WYSE Academic Challenge
2014 Regional Physics Exam
SOLUTION SET

1. **Correct answer: B**

For a particle, the time rate of change of its linear momentum is equal to the net force acting on the particle. This is a statement of Newton's 2nd Law.

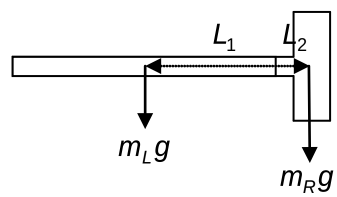
$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}.$$

And if mass is constant,

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d(\vec{v})}{dt} = m\vec{a}.$$

2. **Correct answer: B**

The sum of the gravitational torques about the center of mass of this small object will be zero. In the following diagram, L_1 and L_2 are the distances from the center of mass of the whole object to the center of mass of the left and right pieces, respectively. The torque equation becomes:



$m_L g L_1(\text{counter clockwise}) + m_R g L_2(\text{clockwise}) = 0$

Thus, $m_L L_1 = m_R L_2$, and $L_1 / L_2 = m_R / m_L$.

Since $L_1 > L_2$, then $m_R > m_L$.

The right piece, R, has greater mass.

3. **Correct answer: C**

First two seconds: $d = \frac{1}{2}gt^2 \rightarrow g = \frac{2d}{t^2} = \frac{2(19.6\,\text{m})}{(2.00\,s)^2} = 9.80\,m/s^2$. This is what one expects if the rock is falling from a cliff on the earth.

First three seconds: $d = \frac{1}{2}gt^2 = \frac{1}{2}(9.80\,m/s^2)(3.00\,s)^2 = 44.1m$

So the distance fallen between 2.00 s and 3.00 s is 44.1 m – 19.6 m = 24.5 m.

4. **Correct answer: E**

$\vec{V}_{boat\ to\ earth} = \vec{V}_{boat\ to\ river} + \vec{V}_{river\ to\ earth}$ Since the velocities form a right triangle,

$V^2_{boat\ to\ earth} = V^2_{boat\ to\ river} + V^2_{river\ to\ earth}$

$(12.0\,\text{km/hr})^2 = V^2_{boat\ to\ river} + (5.00\,\text{km/hr})^2$

$V_{boat\ to\ river} = 10.9\,\text{km/hr}$

5. **Correct answer: A**

Since momentum of the two mass system is conserved:

$$(12.0\,\text{kg})V_{12kg,init} + (22.0\,\text{kg})V_{22kg,init} = (12.0\,\text{kg})V_{12kg,final} + (22.0\,\text{kg})V_{22kg,final}$$
$$(12.0\,\text{kg})V_{12kg,init} + (22.0\,\text{kg})(0) = (12.0\,\text{kg})(-4.00\,\text{m/s}) + (22.0\,\text{kg})(15.0\,\text{m/s})$$
$$V_{12kg,init} = 23.5\,\text{m/s}$$

6. **Correct answer: D**

$$\left|F_{ave\,on\,22\,kg}\right| = m|a| = m\left|\frac{\Delta V}{\Delta t}\right| = (22.0\,kg)\left|\frac{(15.0-0.00)\text{m/s}}{0.280\,\text{s}}\right| = 1180\,\text{N}$$

7. **Correct answer: C**

Response C is a statement of Newton's First Law:

If $\sum_i \mathbf{F}_i = 0$, then the velocity of the object must be constant.

While response D is true:

The acceleration of an object is given by $\left(\sum_i \mathbf{F}_i\right)/m$;

it is a statement of Newton's Second Law, not his first.

8. **Correct answer: B**

At the top point of its path, the ball has only a horizontal (or x) component of velocity.

$$x = 0 + (13.2\text{ m/s})\cos\left(26.5^{o}\right)t = x_i + V_{ix}t$$
$$V_{ix} = (13.2\text{ m/s})\cos\left(26.5^{o}\right) = 11.8\text{ m/s}$$

9. **Correct answer: D**

The first bounce after t = 0, occurs when y is again 0.119 m.

$$0.119\text{ m} = 0.119\text{ m} + (13.2\text{ m/s})\sin\left(26.5^{o}\right)t - \frac{1}{2}\left(9.80\text{ m/s}^2\right)t^2$$

$$0 = \left[(13.2\text{ m/s})\sin\left(26.5^{o}\right) - \frac{1}{2}\left(9.80\text{ m/s}^2\right)t\right](t)$$

$$t = 0, \quad or \quad t = \frac{(13.2\text{ m/s})\sin\left(26.5^{o}\right)}{4.90\text{ m/s}^2} = 1.20\,\text{s}$$

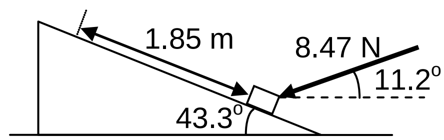
10. **Correct answer: B**

$$V_{ave} = \frac{\text{Total distance traveled}}{\text{total trip time}} = \frac{126\text{ km}}{3.00\text{ hr}} = 42.0\text{ km/hr}$$

11. **Correct answer: E**

Which scientist first devised and did experiments involving an apparatus which used charged oil droplets to determine the elementary charge of an electron? – Robert Millikan – The experiment is known as "the Millikan oil drop experiment".

12. **Correct answer: A**

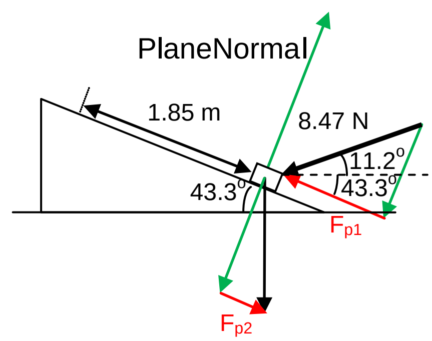


$$W = Fd\cos(\theta) = (8.47\,\text{N})(1.85\,\text{m})\cos(43.3^\circ + 11.2^\circ) = 9.10\,\text{J}$$

13. **Correct answer: D**

$$\Delta GPE = mg\Delta h = (0.510\text{ kg})(9.80\text{ m/s}^2)(1.85\text{ m})\sin(43.3^\circ) = 6.34\text{ J}$$

14. **Correct answer: A**



Since the object accelerates along the plane, the net force must be along the plane, so find the components of the applied force and the gravitational force parallel to the plane. (Note: The normal force from the plane is canceled by the normal components of gravity and the 8.47 N force, since the object does not accelerate perpendicular to the plane's surface.)

$$F_{\textit{parallel to plane}} = F_{p1} - F_{p2} = (8.47\text{ N})\cos(43.3^\circ + 11.2^\circ) - (0.510\text{ kg})(9.80\text{ m/s}^2)\sin(43.3^\circ) = 1.49\text{ N}$$

Note:

$$F_{\textit{perpendicular to plane}} = -(8.47\text{ N})\sin(43.3^\circ + 11.2^\circ) - (0.510\text{ kg})(9.80\text{ m/s}^2)\cos(43.3^\circ) + \textit{PlaneNormal} = 0$$

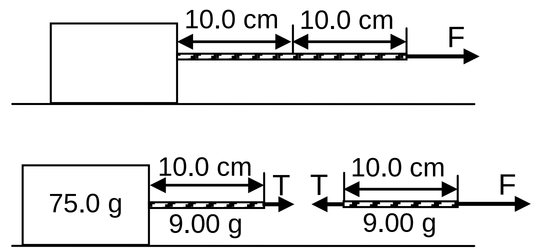
15. **Correct answer: E**

Assuming that a basketball is in free flight, when it is at the top of its parabolic trajectory, the basketball has a downward acceleration and a horizontal velocity. Since the force due to gravity is always downward, the acceleration of the basketball is 9.80 m/s^2 downward at all points along the trajectory. Since, ignoring friction, there is no force in the horizontal direction, the velocity in the horizontal direction is constant and equal to the horizontal component of velocity the ball had just after it was released.

16. **Correct answer: A**

$$\textit{Battery Energy} = \Delta GPE + \Delta KE + \textit{Other}$$
$$240.\text{W}\cdot\text{h} = mg\Delta h + 0 + \textit{Other}$$
$$240.\text{J/s}\cdot 3600\text{ s} = (950.\text{N})(368.\text{m}) + \textit{Other}$$
$$864\text{ kJ} = 349.6\text{kJ} + \text{Other}$$
$$\textit{Other} = 514.4\text{ kJ}$$
$$\text{Other} / 864\text{ kJ} = 59.5\%$$

17. **Correct answer: C**



Using Newton's 2nd Law, the acceleration of the system is

$$a = \frac{F}{75.0\,\text{g} + 18.0\,\text{g}} = \frac{F}{93.0\,\text{g}}.$$

Applying Newton's 2nd Law to the right half of rope:

$$F - T = (9.00\,g)a = (9.00\,g)\left(\frac{F}{93.0\,\text{g}}\right)$$

$$F - (9.00\,g)\left(\frac{F}{93.0\,\text{g}}\right) = T$$

$$T = \left(1 - \frac{9.00}{93.0}\right)F = 0.903\,\text{F}$$

18. **Correct answer: C**

Using the work-energy theorem:

$$W = \Delta Energy_{system}$$

$$Fd\cos(\theta) = \Delta KE + \Delta GPE$$

$$F(0.237\text{ m})\cos(180^o) = \left[0 - \frac{1}{2}(0.0289\text{ kg})(400.\text{ m/s})^2\right] + \left[0 - (0.0289\text{ kg})(9.80\text{ m/s}^2)(389\text{ m})\right]$$

$$F = 10.2\text{ kN}$$

19. **Correct answer: B**

Using the definition of work, $W_{\text{done by F}} = Fd\cos(\theta_{\text{between F and d}})$

$$W = W_{spring} + W_{gravity} = \left[\int_0^{0.150\,m} F_{spring}\cos(180^o)dx\right] + \left[F_{gravity}d\cos(0^o)\right]$$

$$W = \left[\int_0^{0.150\,m} kx\cos(180^o)dx\right] + \left[m_{block}gd\cos(0^o)\right]$$

$$W = \left[-\frac{1}{2}kx^2\Big|_0^{0.150\,m}\right] + \left[m_{block}gd\right] = \left[-\frac{1}{2}(125\text{ N/m})(0.150m)^2\right] + \left[(0.248\text{ kg})(9.80\text{ m/s}^2)(0.150\,m)\right]$$

$$W = -1.04\text{ J}$$

Note that the work done on the block by the spring equals the negative of the change in energy stored in the spring. The negative comes about because the spring force on the block is in the opposite direction of the motion of the block. Also note that the work done by gravity equals the negative of the change in gravitational potential energy of the block.

20. **Correct answer: B**

$$P_{ave} = \frac{\Delta KE}{t} = \frac{\frac{1}{2}mV_f^2 - \frac{1}{2}mV_i^2}{t} = \frac{\frac{1}{2}mV_f^2 - 0}{t} = \frac{mgV_f^2}{2gt} = \frac{(1.57\times10^4\text{ N})(16.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(7.20\text{ s})} = 28.5kW$$

21. **Correct answer: E**

$$x_{cm} = \frac{(2.00M)(0.00\text{ cm}) + (1.00M)(3.00\text{ cm})}{(3.00M)} = 1.00\text{ cm}$$

$$y_{cm} = \frac{(2.00M)(4.00\text{ cm}) + (1.00M)(0.00\text{ cm})}{(3.00M)} = 2.6(6)\text{ cm}$$

$$\text{Distance from origin} = \sqrt{(x_{cm})^2 + (y_{cm})^2} = 2.85\text{ cm}$$

22. **Correct answer: D**

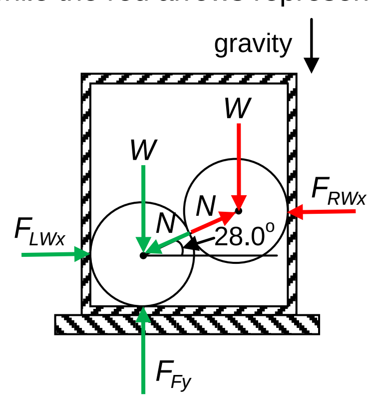
$$a_{centripetal} = \frac{V^2}{R} = \frac{(R\omega)^2}{R} = R\omega^2$$

$$\omega = \sqrt{\frac{a_{centripetal}}{R}} = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_{centripetal}}{R}} = \frac{1}{2\pi}\sqrt{\frac{(12.0)(9.80\text{ m/s}^2)}{18.0\text{ m}}} = 0.407\text{ rev/s}$$

23. **Correct answer: D**

The green arrows represent the forces acting on the left cylinder, while the red arrows represent the forces acting on the right cylinder.



The sum of forces acting on the right cylinder must equal zero.

1) $\text{x-direction:} \quad -F_{RWx} + N\cos(28.0^o) = 0$

2) $\text{y-direction:} \quad -W + N\sin(28.0^o) = 0$

The sum of forces acting on the left cylinder must equal zero.

3) $\text{x-direction:} \quad F_{LWx} - N\cos(28.0^o) = 0$

4) $\text{y-direction:} \quad -W - N\sin(28.0^o) + F_{Fy} = 0$

The sum of forces acting on the two cylinder combination must be zero:

5) $\text{x-direction:} \quad F_{LWx} - F_{RWx} = 0 \qquad 1) + 3)$

6) $\text{y-direction:} \quad -2W + F_{Fy} = 0 \qquad 2) + 4)$

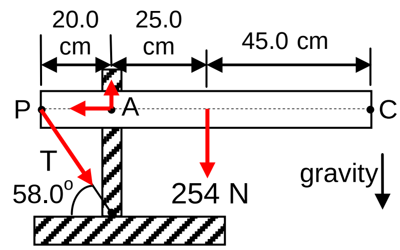
We want the force of the left wall (LW) on the left cylinder, so solving for N in equation 2) and substituting into 3), one finds:

4) $N\sin(28.0^o) = W \quad \rightarrow \quad N = \dfrac{W}{\sin(28.0^o)}$

3) $F_{LWx} = N\cos(28.0^o) = \dfrac{W}{\sin(28.0^o)}\cos(28.0^o) = 1.88W$

24. **Correct answer: E**

The red arrows show the forces acting on the plank. Since the plank is in static equilibrium, the sum of torques about point A must be zero. Since the pin at A is frictionless, the only forces causing a torque with respect to A are the weight of the plank and the tension in the cord.



$$T\sin(58.0^o)(20.0\text{ cm})\text{CCW} + (254\text{ N})(45.0\text{ cm} - 20.0\text{ cm})\text{CW} = 0$$
$$T\sin(58.0^o)(20.0\text{ cm}) = (254\text{ N})(45.0\text{ cm} - 20.0\text{ cm})$$
$$\text{T} = 374\text{ N}$$

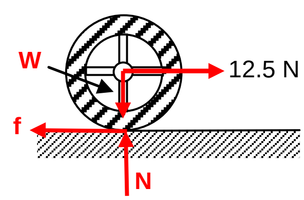
25. **Correct answer: A**

$$V_f^2 = V_i^2 + 2gd = 0 + 2gd$$
$$d = \left(\frac{1}{2g}\right)V_f^2 = \left(\frac{1}{19.6\text{ m/s}^2}\right)V_f^2 = \left(0.0510\frac{s^2}{m}\right)V_f^2$$
$$d = \left(0.0510\frac{s^2}{m}\right)V_f^2$$
$$y = (.....slope....)x$$
$$slope = 5.10\times10^{-2}\text{u}$$

26. **Correct answer: C**

$$\omega_f = \omega_i + \alpha t$$
$$\alpha = \frac{(\omega_f - \omega_i)}{t} = \frac{(15.0\text{ rad/s} - (-12.0\text{ rad/s}))}{5.75\text{ s}} = 4.70\text{ rad/s}^2$$

27. **Correct answer: A**



Using Newton's 2nd Law in the horizontal direction:

$$12.5\text{ N} - \text{f} = (16.0\text{ kg})(0.369\text{ m/s}^2)$$
$$f = 6.60\text{ N}$$

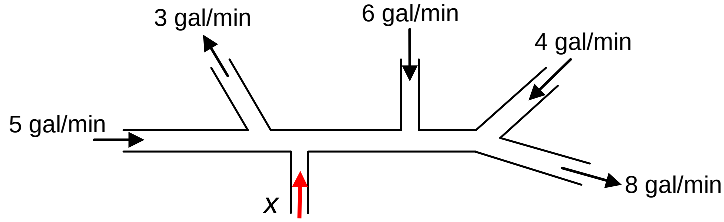
28. **Correct answer: A**

$$\alpha = \frac{\text{a}}{\text{R}} = \frac{0.369\text{ m/s}^2}{\frac{1}{2}(0.667\text{m})} = 1.11\text{rad/s}^2$$

29. **Correct answer: D**

The sum of inward currents = 0.

(Let in = positive, out = negative.)

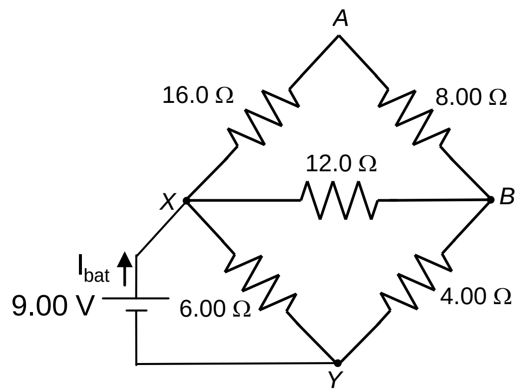


$+(5\text{ gal/min}) - (3\text{ gal/min}) + (6\text{ gal/min}) + 4\text{ gal/min}) - (8\text{ gal/min}) + x = 0$

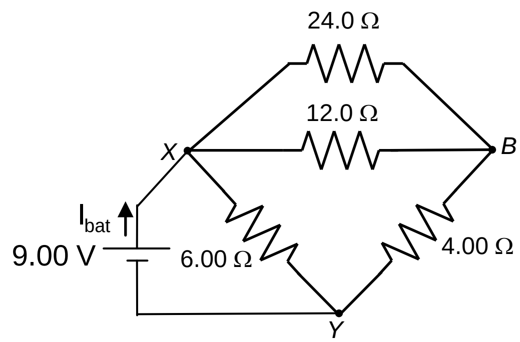
$x = -4\text{ gal/min}$

So 4 gal/min leaves the last pipe.

30. **Correct answer: E**

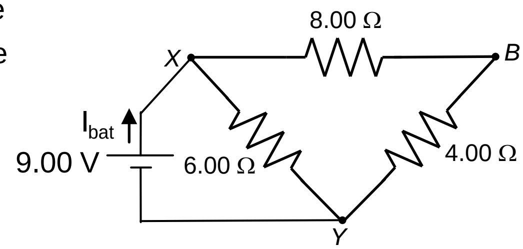


The 16.0 Ω resistor is in series with the 8.00 Ω resistor, since ONLY those two resistors connect to node A. They can be replaced with a 24.0 Ω resistor which will not cause the current in any branch to change.



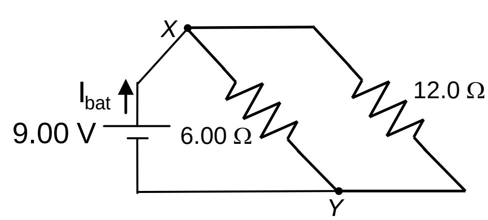
The resulting 24.0 Ω resistor is in parallel with the 12.0 Ω resistor, since those two resistors have nodes X and B in common. They can be replaced with a resistor of value, $\frac{24.0 \times 12.0}{24.0 + 12.0}\,\Omega = 8.00\Omega$.

This change will maintain the voltage difference from X to B, so that the currents in all the remaining branches will not be altered.



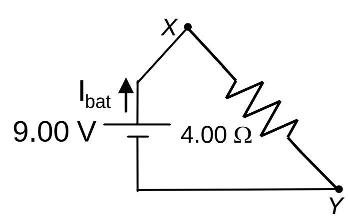
Now the resulting 8.00 Ω resistor is in series with the 4.00 Ω resistor, since ONLY those two resistors have node B in common. They can be replaced with a resistor of value, $(8.00 + 4.00)\,\Omega = 12.0\,\Omega$.

This change will maintain the currents in all the branches that remain.



Since the 12.0 Ω resistor and the 6.00 Ω resistor have nodes X and Y in common, they are in parallel. They can be replaced with a resistance of value $\frac{6.00 \times 12.0}{6.00 + 12.0}\,\Omega = 4.00\,\Omega$.

This change leaves the value of I_{bat} unchanged.



$$I_{bat} = \frac{9.00\text{ V}}{4.00\,\Omega} = 2.25\text{ A}$$

31. **Correct answer: E**

Using Archimedes principle, the bouyant force acting on a submerged object equals the weight of the displaced fluid.

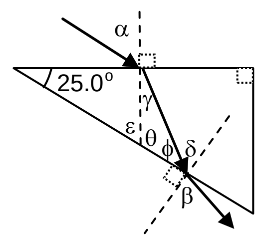
$$\text{Volume of displaced fluid} = (0.750)(1260\ \text{cm}^3) = 945\ \text{cm}^3$$
$$\text{Mass of displaced fluid} = (945\ \text{cm}^3)(1.00\ \text{g/cm}^3) = 945\ \text{g}$$
$$\text{Weight of displaced fluid} = (945\ \text{g})(980\ \text{cm/s}^2)$$

This weight of displaced fluid can support the same in load weight.

$$(m_{lead} + m_{can})(980\ \text{cm/s}^2) = (945\ \text{g})(980\ \text{cm/s}^2)$$
$$(m_{lead} + 152\ \text{g})(980\ \text{cm/s}^2) = (945\ \text{g})(980\ \text{cm/s}^2)$$
$$m_{lead} = 793\ \text{g}$$

32. **Correct answer: B**

Using Snell's Law and geometry:



$$1.00\sin(\alpha) = 1.32\sin(\gamma)$$
$$1.00\sin(20.0^o) = 1.32\sin(\gamma)$$
$$\gamma = 15.02^o$$
$$\phi + (\gamma + 90^o) + 25^o = 180^o \quad (\text{sum of angles in triangle} = 180^o)$$
$$\phi = 49.98^o$$
$$\delta + \phi = 90.0^o$$
$$\delta = 40.02^o$$
$$1.32\sin(\delta) = 1.00\sin(\beta)$$
$$\beta = 58.1^o$$

33. **Correct answer: D**

$$P = \sigma e A_{surface}\left(T^4_{surroundings} - T^4_{object}\right), \quad \text{T must be in Kelvin}$$
$$\text{P} = \sigma e 4\pi R^2\left(T^4_{surroundings} - T^4_{object}\right)$$
$$P = \left(5.6704\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(0.780)(4\pi)(0.250\ \text{m})^2\left[(129+273.15)^4 - (29+273.15)^4\right] = 619\ \text{W}$$

34. **Correct answer: D**

$$F = ma$$

$$qvB = m\frac{v^2}{R}$$

$$qB = m\frac{v}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right) = m\left(\frac{2\pi}{T}\right)$$

$$m = \frac{2eBT}{2\pi} = \frac{\left(1.602\times10^{-19}C\right)\left(0.164T\right)\left(0.0234s/6\right)}{\pi} = 3.26\times10^{-23}kg$$

35. **Correct answer: B**

$$d_o = ?$$

$$R = 48.0\ \text{cm}$$

$$\frac{h_i}{h_o} = 2.80$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = 2.80$$

$$d_i = -2.80 d_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{1}{R/2} = \frac{2}{R}$$

$$\frac{1}{d_o} + \frac{1}{-2.80 d_o} = \frac{2}{R}$$

$$\frac{1}{d_o}\left(1 - \frac{1}{2.80}\right) = \frac{1}{24.0\ \text{cm}}$$

$$d_o = 15.4\ \text{cm}$$