



Worldwide Youth in Science and Engineering

## 2014 Academic Challenge

### MATHEMATICS TEST - SECTIONAL

This Test Consists of 40 Questions

#### Mathematics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader

Linda Wiggins, Illinois State University – Author

Matthew Childers, Illinois State University – Author

Sandra Cox, Kaskaskia College – Reviewer

Mary Weaver, WYSE – Coordinator of Test Production

#### GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

**\*\*\* TIME: 40 MINUTES \*\*\***

**DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!**

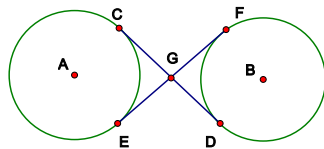
© 2014 Worldwide Youth in Science and Engineering

“WYSE”, “Worldwide Youth in Science and Engineering” and the “WYSE Design” are service marks of and this work is the Copyright © 2014 of the Board of Trustees of the University of Illinois at Urbana - Champaign.

All rights reserved.

WYSE Academic Challenge  
Mathematics Test (Sectional) – 2014

1. Three invertible matrices of the same size are multiplied together. Which of the following is not necessarily true of the product?
  - a) The determinant is not zero.
  - b) The determinant is not negative.
  - c) The result has the same dimensions.
  - d) The product is invertible.
  - e) All of the above must be true.
  
2. A friend borrows \$2000 from you and agrees to pay you back in 6 years paying you interest of 1.5% compounded monthly. What is the total interest the friend will pay you in 6 years?
  - a) \$2188.23
  - b) \$2180.00
  - c) \$2030.22
  - d) \$188.23
  - e) \$30.22
  
3. Given three vectors  $A = \langle 1, 7 \rangle$ ,  $B = \langle 2, 4 \rangle$  and  $C = \langle -4, 6 \rangle$ , what would be their equilibrant vector?
  - a)  $\langle -1, 17 \rangle$
  - b)  $\langle 1, -17 \rangle$
  - c)  $\langle 7, 5 \rangle$
  - d)  $\langle -5, 9 \rangle$
  - e)  $\langle -8, 168 \rangle$
  
4. The pulley system in a certain town clock tower with a belt of negligible thickness is wrapped around two identical wheels each with a radius of 0.5 feet. The belt criss-crosses at a point G between the two circles so that the measure of the angle CGE is 60 degrees. Segments CGD and EGF are considered to be tangent segments. See diagram below.



On a particular day it was found that a mouse traveled clockwise on a moving belt starting at point E and ending at point C. How far did the mouse travel? Round to the nearest inch

- a) 2 in
  - b) 12 in
  - c) 13 in
  - d) 25 in
  - e) 30 in
- 
5. Given  $\frac{A}{B+C} = \frac{B}{A+C}$  where  $A > B > C > 0$ , solve for A.
    - a)  $A = B^2$
    - b)  $A = B + C$
    - c)  $A = \frac{-C \pm \sqrt{C^2 - 4BC + B^2}}{2}$
    - d)  $A = \frac{-C \pm \sqrt{C^2 + 4BC + B^2}}{2}$
    - e) no solution

6. What is the derivative with respect to  $x$  of  $y = \sin(x^2 + 3x)$ ?
- $\cos(2x + 3)$
  - $\cos(x^2 + 3x)$
  - $-\cos(x^2 + 3x)$
  - $(2x + 3)\cos(x^2 + 3x)$
  - $(2x + 3)\sin(x^2 + 3x)$
7. Find the quotient of  $\frac{1}{(x+y)^2} - \frac{1}{x^2}$  and  $y$ . Simplify as much as possible.
- $\frac{2xy + y^2}{x^4 - 2x^3y - x^2y^2}$
  - $\frac{2 - y^2}{4x^2 + 2x^3 + xy}$
  - $\frac{2xy - y^2}{4x^2y^3}$
  - $\frac{2xy + y^2}{x^4 - 2xy + y^2}$
  - $\frac{-2x - y}{x^4 + 2x^3y + x^2y^2}$
8. A cube has one inch edges. Determine the distance between opposite vertices (vertices that do not share either an edge or a face). Round your answer to the nearest hundredth of an inch.
- 1.41 in
  - 1.50 in
  - 1.73 in
  - 1.83 in
  - 2.00 in
9. Let  $r = 2 + 3\theta$  with  $\theta$  in radians. What is the distance, in radians, between successive turns of the spiral?
- $3\pi$
  - $\frac{\pi}{3}$
  - $6\pi$
  - $\frac{2\pi}{3}$
  - 3
10. Find the volume of a cube if it takes 12 tubes of paint to paint the surface area of the cube when it is known that a tube of paint covers 4.5 square feet.
- 19 cu. ft.
  - 24 cu. ft.
  - 27 cu. ft.
  - 30 cu. ft.
  - 45 cu. ft.
11. \$6000 is compounded quarterly at 9% APR. If the account is left dormant for seven years, how much interest will have accrued? Please round to the nearest cent.
- \$1011.23
  - \$4968.23
  - \$5187.27
  - \$11187.27
  - \$61002.84

12. You own a triangular parcel of land with an area of 180 square feet. You will be planting 3 spruce trees, one at each corner of the triangular parcel. The first tree is planted 9 ft. from the second tree. The third tree is planted 41 ft. from the second tree. How far is the third tree from the first?
- a) 40 ft.                      b) 32 ft.                      c) 25 ft.                      d) 20 ft.                      e) 12 ft.
13. A ten foot ladder is leaned up against a wall that is eight feet tall. Determine the angle the ladder makes with the ground. Round your answer to the nearest degree.
- a)  $45^\circ$                       b)  $50^\circ$                       c)  $53^\circ$                       d)  $57^\circ$                       e)  $60^\circ$
14. What is the eccentricity of the ellipse  $4x^2 + 9y^2 = 25$ ? Please round your answer to the nearest hundredth.
- a) 0.00                      b) 0.56                      c) 0.75                      d) 1.00                      e) 1.34
15. Find the product of  $6x^3 - 14x^2 + 22x - 6$  and  $\frac{1}{6x - 2}$ .
- a)  $6x^3 - 14x^2 + 28x - 8$   
b)  $x^2 - 2x + 3$   
c)  $6x^3 - 14x^2 + 16x$   
d)  $x^3 - 2x^2 + 3x - 4$   
e)  $6x^3 - 20x^2 + 3$
16. In a large group of students, 80% of them have jobs, and 70% of those that have jobs also own their own car. If 60% of all the students own their own car, what percent of students that own their own car also have jobs? Round your answer to the nearest percent.
- a) 50%  
b) 53%  
c) 70%  
d) 93%  
e) Situation can't exist
17. A triangle is created with sides 3 and 5 and their included angle has a (radian) measure of  $\frac{6\pi}{7}$ . What is the area of this triangle? Please round your answer to the nearest tenth of a square unit.
- a) 0.4                      b) 1.7                      c) 3.3                      d) 6.8                      e) 7.5
18. Water is continuously collected in a water fountain's holding tank such that during a given hour, only 92% as much water is collected as in the previous hour. If 28 gallons are collected in the first hour, what must be the minimum capacity of the tank?
- a) 350 gal                      b) 280 gal                      c) 125 gal                      d) 64 gal                      e) 30 gal

19. In a 5-12-13 right triangle, what is the length of the altitude to the hypotenuse? Round to the nearest tenth of a unit.
- a) 0.2                      b) 2.4                      c) 2.6                      d) 4.6                      e) 5.0
20. The function  $h(t) = -16t^2 + 80t + 5$  takes  $t$ , the number of seconds since a ball was tossed upwards, and gives  $h$ , the distance in feet from the ball to the ground. If the ball is falling at a speed of 20 feet per second, determine how many seconds it has been since the ball was tossed. Round your answer(s) to the nearest tenth of a second.
- a) 1.9 seconds  
 b) 3.1 seconds  
 c) 3.8 seconds  
 d) 4.8 seconds  
 e) The ball never achieves this speed
21. In seven card stud, how many ways can you draw exactly two pairs?
- a) 2,304  
 b) 12,870  
 c) 154,440  
 d) 29,652,480  
 e) 1,423,319,040
22. Solve the following system of equations for the variable  $x$  by using determinants. Provide the set up and the value of the numerator's determinant.

$$\begin{aligned} 2x + y + 2z &= 8 \\ 3x - 2y - 4z &= 5 \\ -2x + 3y + 4z &= -3 \end{aligned}$$

a)  $x = \frac{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} 8 & 1 & 2 \\ 5 & -2 & -4 \\ 3 & 3 & 4 \end{vmatrix}}; 14$

b)  $x = \frac{\begin{vmatrix} 8 & 1 & 2 \\ 5 & -2 & -4 \\ -3 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}}; 42$

c)  $x = \frac{\begin{vmatrix} 8 & 1 & 2 \\ 3 & 5 & -4 \\ -2 & 3 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 5 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}}; 31$

d)  $x = \frac{\begin{vmatrix} 2 & 8 & 2 \\ 3 & 5 & -4 \\ -2 & -3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}}; -14$

e)  $x = \frac{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 8 \\ 3 & -2 & 5 \\ -2 & 3 & -3 \end{vmatrix}}; 14$

23. Find a solution for the equation  $6 \sin x - 3 \tan x = 0$  where  $0 \leq x \leq 2\pi$ .
- a)  $\frac{7\pi}{4}$                       b)  $\frac{5\pi}{4}$                       c)  $\frac{2\pi}{3}$                       d)  $\frac{\pi}{2}$                       e)  $\frac{\pi}{3}$
24. A large tank has two input valves and one drain. The first input valve can fill the tank in 50 minutes, the second valve can fill the tank in 40 minutes, and the drain can empty the tank in 25 minutes. If the first valve is turned on at noon, the second turned on at 12:10 PM, and the drain opened at 12:20 PM, determine when the tank is either completely empty or completely full. Round to the nearest minute.
- a) Empties at 1:15 PM  
 b) Fills at 1:05 PM  
 c) Fills at 1:10 PM  
 d) Fills at 1:15 PM  
 e) Fills at 1:30 PM
25. On a reality show, sixteen people are pitted against one another and, in musical chairs fashion, one at a time is eliminated from the competition until a winner is selected from the last two. If a random group of three is chosen, how many elimination orders may be formed which place our triumvirate in the top three?
- a) 3,360  
 b) 6,227,020,800  
 c) 37,362,124,800  
 d) 3,487,131,648,000  
 e) 20,922,789,888,000
26. Compare the two parametric equations, I and II, below. Then choose the statement that best represents the characteristics of the two equations.
- I.  $x = 3 \cos(-t)$   
 $y = 2 \sin(-t)$
- II.  $x = 3 \cos(-2t)$   
 $y = 2 \sin(-2t)$
- a) I is an ellipse while II is a circle. They travel at the same rate.  
 b) I and II are both parabolas that concave downward. II concaves downward twice as fast as I.  
 c) I and II represent the same ellipse. II moves twice as fast clockwise around the ellipse as I.  
 d) I and II represent the same circle. I moves twice as fast counter clockwise around the circle as II.  
 e) I and II represent cardioids where II is twice as large as I.
27. Thirteen keys are on a keyring, including one set of triplicate keys. What is the probability that the triplicate keys are not in a group of three together? Please round to the nearest percent.
- a) 4%                      b) 50%                      c) 77%                      d) 96%                      e) 100%

28. Given  $3^x = \frac{1}{81}$ , solve for  $x$ .
- a)  $x = -4$       b)  $x = 4$       c)  $x = -12$       d)  $x = 16$       e)  $x = -27$
29. An ice cream cone shaped object is created by placing a hemisphere on top of a circular cone. If the hemisphere is three inches in diameter and the entire object measures eight inches from top to bottom, what is the volume of the object? Round your answer to the nearest cubic inch.
- a) 22 cu in      b) 29 cu in      c) 33 cu in      d) 75 cu in      e) 104 cu in
30. Which of the following is not preserved by a size change transformation?
- a) Angle measure  
b) Betweenness  
c) Collinearity  
d) Distance  
e) All of the above are preserved
31. Given  $\frac{1}{2}\log(x + 4) + \log 5 = 1$ , solve for  $x$
- a)  $x = -4$       b)  $x = 0$       c)  $x = 12$       d)  $x = 21$       e)  $x = 32$
32. A cylinder is circumscribed about a sphere with radius 12". What percentage of its volume is contained within the sphere? Round to the nearest percent.
- a) 25%      b) 50%      c) 60%      d) 67%      e) 100%
33. If the diagonals of a quadrilateral bisect each other, which of the following must be true about the quadrilateral?
- a) The quadrilateral must have all four sides the same length  
b) The quadrilateral must have all four angles be right angles  
c) The opposite sides of the quadrilateral must be parallel  
d) All of the above are required to be true  
e) None of the above are required to be true
34. Perform the indicated operation and simplify the complex number to its rectangular form:  
 $-\sqrt{-100} + 3i$
- a)  $10 + 3i$       b)  $-10 + 3i$       c)  $-7i$       d)  $13i$       e)  $-3$
35. What is the remainder when  $P(x) = x^3 - 6x + 11$  is divided by  $Q(x) = x + 7$ ?
- a) -290      b) -4      c) 4      d) 18      e) 102

36. A restaurant menu has a family feast option where one selects 3 side dishes and 2 main entrées. If there are 9 options for side dishes and 6 options for main entrées with no option of doubling up on any side or entrée, how many ways can the family feast be made?
- a) 54                      b) 324                      c) 972                      d) 1260                      e) 26,244
37. When a company made 2000 items, they spent \$3000. When they made 3000 items, they spent \$3500. Assuming a linear relationship between items made and cost, how much must the company spend if they want to make 3500 items?
- a) \$3575                      b) \$3750                      c) \$3800                      d) \$4000                      e) \$5250
38. Which of the following is a solution to  $(x - 9)(x - 1) = -16$  ?
- a) -7                      b) -4                      c) 1                      d) 5                      e) 9
39. Given  $y = \ln \frac{e^x}{e^x + 2}$ , find  $\frac{dy}{dx}$ .
- a)  $\frac{dy}{dx} = \frac{-2}{e^x + 2} + x$
- b)  $\frac{dy}{dx} = \frac{e^x + 2}{e^x} - 1$
- c)  $\frac{dy}{dx} = x - \frac{e^x}{e^x + 2}$
- d)  $\frac{dy}{dx} = \frac{-e^x + 2}{2}$
- e)  $\frac{dy}{dx} = \frac{2}{e^x + 2}$
40. A game club consisting of Andrea (f), Benedict (m), Cheryl (f), Don (m), Elise (f), and Felicia (f) recently held three fundraising events: a rummage sale, a game-a-thon, and a fashion show. Three members went to only one event, three members went to two events, and no one shared attendance schedules. Also:
- I. Each member earned a multiple of \$10 at each event with no event values repeated
  - II. The overall total was \$450, and all event totals and individual totals were different
  - III. The lowest event total was \$20 less than the highest event total
  - IV. Benedict made \$40 more at the rummage sale than he did at the game-a-thon
  - V. Elise's total of \$50 from the rummage sale and fashion show was the lowest total
  - VI. At Felicia's only event, she made the same as the other two girls there combined
  - VII. Andrea made \$50 less than Don at her 1<sup>st</sup> event, but \$40 more than Elise at her 2<sup>nd</sup> event
- Determine who earned the most money for the club.
- a) Andrea                      b) Benedict                      c) Cheryl                      d) Don                      e) Felicia