WYSE MATH SECTIONAL 2014 SOLUTIONS

1. Ans B: (a) and (d) are equivalent. Therefore since the determinant of the product of matrices is the product of their determinants, it too cannot be zero. If three square matrices of the same dimensions are multiplied together, the result has the same dimensions. The only thing which isn't necessarily true is (b), as if there are three matrices with a negative determinant or a set of two with positive determinant and one with negative determinant, the product would have a negative determinant.

2. Ans D: Using $FV = PV\left(1+\frac{r}{m}\right)^{mt}$ and $INT = FV - PV$, we get $FV = 2000\left(1+\frac{0.015}{12}\right)^{12(6)}$. $FV = 2188.225579$. $INT = 2188.225579 - 2000$. So the interest is about $188.23.

3. Ans B: The equilibrant vector to a set of vectors is the opposite of their sum as it serves to "balance" them. The sum of these three vectors is (a) so the equilibrant would be (b).

4. Ans D: Since $\angle CGE$ is 60°, then $\angle CAE$ is 120° which represents $\frac{1}{3}$ angle measure of the circle. So arc CE represents $\frac{1}{3}$ the circumference of circle A. Then arc EC represents $\frac{2}{3}$ circumference of the circle. $C = 2\pi(6) \Rightarrow 12\pi$. Then $\frac{2}{3}$ the circumference is $8\pi \approx 25in$.

5. Ans E: $\frac{A}{B+C} = \frac{B}{A+C} \Rightarrow A(A+C) = B(B+C) \Rightarrow A^2 + AC = B^2 + BC \Rightarrow$ $A^2 + AC - B^2 - BC = 0$. $A = \frac{-C \pm \sqrt{C^2 - 4(1)(-B^2 - BC)}}{2(1)} \Rightarrow A = \frac{-C \pm \sqrt{C^2 + 4BC + 4B^2}}{2} \Rightarrow$ $A = \frac{-C \pm \sqrt{(C+2B)^2}}{2} \Rightarrow A = \frac{-C \pm (C+2B)}{2}$. Giving us solutions $A = B$ and $A = -C - B$, both of which violate the original assumptions. Therefore, we have no solution.

6. Ans D: By the Chain Rule,
$\frac{d}{dx}\left[\sin\left(x^2+3x\right)\right] = \cos\left(x^2+3x\right) \bullet \frac{d}{dx}\left[x^2+3x\right] = (2x+3)\cos\left(x^2+3x\right)$.

7. Ans E: $\frac{1}{(x+y)^2} - \frac{1}{x^2} \Rightarrow \frac{x^2-(x+y)^2}{x^2(x+y)^2} \Rightarrow \frac{-\left(2xy+y^2\right)}{x^4+2x^3y+x^2y^2}$. Then $\frac{-\left(2xy+y^2\right)}{x^4+2x^3y+x^2y^2} \div y$ gives us $\frac{-\left(2xy+y^2\right)}{x^4+2x^3y+x^2y^2} \bullet \frac{1}{y} \Rightarrow \frac{-y(2x+y)}{y\left(x^4+2x^3y+x^2y^2\right)} \Rightarrow \frac{-2x-y}{x^4+2x^3y+x^2y^2}$.

8. Ans C: Use Pythagorean theorem twice, once to find the diagonal of a face, then to find the distance between the opposing vertices. The diagonal of a face is $\sqrt{2}$, the distance between the opposing vertices is $\sqrt{3} \approx 1.73205$.

9. Ans C: $r = a + b\theta$ has a distance between successive turns of $2\pi b$.

10. Ans C: $6s^2 = 4.5(12) \Rightarrow s^2 = 9 \Rightarrow s = 3$. Then $V = 3^3 \Rightarrow V = 27$.

11. Ans C: If P dollars are invested at an interest rate of r (written as a decimal, not a percentage), compounded n times per year for t years, the approximate total in the account after that time is $A = P\left(1+\frac{r}{n}\right)^{nt}$. In this scenario, P = 6000, r = .09, n = 4 and t = 7. Please note that this would give us (d), but that counts the principal as well as the interest.

12. Ans A: Suppose the 3rd tree is (c) 25 ft. Then using Heron"s formula we find that $s = 0.5(9+41+25) \Rightarrow 37.5$. So $A = \sqrt{37.5(28.5)(-3.5)(12.5)}$. But we can't have a negative under the radical so the distance must be greater than 25 ft. We then assign the distance to be 32 ft. Here $s = 0.5(9+41+32) \Rightarrow 41$. Then the area is $A = \sqrt{41(32)(41-41)(9)}$. But this produces 0. The distance must be greater than 32 ft. The distance must be 40 ft. With this length the value $s = 0.5(9+41+40) \Rightarrow 45$. The area is $A = \sqrt{45(36)(4)(5)} \Rightarrow 180$. This is the given area. The distance is 40 ft.

13. Ans C: Simply solve $\sin\theta = \frac{8}{10}$ and get the angle as 53.1301 degrees.

14. Ans C: For an ellipse, the eccentricity is $e = \sqrt{\frac{a^2 - b^2}{a^2}}$. It's easier to find a and b if the ellipse is placed in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (We'll see that the major axis is on the x side a little bit down the road.) First, we divide the equation by 25 to get $\frac{4}{25}x^2 + \frac{9}{25}y^2 = 1$. This is equivalent to $\frac{x^2}{25/4} + \frac{y^2}{25/9} = 1$. So $a^2 = \frac{25}{4}$ and $b^2 = \frac{25}{9}$, which means $e = \sqrt{\frac{25/4 - 25/9}{25/4}}$. By multiplying the numerator and denominator of the fraction by 36 and then subsequently dividing each by 25, we get $e = \sqrt{\frac{9-4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$.

15. Ans B: $\frac{6x^3 - 14x^2 + 22x - 6}{6x - 2} \Rightarrow \frac{2(3x-1)(x^2 - 2x + 3)}{2(3x-1)} \Rightarrow x^2 - 2x + 3$.

16. Ans D: Out of all students, 0.80*0.70, or 56% both own their own car and have jobs. Out of the 60% that have cars, this represents 56/60, or about 93%.

17. Ans C: The area of a triangle when two sides and the angle included between them are given is $A = \frac{1}{2}ab\sin\theta$.

18. Ans A: $r = 0.92$ and $a_1 = 28$. Then $S = \frac{28}{1-0.92} \Rightarrow \frac{28}{0.08} \Rightarrow 350$.

19. Ans D: Let θ be the angle opposite the side with measure 5. Let a be the measure of the altitude. On the large triangle, the tangent of θ will be $\frac{5}{12}$. On the small triangle which is created with a hypotenuse of 5, the tangent of θ will be $\frac{x}{a}$. So $\frac{x}{a} = \frac{5}{12}$ and $x = \frac{5}{12}a$. By the Pythagorean theorem on that small triangle, $a^2 + x^2 = 5^2$ and thus $a^2 + \left(\frac{5}{12}a\right)^2 = 25$. Then $a^2 + \frac{25}{144}a^2 = 25$ and $\frac{169}{144}a^2 = 25$. So $a^2 = \frac{3600}{169}$ and $a = \frac{60}{13}$.

20. Ans B: $h'(t) = -32t + 80t$ tracks speed. Solving $-32t + 80 = -20$ gives us $t = 3.125$.

21. Ans D: There are C(13, 2) ways to pick the denominations which will be paired and then there are C(4, 2) ways to pick the two cards of those denominations. There are C(11, 3) ways to pick the three denominations of which there are single cards chosen, and four ways to choose the single cards from each of those denominations. Then by the multiplication principle, there are $C(13,2) \bullet C(11,3) \bullet C(4,2)^2 \bullet 4^3 = 29{,}652{,}480$ ways to choose the hand with two pairs in it.

22. Ans B: Using Cramer's Rule, $x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \Rightarrow \frac{\begin{vmatrix} 8 & 1 & 2 \\ 5 & -2 & -4 \\ -3 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}}$. The determinant of the numerator is $\begin{vmatrix} 8 & 1 & 2 \\ 5 & -2 & -4 \\ -3 & 3 & 4 \end{vmatrix} \Rightarrow 8(-8+12) - 1(20-12) + 2(15-6) = 42$.

23. Ans E: $6\sin x - 3\tan x = 0 \Rightarrow 2\sin x = \tan x \Rightarrow 2\sin x = \frac{\sin x}{\cos x} \Rightarrow 2\sin x \cos x = \sin x$. Then $\sin 2x = \sin x$. So $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$. Only $\frac{\pi}{3}$ is one of the listed solutions.

24. Ans E: The first valve fills the tank at a rate of 1/50 = 0.02 tank per minute, the second fills at a rate of 1/40 = 0.025 tank per minute, and the drain empties the tank at 1/25 = 0.04 tank per minute. Since the two valves have a greater combined rate than the drain, the tank will fill when all three are open. In the first 10 minutes, the tank fills to 0.02*10 = 0.2 tank. After 20 minutes, the tank is up to 0.2 + 10*(0.02 + 0.025) = 0.65 tank. Solving the equation 1 = 0.65 + x*(0.02 + 0.025 – 0.04) tells us that the tank fills up 70 minutes after the drain was opened, which would be 1:30 PM.

25. Ans C: There are 13! ways to order the ones who aren't the top three and 3! ways to order the top three. The product of 13! and 3! is (c).

26. Ans C: The parametric equation (I) produces $\frac{x}{3} = \cos(-t)$ and $\frac{y}{2} = \sin(-t)$. Using the identity $\cos^2(-t) + \sin^2(-t) = 1$, by substitution we have $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$. Similarly the second parametric equation produces $\frac{x}{3} = \cos(-2t)$ and $\frac{y}{2} = \sin(-2t)$. Using the identity $\cos^2(-2t) + \sin^2(-2t) = 1$, substitution gives us $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$. Both give the same equation for an ellipse. Finally, making a table of points for both parametric equations we find the following:

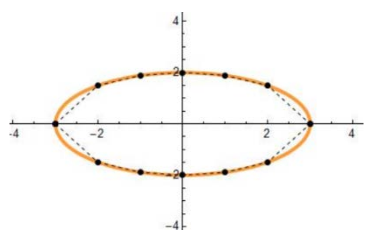
I.

t	x	y
0	3	0
$\frac{\pi}{2}$	0	-2
π	-3	0

II.

t	x	y
0	3	0
$\frac{\pi}{2}$	-3	0
π	3	0

Using the graph below, we can see the first parametric equation travels a quarter of the ellipse in a clockwise direction from time t = 0 to t = $\frac{\pi}{2}$ while the second equation, travels half of the ellipse in the same time.



27. Ans D: There are 12! possible arrangements of those keys in a ring. There are 10! ways to arrange the non-triplicate keys and 3! ways to arrange the triplicate keys. Thus there are 21,772,800 ways to arrange the keys in a way that the three are next to one another, so there's a 1 in 22 chance that they're next to one another (and a 21 in 22 chance that they're not all found together).

28. Ans A: $\frac{1}{81} \Rightarrow 81^{-1} \Rightarrow \left(3^4\right)^{-1} \Rightarrow 3^{-4}$. Then $3^x = 3^{-4} \Rightarrow x = -4$.

29. Ans A: For a hemisphere, $V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$, and for a cone, $V = \frac{1}{3}\pi r^2 h$. The given information tells us r = 1.5 and h = 6.5, so $V = \frac{1}{2} \cdot \frac{4}{3}\pi \cdot 1.5^3 + \frac{1}{3}\pi \cdot 1.5^2 \cdot 6.5 \approx 22.384$.

30. Ans D: Angle measure, betweenness and collinearity are all preserved. Distance is not as "size change" means that distances will be changed.

31. Ans B: $\frac{1}{2}\log(x+4)+\log 5=1 \Rightarrow \log\sqrt{x+4}+\log 5=1 \Rightarrow \log 5\sqrt{x+4}=1$. Rewrite as an exponential equation we have $10^1=5\sqrt{x+4} \Rightarrow 2=\sqrt{x+4} \Rightarrow x=0$.

32. Ans D: The radius of the cylinder would be 12" while its height would be 24". So, for the cylinder, the volume would be $V=\pi r^2 h=3456\pi$ cubic inches. For the sphere, the volume would be $V=\frac{4}{3}r^3=2304\pi$ cubic inches, which is two-thirds of the volume of the cylinder.

33. Ans C: The quadrilateral must be a parallelogram due to similar triangles, but none of the other properties are guaranteed simply from having the diagonals bisected.

34. Ans C: $-\sqrt{-100}+3i \Rightarrow -\sqrt{100}\sqrt{-1}+3i \Rightarrow -10i+3i \Rightarrow -7i$.

35. Ans A: By the Remainder Theorem, when a polynomial P(x) is divided by a simple linear function Q(x) = x – c, P(c) is the remainder. P(-7) = -290.

36. Ans D: ${}_9C_3 \bullet {}_6C_2=1260$

37. Ans B: The additional cost per item is (3500-3000)/(3000-2000) = $0.5 per item. For the additional 500 items, it will cost 500*0.5 = $250 above the $3500, or $3750. A quick check also reveals that the equation is cost = 0.5 * items + 2000

38. Ans D: We first need to multiply out the left-hand side to get $x^2-10x+9=-16$. If we add 16 to both sides, we get $x^2-10x+25=0$. Factoring the left-hand side gives us $(x-5)^2=0$. Then x has to be 5.

39. Ans E: $y=\ln\frac{e^x}{e^x+2} \Rightarrow \ln e^x-\ln\left(e^x+2\right) \Rightarrow y=x-\ln\left(e^x+2\right)$. $\frac{dy}{dx}=1-\frac{e^x}{e^x+2}$ $\Rightarrow \frac{e^x+2-e^x}{e^x+2}=\frac{2}{e^x+2}$.

40. Ans A: Based on I, II, and III, the nine individual event values must be $10, $20, $30, $40, $50, $60, $70, $80, and $90. The three event totals must be $140, $150, and $160. Based on V, the six individual totals must be $50, $60, $70, $80, $90, and $100. First place who was at what event(s). B was at RS and GT, and E was at RS and FS. Since A was also at two events, she must be at FS and GT. F must be at FS. D must have been at GT with A, which means C was at RS. If we call E's FS value x, then A's FS value is x + 40, and F's FS value is x + (x + 40). Adding these up gives us 4x + 80. If we set this equal to the three possible totals, 4x + 80 = 140 gives x = 15, 4x + 80 = 150 gives x = 17.5, and 4x + 80 = 160 gives x = 20, the only acceptable result. E's FS value is $20, A's FS value is $60, and F's FS value is $80. This means E's RS value must be $30. To have a gap of $50 between A and D at GT, A's GT value must be $40 and D's must be $90. To get a gap of $40 for B 's event values, $10 must be his GT value to make the total $140, leaving $50 for his RS value. By default, C's RS value is $70. For the final totals, we have E = $50, B = $60, C = $70, F = $80, D = $90, and A = $100.