Worldwide Youth in Science and Engineering

2014 Academic Challenge

MATHEMATICS TEST - REGIONAL

This Test Consists of 40 Questions

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ◐ , ◑ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** TIME: 40 MINUTES ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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1. What is the quantity which results when the number of vertices in an octahedron is subtracted from the number of vertices in a cube?

    a) -2   b) 0   c) 2   d) 6   e) 8

2. A breakfast bar has the following selection of doughnuts: 8 glazed, 15 chocolate frosted, and 10 jelly filled. Your friend has been selected to choose 9 doughnuts. What is the probability that your friend selected 3 glazed, 2 chocolate frosted, and 4 jelly doughnuts? Round to 3 decimal places.

    a) 0.015   b) 0.017   c) 0.020   d) 0.027   e) 0.032

3. Given the equation $P + QR = PR + T$, solve for P.

    a) $P = T - QR + R - 1$

    b) $P = \dfrac{T - QR}{1 - R}$

    c) $P = \dfrac{T + QR}{1 + R}$

    d) $P = \dfrac{T + QR}{R}$

    e) $P = \dfrac{QR - T}{R}$

4. When the following two matrices are multiplied, what dimensions do the resultant matrix take on, if it even exists?

    $$\begin{bmatrix} 1 & 3 & 4 \\ -7 & 6 & x \\ 8 & y & 3 \end{bmatrix} \begin{bmatrix} 1 & z & 3 \\ 4 & 2 & 8 \end{bmatrix}$$

    a) 2 x 3   b) 3 x 2   c) 3 x 3   d) 2 x 2   e) Not a matrix.

5. Find the corresponding rectangular coordinate for the polar coordinate $\left(2, \dfrac{\pi}{6} \right)$

   a) $\left(\sqrt{3}, \dfrac{1}{2} \right)$ b) $\left(\dfrac{\sqrt{3}}{2}, 1 \right)$ c) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right)$ d) $\left(\sqrt{3}, 1 \right)$ e) $\left(-\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2} \right)$

6. What is $i^{504} + i^{506}$?

   a) -2i   b) 2i   c) 2   d) -2   e) 0

7. Sheldon, Leonard, and Raj are standing on a circle so that the chord made from Leonard to Sheldon contains the arc length of $100°$. The chord made from Leonard to Raj contain an arc length of $140°$. The chord made from Raj to Sheldon contains the arc length of $120°$. If Howard is standing behind Sheldon so that he is not on the circle but perfectly aligned so that Leonard, Sheldon and Howard are on the same line, find the angle measure formed from Raj, Sheldon and Howard where Sheldon is the vertex point of the angle. See diagram below. The diagram is not drawn to scale.
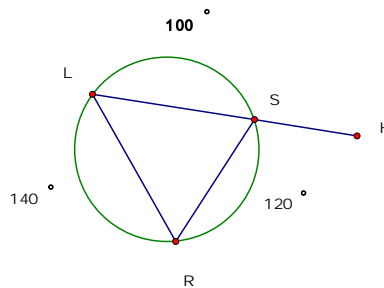
   a) $120°$ b) $110°$ c) $80°$ d) $70°$ e) $60°$

8. A six sided figure is created using two squares and two equilateral triangles as shown below. If the outer perimeter of the figure is 1 inch, what is its area? Round your answer to the nearest hundredth of a square inch.
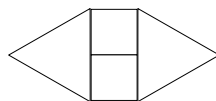
   a) 0.05 sq in   b) 0.06 sq in   c) 0.07 sq in   d) 0.08 sq in   e) 0.10 sq in

9. What is the sum of the coefficients of $(x - y)^5$?

   a) 0   b) 2   c) 6   d) 32   e) 64

10. Andy has a trapezoid shaped flower garden with the shortest base (row) side measuring 7 yards and the median measuring 10 yards. If each plant in his garden is planted 1 foot apart in rows that are horizontal to the base, how many more plants are planted in the larger base row than the shortest row of Andy's trapezoid garden?

   a) 3   b) 6   c) 9   d) 13   e) 18

11. A girl would like to determine how tall a dirt pile is above ground level without resorting to digging. She puts a flag on top of the pile and then walks some distance away. At first she has to look up six degrees from eye level to see the flag. After moving ten feet closer to the pile, she now has to look up eight degrees from eye level. After determining how high the flag is above eye level, the girl adds on five feet to account for her height. Determine how tall the dirt pile must be to the nearest foot.

    a) 9 ft   b) 12 ft   c) 17 ft   d) 30 ft   e) 40 ft

12. In an arithmetic sequence, S_n, we have that $S_1 = 43$ while $S_{15} = 14$. What is $\sum_{n=1}^{\infty} S_n$? Round to the nearest integer if necessary.

    a) 0   b) 1   c) 558   d) $-\infty$ e) undetermined

13. Find the perimeter of a regular hexagon inscribed in a circle if the radius is 10 inches.

    a) 60 in   b) $30\sqrt{3}$ in   c) $30\sqrt{5}$ in   d) 30 in   e) 24 in

14. In how many distinguishable ways can the letters from the word "iconoclasm" be reordered?

    a) 8   b) 302,400   c) 604,800   d) 907,200   e) 3,628,800

15. Simplify the quotient $\dfrac{-6^2 m^2 n - 12mn^2}{6mn}$.

    a) $6m - 2n$ b) $2n - 6m$ c) $-6m - 2n$ d) $6n - 2m$ e) $-6m - 12mn^2$

16. In a group of 200 people, 50 of them have visited Europe, 25 have visited Africa, and 20 have visited both continents. If we randomly pick one of these people knowing they have not been to Africa, what is the probability they have not been to Europe either? Round your answer to the nearest percent.

    a) 73%   b) 83%   c) 86%   d) 88%   e) 97%

17. A gondolier rows 4.5 mph with the current behind him and 2.5 mph against the current. How long will it take him to row 63 miles upstream and subsequently make the return trip? Round to the nearest hour.

    a) 14   b) 18   c) 25   d) 36   e) 39

18. A candy machine can dispense no more than 40 peanut m&m's per minute. If it operates for 8 minutes and a second candy machine operates for 5 minutes, what must be the dispense rate of the second machine if the total of peanut m&m's dispensed from both machine is 480?

    a) At least 32 peanut m&m's per minute.
    b) At most 96 peanut m&m's per minute.
    c) At least 64 peanut m&m's and no more than 96 peanut m&m's per minute.
    d) At least 32 peanut m&m's and no more than 96 peanut m&m's per minute.
    e) Cannot be determined.

19. Some money was placed in a bank account that compounds monthly at an annual interest rate of 2.5%. If there is $20,000 in the bank after five years of accumulating interest, how much was initially deposited? Round your answer to the nearest cent.

    a) $5,804.16
    b) $17,339.98
    c) $17,500.00
    d) $17,652.23
    e) $17,677.09

20. How many diagonals does an octagon have?

    a) 7   b) 14   c) 20   d) 21   e) 28

21. Solve the equation $x - \sqrt{x+4} = 8$ for x.

    a) $x = 5$ b) $x = 6$ c) $x = 12$ d) $x = 5,12$ e) $x = 6,12$

22. Eight elves can make either 48 cookies and 24 cakes or 72 cookies and 16 cakes in a 48 hour period. If he forwent making cookies, how many complete cakes could one elf make in 48 hours?

    a) 5   b) 9   c) 10   d) 15   e) 25

23. Find two expressions that are factors of $f(x) = 4x^4 - 4x^3 + 23x^2 + x - 6$

    a) $2x+3$, $x-2$
    b) $2x-1, 2x+1$
    c) 4, $x^2 - x + 6$
    d) $4, 2x-1$
    e) Does not factor

24. Determine the domain of the function $f(x) = \dfrac{1}{\sqrt{x-1}} - \dfrac{2}{\sqrt{5-x}}$

    a) $\varnothing$ b) $(-\infty, \infty)$ c) $(5, \infty)$ d) $[1, 5]$ e) $(1, 5)$

25. What do we call a kite with diagonals of equal length?

    a) square
    b) rectangle
    c) rhombus
    d) isosceles trapezoid
    e) It is no longer a kite at this point.

26. Find the value K when the $\det\left(\begin{bmatrix} -5 & 4 & 5 \\ 2 & 4 & K \\ 1 & 4 & 2 \end{bmatrix}\right) = 84$.

    a) $K = 5$ b) $K = 4$ c) $K = 0$ d) $K = -4$ e) $K = -7$

27. A cylinder with a square cross section has a volume of 100 cubic inches. Determine its surface area to the nearest square inch.

    a) 22 sq in   b) 77 sq in   c) 80 sq in   d) 119 sq in   e) 126 sq in

28. Four of the following five expressions are equivalent. Choose the one which is not.

    a) $\sin(4x)$
    b) $2\sin(2x)\cos(2x)$
    c) $4\sin(x)\cos(x)\cos(2x)$
    d) $\sin^2(2x) - \cos^2(2x)$
    e) $\sin(3x)\cos(x) + \cos(3x)\sin(x)$

29. Find x given $\left(5^{x-2}\right)^{-3} = \dfrac{(125)^{-\frac{1}{3}x}}{25}$.

    a) $x = -\dfrac{3}{5}$ b) $x = -\dfrac{1}{3}$ c) $x = 4$ d) $x = 5$ e) $x = \dfrac{25}{3}$

30. By the rational root theorem, which of the following is not a possible rational root for $f(x) = 4x^6 + 3x^5 + 12x - 851$?

    a) 37/4   b) 3   c) 23/2   d) -37/2   e) -851

31. Find the equation of the parabola with focus (6 , 2) and directrix y = 0.

    a) $x^2 = 6y$

    b) $y = 6x^2 + 1$

    c) $x = y^2$

    d) $y = \dfrac{1}{2}x^2 - 6x + 1$

    e) $y = \dfrac{1}{4}x^2 - 3x + 10$

32. Angle A and angle B are supplementary. Angle B and angle C are congruent. Angle C and angle D are acute. Which of the following <u>must</u> be true about angles A and D?

    a) A and D must be complementary
    b) A and D must be supplementary
    c) A and D must be congruent
    d) A and D must be acute
    e) None of the above are required to be true

33. What is the angle of elevation (in degrees) from the top of a 6'1" person to the top of the Eiffel Tower, which is 1063 feet above ground? Assume the person is standing 100 yards away and round the angle of elevation to the nearest degree.

    a) $16°$ b) $52°$ c) $74°$ d) $85°$ e) $89°$

34. A prop plane is 1850 feet above the ground and south of the Cincinnati Bengals' football stadium during their football game. The angle of depression of the north goal line from the plane is 58.5 degrees. How far is the observer in the plane from the goal line? Round to the nearest foot.

    a) 4000 ft.   b) 3700 ft.   c) 2938 ft.   d) 2170 ft.   e) 1088 ft.

35. We currently have 6 gallons of a mixture that is 30% syrup and 70% water. We want to end up with 10 gallons of a mixture that is 25% syrup and 75% water. We have pure water and pure syrup available. How much of each do we have to add to get the required result? Round your answer to the nearest tenth of a gallon.

    a) 0.4 gallons syrup, 3.6 gallons water
    b) 0.5 gallons syrup, 3.5 gallons water
    c) 0.7 gallons syrup, 3.3 gallons water
    d) 0.8 gallons syrup, 3.2 gallons water
    e) 1.0 gallons syrup, 3.0 gallons water

36. Let the x-coordinate of an object at time t be x = sin t and the y-coordinate be y = cos 2t. What is the maximum distance from (0, 0) possibly achieved by this object? Round to the nearest tenth of a unit.

    a) 0   b) 1   c) 1.4   d) 1.5   e) 2

37. Determine $\sec\theta$ if the terminal side of θ passes through $\left(-1, \sqrt{3}\right)$. Round to 3 decimal places.

 a) 0.866 b) -2.000 c) -0.500 d) 0.500 e) -0.577

38. What is $\lim\limits_{x\to\infty} \dfrac{\log\left(4x^2 - 4x + 1\right)}{\log\left(2x - 1\right)}$?

 a) 2 b) ∞ c) 0 d) $-\infty$ e) 4

39. Given the numbers 5,2,6,4,7,4,7,2,8,9,4,10,9,1,3 find the median.

 a) 5 b) 5.5 c) 6 d) 7.5 e) 9

40. Four boys, Aaron, Brian, Carlos, and Damon, participated in a School Festival Olympiad. The three head-to-head events: pugil sticks, sack race, and bag toss, each had different pairings. The three individual events were pie eating, the obstacle course, and the egg carry. Participants got two points for winning a head-to-head event or finishing an individual event, one point for tying a head-to-head event, and no points for losing a head-to-head event or failing an individual event.

 I. Each boy got a different head-to-head total
 II. Aaron got twice as many ties in head to head as Damon
 III. The two boys that finished the Pie eating tied each other in the Bag Toss
 IV. Carlos won his Sack Race and Bag Toss, but tied with Brian in Pugil sticks
 V. Only Damon finished the Obstacle course, and only Aaron finished the Egg carry

 Determine who got the most points and won the Olympiad

 a) Brian
 b) Carlos
 c) Aaron and Brian tied for first
 d) Brian and Carlos tied for first
 e) Carlos and Damon tied for first