP}=e^{-kx}$

$$\Rightarrow \ln\left(\frac{A-P}{BP}\right)=-kx \Rightarrow x=\frac{\ln\left(\frac{A-P}{BP}\right)}{-k}.$$

30. Ans B: We can rewrite the first equation as follows:

$$x=\frac{3}{\sec^2 t}=3\cos^2 t=3(1-\sin^2 t)=3-3\sin^2 t=3-3y^2.$$

31. Ans A: The volume of a sphere is given by $V=\frac{4}{3}\pi r^3$. Implicitly differentiate with respect to time t we have $\frac{dv}{dt}=4\pi r^2\frac{dr}{dt}$. Substituting in the given values we find

$$\frac{dv}{dt}=4\pi\,(3\ in.)^2(\frac{1\ in.}{2\sec}) \Rightarrow \frac{dv}{dt}=18\pi\ \ in^3/\sec$$

32. Ans D: For a regular polygon with n sides, use $\frac{180(n-2)}{n}$. If n = 12, angle = 150°.

33. Ans C: The two absolute value functions are continuous over the real numbers, so we need only find where they are equal and then test the regions between these x-values. We need to set the insides equal to solve for one of these points and then set the opposite of one of the insides equal to the other inside to solve for the other.

$$3-5x=6x \Leftrightarrow 3=11x \Leftrightarrow x=\frac{3}{11} \text{ and } 3-5x=-6x \Leftrightarrow 3=-x \Leftrightarrow x=-3$$

If we test each of the regions, we notice that the left and right regions are the two where the inequality holds.

34. Ans E: $4(x^2+y^2)-x+y=0$. Now substitute $r^2=x^2+y^2$, $y=r\sin t$, and $x=r\cos t$. We have $4r^2-r\cos t+r\sin t=0 \Rightarrow r(4r-\cos t+\sin t)=0 \Rightarrow 4r-\cos t+\sin t=0$. Solving for r we have $r=\dfrac{\cos t-\sin t}{4}$.

35. Ans C: First, figure out how many golf shots it will take in order to get 66 revolutions. It will take 264 accurate shots to get there (66 times 4). If we multiply that by 6, we get 1584 Daly shots to get those 66 revolutions. Dividing that by 4, we get 396 sleeves of balls to get the first 66 revolutions done. That means that he needs to crack open at least one more sleeve in order to get the last eighth of a revolution.

36. Ans B: Separate into two simple integrations we have

$$\int 6x^2dx \quad + \quad \int 3e^x dx \Rightarrow \frac{6}{2+1}x^{2+1} \quad + \quad 3e^x \quad + \quad c \Rightarrow 2x^3+3e^x+c$$

37. Ans D: The left side is the series $1+2+\ldots+(x+1)$. If an arithmetic series is of the form $a+(a+r)+(a+2r)+\ldots+(a+nr)$, it has the sum $(n+1)\cdot a+\frac{1}{2}r\cdot n\cdot(n+1)$. In our series, a = 1, r = 1, and n = x, so it has a sum equal to $(x+1)\cdot 1+\frac{1}{2}\cdot 1\cdot x\cdot(x+1)$. Now solve for x. $(x+1)\cdot 1+\frac{1}{2}x(x+1)=c \Rightarrow$

$x+1+\frac{1}{2}x^2+\frac{1}{2}x=c \Rightarrow \frac{1}{2}x^2+x+\frac{1}{2}x+1-c=0 \Rightarrow \frac{1}{2}x^2+\frac{3}{2}x+(1-c)=0$.

Use the quadratic equation to get $x=\dfrac{-\frac{3}{2}\pm\sqrt{\left(\frac{3}{2}\right)^2-4\cdot\frac{1}{2}\cdot(1-c)}}{2\cdot\frac{1}{2}}=-\frac{3}{2}\pm\sqrt{\frac{1}{4}+2c}$

38. Ans C: As $9e^x$ is its own derivative, the function in question takes on the form $f(x)=9e^x+C$ for some constant C. We know that $f(0)=0=9e^0+C=9+C$ so C = -9. Thus we have that $f(x)=9e^x-9$ so $f(3)=9e^3-9\approx 172$.

39. Ans D: The parabolas intersect at (1 , 1) and (2 , 0). Then the lower and upper bounds of the integral are 1 and 2 respectively. We solve

$$\int_1^2 (2x-x^2)-(x^2-4x+4)\,dx \Rightarrow \int_1^2 -2x^2+6x-4 \Rightarrow \frac{-2}{3}x^3+3x^2-4x\Big|_1^2 \Rightarrow -\frac{4}{3}+\frac{5}{3}=\frac{1}{3}$$

40. Ans A: By i, the order must include the sequence boy, Dana, boy. Since iv says Amy goes after someone, she must be last, so the third presentation is done by a boy talking about reptiles. iii forces the first presentation to be about birds and the second to be about fish (only open pair), this means the presentation about mammals is last. By ii, Bill must therefore be doing the third presentation on reptiles. So overall, order is 9:00: Charles, birds; 9:10: Dana, fish; 9:20: Bill, reptiles; 9:30: Amy, mammals.