2013 Academic Challenge
State Physics Exam Solutions

1. **Correct response: A**

$$a = \frac{v - v_0}{t} = \frac{(25.0\,\text{m/s}) - (15.0\,\text{m/s})}{10.0\,\text{s}} = 1.00\,\text{m/s}^2$$

2. **Correct response: B**

$$x_1 = x_0 + \tfrac{1}{2}(v_1 + v_0)t_1 = 0 + \tfrac{1}{2}(15.0\text{ m/s} + 25.0\text{ m/s})(10.0\text{ s}) = 200.\text{ m}$$

3. **Correct response: A**

$$6.00\times10^8\,\text{gloobs} = 24.0\text{ hr}\times\frac{3600\text{ s}}{1\,\text{hr}} \quad\Rightarrow\quad 1.00\text{ gloob} = 1.44\times10^{-4}\text{ s}$$

$$5.00\times10^4\ \frac{\text{thripps}}{\text{gloob}} = 2.9979\times10^8\ \text{m/s} \quad\Rightarrow$$

$$1.00\text{ thripps} = \frac{2.9979\times10^8\ \text{m}\cdot\text{gloob/s}}{5.00\times10^4}\times\frac{1.44\times10^{-4}\ \text{s}}{1.00\text{ gloob}} = 0.863\,\text{m}$$

4. **Correct response: B**

Using a coordinate system with east in the +x direction and north in the +y direction, the total displacement components are

$$d_x = 400.\text{ m} \quad \text{and} \quad d_y = +100.\text{ m} - 400.\text{ m} = -300.\text{ m}$$

The magnitude of the displacement is

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(400.\text{ m})^2 + (-300.\text{ m})^2} = 500.\text{ m}$$

5. **Correct response: B**

Average speed is total path length divided by total travel time:

$$\overline{v} = \frac{\text{path length}}{\text{travel time}} = \frac{100.\text{ m} + 400.\text{ m} + 400.\text{ m}}{15.0\text{ s} + 80.0\text{ s} + 90.0\text{ s}} = 4.86\text{ m/s}$$

6. **Correct response: E**

The only horizontal component of the force on the 6.00 kg block is the 30.0 N tension force. The horizontal acceleration of the 6.00 kg block is

$$a = \frac{F}{m} = \frac{30.0\text{ N}}{6.00\text{ kg}} = 5.00\text{ m/s}^2$$

The block of mass m has the same magnitude acceleration, but in the downward direction. The total vertical force on that block is the upward tension force and the downward gravitational force. By Newton's 2nd Law,

$$30.0\text{ N} - mg = ma \quad\Rightarrow\quad m = \frac{30.0\text{ N}}{a + g} = \frac{30.0\text{ N}}{-5.00\text{ m/s}^2 + 9.80\text{ m/s}^2} = 6.25\text{ kg}$$

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7. **Correct response: D**

Using a coordinate system with east in the +x direction and north in the +y direction, the total displacement components are

$$d_x = (200.\ \mathrm{mi})\cos(30.0°) + (400.\ \mathrm{mi})\cos(-10.0°) = 567.\ \mathrm{mi}$$

$$d_y = (200.\ \mathrm{mi})\sin(30.0°) + (400.\ \mathrm{mi})\sin(-10.0°) = 30.5\ \mathrm{mi}$$

The magnitude of the displacement is

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(567.\ \mathrm{m})^2 + (30.5\ \mathrm{m})^2} = 568.\ \mathrm{m}$$

8. **Correct response: E**

$$F = bx^3 \quad \Rightarrow \quad b = \frac{F}{x^3}$$

The SI units of the right hand side if this equation are

$$\frac{\mathrm{N}}{\mathrm{m}^3} = \frac{\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}}{\mathrm{m}^3} = \frac{\mathrm{kg}}{\mathrm{m}^2 \cdot \mathrm{s}^2}$$

9. **Correct response: A**

Using a coordinate system with east in the +x direction and north in the +y direction,

$$W = \vec{F} \cdot \vec{d} = F_x d_x + F_y d_y = (30.0\ \mathrm{N})\big(10.0\ \mathrm{m}\cos(-10.0°)\big) + (40.0\ \mathrm{N})\big(10.0\ \mathrm{m}\sin(-10.0°)\big) = 59.8\ \mathrm{J}$$

10. **Correct response: E**

By definition, the work done by a conservative force is independent of the path followed. By the Work-Energy Theorem, the change in kinetic energy is also independent of the path followed.

11. **Correct response: B**

$$\Delta K + \Delta U = 0 \quad \Rightarrow \quad \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 + \Delta U_{gravitational} = 0 \quad \Rightarrow$$

$$\Delta U_{gravitational} = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 = 0 - 0 + 0 - \tfrac{1}{2}(400.\ \mathrm{N/m})(0.200\ \mathrm{m})^2 = -8.00\ \mathrm{J}$$

12. **Correct response: B**

Using a coordinate system with in the +x direction toward the right and the +y direction upward,

$$\vec{F}_{net} = m\vec{a} \quad \Rightarrow \quad F_{net\ x} = (15.0\ \mathrm{N})\cos 40.0° - 8.00\ \mathrm{N} = (2.00\ \mathrm{kg})\,a_x$$

$$\Rightarrow \quad a_x = \frac{(15.0\ \mathrm{N})\cos 40.0° - 8.00\ \mathrm{N}}{(2.00\ \mathrm{kg})} = 1.75\ \mathrm{m/s^2}$$

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13. **Correct response: A**

The block does not accelerate vertically, so the total vertical force is zero. Using a coordinate system with in the +x direction toward the right and the +y direction upward,

$$F_{y\ net} = -15.0\ \text{N} \sin 40.0° - (2.00\ \text{kg})(9.80\ \text{m/s}^2) + F_{normal} = 0 \quad \Rightarrow \quad F_{normal} = 29.2\ \text{N}$$

$$F_{kinetic\ friction} = \mu_k F_{normal} \quad \Rightarrow \quad \mu_k = \frac{F_{kinetic\ friction}}{F_{normal}} = \frac{8.00\ \text{N}}{29.2\ \text{N}} = 0.274$$

14. **Correct response: D**

By the impulse momentum relation and using a coordinate system with east as the positive x direction

$$\vec{F} \cdot t = \Delta\vec{p} \quad \Rightarrow \quad \vec{F} = \frac{\Delta\vec{p}}{t} = \frac{(200\ \text{kg}\cdot\text{m/s}) - (-800\ \text{kg}\cdot\text{m/s})}{2.00\ \text{s}} = 500.\ \text{N}$$

15. **Correct response: C**

Average velocity is total displacement divided by total travel time:

$$\bar{\vec{v}} = \frac{\text{displacement}}{\text{travel time}} = \frac{-2.00\ \text{m}}{3.00\ \text{s}} = -0.67\ \text{m/s}$$

16. **Correct response: C**

Average speed is total path length divided by total travel time:

$$\bar{v} = \frac{\text{path length}}{\text{travel time}} = \frac{1.00\ \text{m} + 3.00\ \text{m} + 4.00\ \text{m}}{3.00\ \text{s}} = 2.67\ \text{m/s}$$

17. **Correct response: B**

This is a statement of the law of conservation of angular momentum.

18. **Correct response: A**

Perfectly elastic means that kinetic energy is conserved during the collision:

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Momentum is also conserved during the collision:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Eliminating v_{2f} from these two equations and solving for m_2:

$$m_2 = m_1 \frac{(v_{1i} - v_{1f})^2}{(v_{1i}^2 - v_{1f}^2)} = (1.00\ \text{kg}) \frac{(5.00\ \text{m/s}^2 - 1.00\ \text{m/s}^2)^2}{(5.00\ \text{m/s}^2)^2 - (1.00\ \text{m/s}^2)^2} = 0.667\ \text{kg}$$

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19. **Correct response: A**

To be in static equilibrium, the sum of the forces and the sum of the torques must be zero.

$$\sum F_{horizontal} = 0 \quad \Rightarrow \quad F_{rope} - F_{wall} = 0 \quad \Rightarrow \quad F_{rope} = F_{wall}$$

$$\sum \tau = 0 \quad \Rightarrow \quad -F_{rope}\left(\frac{L}{3.00}\right)\sin 60.0° - mg\left(\frac{L}{2.00}\right)\sin 30.0° + F_{wall} L \sin 60.0° = 0$$

$$\Rightarrow \quad F_{rope} = \frac{(3.00)(4.00\text{ kg})(9.80\text{ m/s}^2)\sin 30.0°}{(4.00)\sin 60.0°} = 17.0\text{ N}$$

Note that the torques were calculated about the contact point of the plank with the floor, and counterclockwise is the positive torque direction.

20. **Correct response: B**

$$\frac{G m_{star} m_{planet}}{R^2} = m_{planet} a = m_{planet}\omega^2 R = m_{planet}\frac{4\pi^2}{T^2} R \quad \Rightarrow$$

$$m_{star} = \frac{4\pi^2}{GT^2} R^3 = \frac{4\pi^2 \left(8.00\times 10^{11}\text{ m}\right)^3}{\left(6.673\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(30.0\text{ yr})^2\left(3.156\times 10^7\text{ s/yr}\right)^2} = 3.38\times 10^{29}\text{ kg}$$

21. **Correct response: A**

$$a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2\left(8.00\times 10^8\text{ km}\right)}{(30.0\text{ yr})^2} = \frac{4\pi^2\left(8.00\times 10^{11}\text{ m}\right)}{\left(9.47\times 10^8\text{ s}\right)^2} = 3.52\times 10^{-5}\text{ m/s}^2$$

22. **Correct response: E**

$$\frac{W}{Q_H} \le \left(1 - \frac{T_C}{T_H}\right) \quad \Rightarrow \quad \frac{Q_H - Q_C}{Q_H} \le \left(1 - \frac{T_C}{T_H}\right) \quad \Rightarrow$$

$$Q_C \ge \frac{Q_H T_C}{T_H} = \frac{(2000.\text{ J})([400. + 273.15]\text{K})}{([1200. + 273.15]\text{K})} = 914.\text{ J}$$

23. **Correct response: B**

$$Q = Q_1 + Q_2 = m_1 C\Delta T_1 + m_2 C\Delta T_2 \quad \Rightarrow \quad C = \frac{Q}{m_1\Delta T_1 + m_2\Delta T_2}$$

$$C = \frac{40.0\text{ Cal}}{(8.00\text{ kg})(15.0\text{ °C} - 20.0\text{ °C}) + (4.00\text{ kg})(15.0\text{ °C} - [-20.0\text{ °C}])} = 0.400\text{ Cal}/(\text{kg}\cdot\text{C°})$$

24. **Correct response: E**

Definition of electric potential

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25. **Correct response: B**

Direct application of Faraday's Law

26. **Correct response: A**

R_2 and R_3 are in parallel. Their equivalent resistance is

$$R_{eqA} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{300.\,\Omega} + \frac{1}{600.\,\Omega}\right)^{-1} = 200.\,\Omega$$

This equivalent resistance is in series with R_1, so their equivalent resistance is

$$R_{eqB} = R_1 + R_{eqA} = 400.\,\Omega + 200.\,\Omega = 600.\,\Omega$$

This equivalent is connected to the voltage source, so the current in R_{eqB} is

$$I = \frac{V}{R_{eqB}} = \frac{6.00\text{ V}}{600.\,\Omega} = 10.0\text{ mA}$$

As R_1 and R_{eqA} are in series, this is also the current through R_{eqA}. Therefore the voltage across R_{eqA} is

$$V_{eqA} = I_{eqA} R_{eqA} = (10.0\text{ mA})(200.\,\Omega) = 2.00\text{ V}$$

As R_2 and R_3 are in parallel, this is also the voltage across R_3.

27. **Correct response: D**

$$\rho_{liquid} V_{submerged}\, g = \rho_{object} V_{object}\, g \quad \Rightarrow \quad \frac{V_{submerged}}{V_{object}} = \frac{\rho_{object}}{\rho_{liquid}}$$

$$\Rightarrow \quad \frac{V_{submerged\,1}}{V_{object1}} = \frac{\rho_{object\,1}}{\rho_{liquid}} \quad \text{and} \quad \frac{V_{submerged\,2}}{V_{object\,2}} = \frac{\rho_{object\,2}}{\rho_{liquid}} \quad \Rightarrow \quad \rho_{object\,2} = \frac{\dfrac{V_{submerged\,2}}{V_{object\,2}}}{\dfrac{V_{submerged\,1}}{V_{object1}}} \rho_{object\,1}$$

$$\rho_{object\,2} = \frac{0.80}{0.40}\left(200.\text{ kg/m}^3\right) = 400.\text{ kg/m}^3$$

28. **Correct response: E**

$$f_n = \frac{nc}{2L} \quad \Rightarrow \quad f_1 = \frac{c}{2L} \quad \Rightarrow \quad c = 2Lf_1 = 2(4.00\text{ m})(50.0\text{ Hz}) = 400.\text{ m/s}.$$

29. **Correct response: C**

$$m = \frac{-s'}{s} \quad \text{and} \quad \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \Rightarrow \quad f = \left(\frac{-m}{s'} + \frac{1}{s'}\right)^{-1} = \frac{s'}{1-m} = \frac{-30.0\text{ cm}}{1-4.00} = 10.0\text{ cm}$$

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30. **Correct response: B**

$$K=(\gamma-1)m_0c^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)m_0c^2=\left(\frac{1}{\sqrt{1-(0.800c)^2/c^2}}-1\right)m_0c^2=0.667m_0c^2$$

31. **Correct response: E**

$$n_A\sin\theta_A=n_B\sin\theta_B\quad\Rightarrow\quad n_B=n_A\frac{\sin\theta_A}{\sin\theta_B}=(1.00)\frac{\sin 40.0°}{\sin 25.0°}=1.52$$

32. **Correct response: B**

$$I=\frac{V}{Z}=\frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}=$$

$$=\frac{20.0\text{ V}}{\sqrt{(12\ \Omega)^2+\left(2\pi\left(4.00\times10^3\text{ Hz}\right)(0.24\text{ H})-\frac{1}{2\pi\left(4.00\times10^3\text{ Hz}\right)\left(0.16\times10^{-6}\text{ F}\right)}\right)^2}}=3.46\text{ mA}.$$

33. **Correct response: B**

The current reaches a maximum rms value at the resonant frequency:

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{(0.24\text{ H})\left(0.16\times10^{-6}\text{ F}\right)}}=812\text{ Hz}$$

34. **Correct response: C**

$$n\lambda=d\sin\theta\quad\Rightarrow\quad d=\frac{n\lambda}{\sin\theta}=\frac{543.\text{ nm}}{\sin 50.0°}=709\text{ nm}$$

35. **Correct response: C**

The electron is a lepton.