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1. Ans C: There are eight vertices in a cube and six in an octahedron.

2. Ans E: There are ${}_{33}C_9$ ways to select the 9 doughnuts. This is 38,567,100 ways. There are ${}_{8}C_3$ ways to select 3 glazed, ${}_{15}C_2$ ways to select 2 chocolate frosted, and ${}_{10}C_4$ ways to select 4 jelly doughnuts. This gives us $\frac{8!}{3!5!}\cdot\frac{15!}{2!13!}\cdot\frac{10!}{4!6!}$. The total ways of making such selection is 1,234,800. The probability is $\frac{1,234,800}{38,567,100} = 0.032$.

3. Ans B: $P - PR = T - QR \Rightarrow P(1-R) = T - QR \Rightarrow \quad P = \frac{T-QR}{1-R}$

4. Ans E: When multiplying two matrices, the left must have dimensions of m x n while the right has dimensions of n x p. This is multiplying a 3 x 3 by a 2 x 3, so the inner dimensions do not match and no product exists.

5. Ans D: From the polar coordinate we know the radius is 2 while the angle θ is $\frac{\pi}{6}$. Then $x = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$ and $y = 2\sin\left(\frac{\pi}{6}\right) = 1$. The coordinate is $\left(\sqrt{3}, 1\right)$.

6. Ans E: $i^{504} = 1 \text{ and } i^{506} = -1$, so their sum would be 0.

7. Ans B: Since $\overset{\frown}{LR} = 140^\circ$, then the inscribed angle $\angle LSR = 70^\circ$. $\angle LSH = 180^\circ$ (a straight angle). Then $\angle RSH = 110^\circ$ (180 – 70).

8. Ans A: If we let x be the side of one of the squares, then each triangle has sides of 2x, and the overall perimeter is 2x + x + 2x + 2x + x + 2x = 10x. Solving for 10x = 1 gives us x = 0.1. The area of each square is $x^2 = 0.01$ square inches, and the area of each triangle is $\frac{1}{2}\cdot 2x \cdot x\sqrt{3} \approx 0.0173$ square inches. This makes the overall area of the region $0.01 + 0.01 + 0.0173 + 0.0173 \approx 0.0546$ square inches.

9. Ans A: By the binomial theorem, $(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$. $1 - 5 + 10 - 10 + 5 - 1 = 0$.

10. Ans E: The median is half the length of the sum of the bases. Then the longer base side can be found by solving for L in the following equation: $\frac{1}{2}(L+7) = 10$. L = 13. The longer base is 13 yards or 39 feet. The shortest row is 7 yards or 21 feet. Larger row has 39 plants while the shortest has 21 plants. The difference is 18 plants.

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11. Ans A: Let h be the height of the pile above eye level, and d the distance from the center of the pile to the girl. Based on angles given in degrees, the two measurements give us $\tan 6 = \frac{h}{d+10}$ and $\tan 8 = \frac{h}{d}$. Solve for d to get $d = \frac{h}{\tan 6} - 10$ and $d = \frac{h}{\tan 8}$.
Substitute and solve for h. $\frac{h}{\tan 6} - 10 = \frac{h}{\tan 8} \Rightarrow \frac{h}{\tan 6} - \frac{h}{\tan 8} = 10 \Rightarrow$
$\left(\frac{1}{\tan 6} - \frac{1}{\tan 8}\right)h = 10 \Rightarrow h = \frac{10}{\frac{1}{\tan 6} - \frac{1}{\tan 8}} \approx 4.168$. When we add the five for eye level, we end up with a dirt pile that is nine feet tall.

12. Ans D: Since it is an arithmetic sequence with a negative common difference, there is an infinite number of increasingly negative terms. So for every negative number, there's a term in the sequence for which the partial sum is less than that number. This is the very definition of a series sum going to negative infinity. Alternatively, there's an infinite number of terms which are less than negative 1 and only a finite number of positive terms, so the sum would go to negative infinity.

13. Ans A: A regular hexagon is made up of 6 equilateral triangles. Since the radius is 10 inches, each side of the hexagon is also 10 inches. Then the perimeter of the hexagon is 10 inches times the 6 sides. The perimeter is 60 inches.

14. Ans D: There are 10 letters in iconoclasm, two of which are repeated twice each. So the number of distinguishable strings is $\frac{10!}{2!2!} = 907{,}200$.

15. Ans C: $\frac{-6^2 m^2 n - 12mn^2}{6mn} \Rightarrow \frac{6mn(-6m - 2n)}{6mn} \Rightarrow -6m - 2n$

16. Ans B: 20 have been to Africa and Europe, 5 Africa only, 30 Europe only, and 145 neither. Of the 175 who have not gone to Africa, 145 have not gone to Europe. $145/175 \approx 0.829$

17. Ans E: Using D = RT and solving for T, we get that the trip upstream takes $\frac{63}{2.5} = 25.2$ hours and the return trip takes $\frac{63}{4.5} = 14$ hours. The total trip thus takes 39.2 hours.

18. Ans D: Let x represent the dispense rate of the first machine in terms of peanut m&m's per minute and y represent the dispense rate of the second machine in terms of peanut m&m's per minute. Then $8x + 5y = 480$. Since the first machine cannot dispense more than 40 peanut m&m's per minute, $0 \le x \le 40$. Solving for x in our equation we have $x = 60 - 0.625y$. Then $0 \le 60 - 0.625y \le 40$. Solving for y in this inequality we find $32 \le y \le 96$ peanut m&m's per minute.

19. Ans D: Using the formula $FV = P\left(1+\dfrac{r}{m}\right)^{mt}$, $20000 = P\left(1+\dfrac{.025}{12}\right)^{12*5}$, $P \approx 17652.23$.

20. Ans C: An n-gon has $\dfrac{n(n-3)}{2}$ diagonals. If n is 8, this would be $\dfrac{8(8-3)}{2} = 20$.

21. Ans C: $x - \sqrt{x+4} = 8 \Rightarrow x - 8 = \sqrt{x+4} \Rightarrow x^2 - 16x + 64 = x + 4 \Rightarrow x^2 - 17x + 60 = 0$. Then $(x-12)(x-5) = 0 \Rightarrow x = 12 \text{ or } x = 5$. Checking for extraneous roots we find that $x \neq 5$. So $x = 12$.

22. Ans A: Let x be the number of hours a cookie takes to make and let y be the number of hours a cake takes to make. A single elf can make 6 cookies and 3 cakes or 9 cookies and 2 cakes in a 48 hour period, so 6x + 3y = 48 while 9x + 2y = 48. By multiplying the first equation through by 3 and the second one through by 2, we get that 18x + 9y = 144 while 18x + 4y = 96. By subtracting the two equations, 5y = 48, so y = 9.6. Thus, each cake takes 9.6 hours to make and by dividing that into the 48 hours, the elf makes five complete cakes.

23. Ans B: Using Des Cartes rule and synthetic division the possible roots are ± 6, ± 4, ± 3, ± 2, ± 1, $\pm \frac{3}{2}$, $\pm \frac{3}{4}$, $\pm \frac{1}{4}$, $\pm \frac{1}{2}$. We will first test -1 to see if it is a root of the polynomial by using synthetic division:

$$\begin{array}{r|rrrrr} -1 & 4 & -4 & 23 & 1 & -6 \\ & & -4 & 8 & -31 & 30 \\ \hline & 4 & -8 & 31 & -30 & 24 \end{array}$$

Since the signs on the bottom alternate there are no negative roots less than -1. Since the bottom row last entry is not 0, we know $x+1$ is not a factor of the polynomial. Next we will try $-\frac{1}{2}$.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 4 & -4 & 23 & 1 & -6 \\ & & -2 & 3 & -13 & 6 \\ \hline & 4 & -6 & 26 & -12 & 0 \end{array}$$

this root is a zero of the polynomial. Therefore $2x+1$ is a factor. Next we will look for a positive root. We will try $\frac{1}{2}$.

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & -6 & 26 & -12 \\ & & 2 & -2 & 12 \\ \hline & 4 & -4 & 24 & 0 \end{array}$$

this root is also a zero of the polynomial. Then $2x-1$ is a factor of the polynomial. Since $(2x-1)(2x+1) = 4x^2 - 1$. We divide the original polynomial by $4x^2 - 1$. Here we find the following factors for the original polynomial: $(2x+1)(2x-1)(x^2 - x + 6)$.

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24. Ans E: Denominators need to be non-zero and radical terms need to be non-negative. The left term gives us $x-1>0$ or $x>1$, the right $5-x>0$ or $x<5$. We need both restrictions in effect, giving a resulting domain of $(1,5)$.

25. Ans A: Quadrilaterals with diagonals of equal length are classified as either squares, rectangles or isosceles trapezoids. Of those, only squares must have congruent adjacent sides.

26. Ans A: Since the det is 84; $\begin{vmatrix} 4 & 5 \\ 4 & K \end{vmatrix} - 4\begin{vmatrix} -5 & 5 \\ 2 & K \end{vmatrix} + 2\begin{vmatrix} -5 & 4 \\ 2 & 4 \end{vmatrix} = 84$. Then
$4K-20-4(-5k-10)+2(-20-8)=84$. So $4K-20+20K+40-56=84$.
$24K=120 \Rightarrow K=5$.

27. Ans D: For a cylinder, $V=\pi r^2 h$. A square cross section means h = 2r, so $V=2\pi r^3$. We were given V = 100, so r = 2.515398. Since $SA=2\pi r^2+2\pi rh=6\pi r^2$, $SA \approx 119.265$.

28. Ans D: a and b are equivalent as we used a double angle formula for sine. We used it again to get c. We used angle addition on a to get e. d is, of course, -cos 4x, using a double angle formula, so it is not equivalent to the other four.

29. Ans C: $\left(5^{x-2}\right)^{-3} = \dfrac{(125)^{-\frac{1}{3}x}}{25} \Rightarrow 5^{-3x+6} = \dfrac{\left(5^3\right)^{-\frac{1}{3}x}}{5^2} \Rightarrow 5^{-3x+6} = 5^{-x-2}$. So
$-3x+6=-x-2 \Rightarrow 8=2x \Rightarrow x=4$.

30. Ans B: First, note that 851 is the product of the prime numbers 23 and 37. Second, the rational root theorem states that, for a polynomial with integral coefficients, all rational roots have numerators which are factors of the constant term (in this case, they would have numerators of either 1, 23, 37 or 851 or their opposites) and their denominators are factors of the leading coefficient (so they could have denominators of 1, 2 or 4 or their opposites). 3 is not possible as none of the possible numerators is a multiple of 3.

31. Ans E: Choose generic point (x , y) on the parabola and equate the distance from point to focus with distance from point to directrix. Then solve for y to find the equation of the parabola. $\sqrt{(x-6)^2+(y-2)^2}=y-0 \Rightarrow (x-6)^2+(y-2)^2=y^2$. Then
$x^2-12x+36+y^2-4y+4-y^2=0 \Rightarrow y=\dfrac{1}{4}x^2-3x+10$.

32. Ans E: Since A and B are supplementary, their measure adds up to 180 degrees. Since B and C are congruent, C must have the same measure as B, so A and C also add up to 180. However, the statement about C and D being acute simply means both have measure less than 90 degrees. This forces A to have a measure between 90 and 180 degrees, so that immediately eliminates a), c), and d). It is possible for b) to be true, but only if C and D have the same measure, which was not required.

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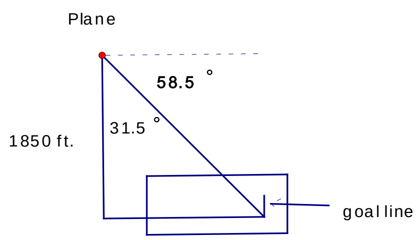
33. Ans C: To find the angle of elevation, draw an imaginary right triangle whose base (which extends from the crest of the head to a place that is 6'1" up the Eiffel Tower) has a length of 300 feet (100 yards) and whose height (which runs up the Eiffel Tower) is 1056 feet, 11 inches (the height of the Eiffel Tower less the height of the human). Thus the tangent of the angle in question would be $\frac{1056'11''}{300'} = \frac{12683''}{3600''}$, and by using the degree version of the arctangent function, we get C.

34. Ans D: Using the diagram below, $\frac{1850}{d} = \cos(31.5°) \Rightarrow d = \frac{1850}{\cos(31.5°)} \Rightarrow d = 2170$



35. Ans C: Let x be the gallons of pure syrup, then create an equation that keeps track of the syrup in the mix: 6*0.3 + x*1 + (4 – x)*0 = 10*0.25. Solve to get x = 0.7.

36. Ans C: Note that the x-value oscillates between -1 and 1, as does the y-value. Thus if we can get x- and y-coordinates whose absolute values are 1, that would give the maximum possible distance of $\sqrt{2}$. Thankfully, in $t = \frac{\pi}{2}$, we have such a value.

37. Ans B: $x = -1, y = \sqrt{3}$. hypotenuse $= \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$. $\sec\theta = \frac{hyp}{adj} = \frac{2}{-1} = -2$.

38. Ans A: Note that $\frac{\log(4x^2 - 4x + 1)}{\log(2x-1)} = \frac{\log(2x-1)^2}{\log(2x-1)}$. For $x > \frac{1}{2}$, this is $\frac{2\log(2x-1)}{\log(2x-1)}$. For any x other than 1, that result is equivalent to 2. So the limit must be 2.

39. Ans A: Numbers in ascending order are 1,2,2,3,4,4,4,5,6,7,7,8,9,9,10. The middle score is the 8th score (found by calculating $\frac{15+1}{2}$). The eighth score is 5.

40. Ans C: First consider h-t-h matches. C beat A and D, and tied B for a point total of 5. For A to have two ties, A must have lost to C and tied B and D for a point total of 2. D's only tie came against A, so the B-D match didn't end in a tie. Since B can't have a point total of 2, B must have won the match. This means B tied A and C and beat D for a point total of 4, and D lost to B and C and tied A for a point total of 1. In individual, D got 2 points from OC and A got 2 points from EC. Since the two who tied BT must be A and B (wasn't C, must include B by IV) that means A and B each get 2 more points from PE. This brings A's total up to 6, B's total up to 6, C's total up to 5, and D's 3.