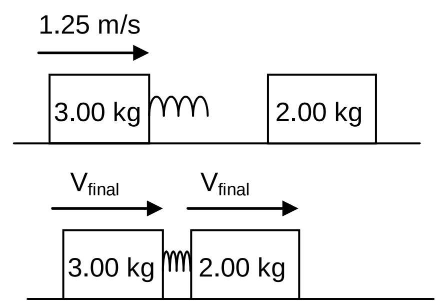
WYSE Academic Challenge
2014 Sectional Physics Exam
SOLUTION SET

1. **Correct answer: E**

Units of Torque / units of power: $\dfrac{[r][F]}{\left(\dfrac{[E]}{[t]}\right)} = \dfrac{[r][F][t]}{[E]} = \dfrac{[r][F][t]}{[F][d]} = \dfrac{[r][t]}{[d]} = \dfrac{m \cdot s}{m} = s$

2. **Correct answer: D**



The net external force (and therefore the impulse) acting on the two block system is zero. For example, the upward normal force that the surface exerts on each block is equal and opposite to the gravitational force on each block. The force that the spring force exerts on the 2.00 kg block is equal and opposite to the force that the spring exerts on the 3.00 kg block. Therefore the momentum of the two block system must be conserved.

$$(m_{3kg}V_{3kg,init}) + (m_{2kg}V_{2kg,init}) = (m_{3kg}V_{3kg,final}) + (m_{2kg}V_{2kg,final})$$
$$(3.00\text{ kg} \cdot 1.25\text{ m/s}) + (2.00\text{ kg} \cdot 0.00\text{ m/s}) = (5.00\text{ kg} \cdot \mathrm{V}_{final})$$
$$\mathrm{V}_{final} = 0.750\text{ m/s}$$

3. **Correct answer: E**

There are no external forces that do work on the system consisting of the two blocks and spring. Therefore the energy of that system must be constant. Thus the initial kinetic energy of the 3.00 kg block must be equal to the kinetic energy of the two blocks in the final state plus the elastic potential energy in this final state. In equation form:

$$\left(\frac{1}{2}m_{3kg}V_{3kg,init}^2\right) = \left(\frac{1}{2}m_{3kg}V_{3,final}^2\right) + \left(\frac{1}{2}m_{2kg}V_{2,final}^2\right) + \left(\frac{1}{2}kx^2\right)$$

4. **Correct answer: C**

At the top of its trajectory, the projectile has only a horizontal component of velocity. This horizontal component of velocity is constant since there are no forces acting on the projectile in the horizontal direction. Thus the horizontal component of velocity at the top is the same as at launch:
$V_{horiz} = (56.0\text{ m/s})\cos(65.1^{o}) = 23.6\text{ m/s}.$
So $V_{top} / V_{launch} = 23.6 / 56.0 = 0.421.$

5. **Correct answer: D**

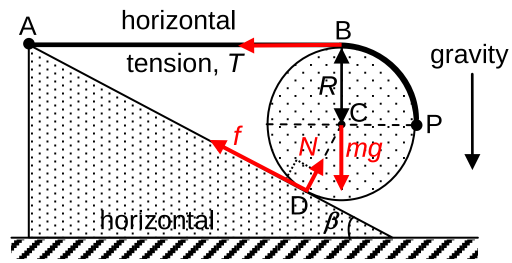
Using conservation of energy, the sum of gravitational potential energy plus kinetic energy is constant.

$$GPE_{initial} + KE_{initial} = GPE_{final} + KE_{final}$$
$$46.0\text{ J} + 74.0\text{ J} = 72.0\text{ J} + KE_{final}$$
$$48.0\text{ J} = KE_{final} = \frac{1}{2}mV_{final}^2 = \frac{1}{2}(3.00\text{ kg})V_{final}^2$$
$$V_{final} = 5.66\text{ m/s}$$

6. **Correct answer: A**

$$GPE_{final} - GPE_{initial} = 72.0\text{ J} - 46.0\text{ J} = mg\Delta h$$
$$\Delta h = \frac{26.0\text{ J}}{(3.00\text{ kg})(9.80\text{ m/s}^2)} = 0.884\text{ m}$$

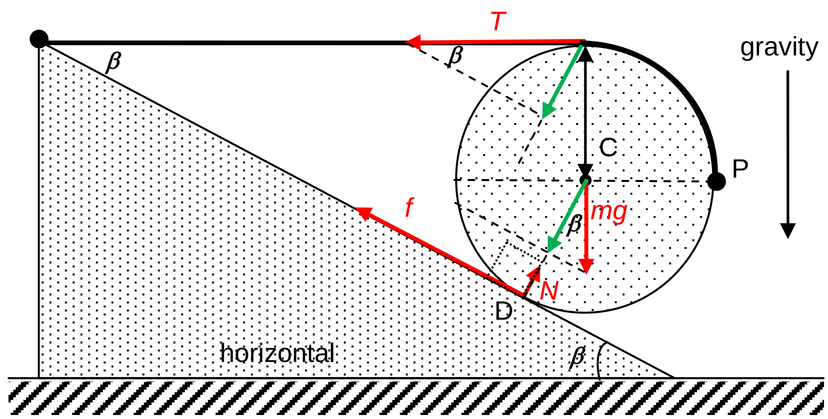
7. **Correct answer: A**



Four forces act upon the cylinder, the tension (T) in the string pulling to the left at B, the frictional force (f) at D acting along the plane as shown, the normal force (N) of the plane at D acting along a line from D to C, and the gravitational force (mg) acting downward at C. With respect to a line perpendicular to the page through point C, N and mg do not produce torques since their lines of action pass through point C. so the only forces producing torques with respect to C are the frictional force and the tension. Since the cylinder is in static equilibrium, the sum of these two torques must be zero.

$$\Sigma Torques_{wrt,C} = RT\sin(90.0^o)ccw + Rf\sin(90.0^o)cw = 0$$
$$RT\,ccw = -Rf\,cw$$
$$RT\,ccw = Rf\,ccw$$
$$RT = Rf$$

8. **Correct answer: B**



Resolving all forces along the normal (to the plane) direction:

Normal: $N\cos(0^o) = N$

Friction: $f\cos(90^o) = 0$

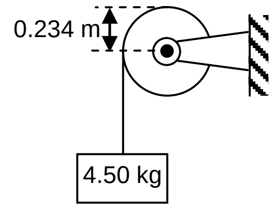
Tension: $-T\sin(\beta)$

Weight: $-mg\cos(\beta)$

The sum of forces in the normal direction must be zero. $N - mg\cos(\beta) - T\sin(\beta) = 0$

9. **Correct answer: D**

Applying Newton's 2nd Law to the falling mass and letting down be positive:

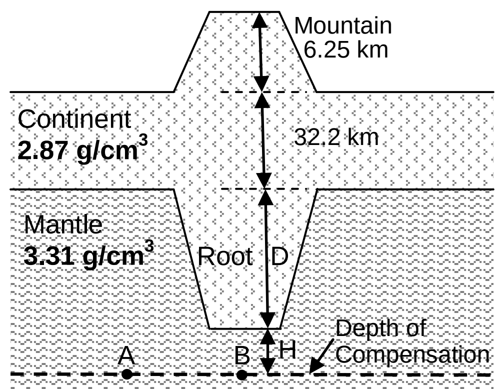


$$mg - T = ma$$
$$T = m(g - a)$$
$$T = (4.50\text{ kg})(9.80\text{ m/s}^2 - 2.00\text{ m/s}^2)$$
$$T = 35.1\text{N, where T is the tension in the rope.}$$

Applying the rotational form of Newton's 2nd Law to the wheel:

$$\tau = I\alpha, \quad \text{(where } \tau = \text{torque, } I = \text{moment of inertia, and } \alpha = \text{angular acceleration.)}$$
$$RT = I\alpha = Ia/R$$
$$I = R^2T/a = (0.234\text{ m})^2(35.1\text{N})/(2.00\text{ m/s}^2) = 0.961\text{kg}\cdot\text{m}^2$$

10. **Correct answer: A**



$$P_A = (1.00\text{ atm}) + \rho_{cont}gH_{cont} + \rho_{mant}g(D+H)$$
$$P_B = (1.00\text{ atm}) + \rho_{cont}g(H_{mtn} + H_{cont} + D) + \rho_{mant}g(H)$$
$$P_A = P_B$$

$$\rho_{cont}gH_{cont} + \rho_{mant}g(D+H) = \rho_{cont}g(H_{mtn} + H_{cont} + D) + \rho_{mant}g(H)$$
$$\rho_{cont}gH_{cont} + \rho_{mant}g(D) = \rho_{cont}g(H_{mtn} + H_{cont} + D)$$
$$Dg(\rho_{mant} - \rho_{cont}) = \rho_{cont}g(H_{mtn} + H_{cont} - H_{cont})$$
$$D = \frac{\rho_{cont}(H_{mtn} + H_{cont} - H_{cont})}{(\rho_{mant} - \rho_{cont})} = \frac{\rho_{cont}(H_{mtn})}{(\rho_{mant} - \rho_{cont})} = \frac{\rho_{cont}(H_{mtn})}{(\rho_{mant} - \rho_{cont})}$$
$$D = \frac{2.87(6.25\text{ km})}{(3.31 - 2.87)} = (40.8\text{ km})$$

11. **Correct answer: E**

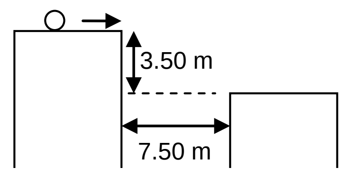
Using Newton's 2nd Law:

$$\boldsymbol{F}_{original} = m\boldsymbol{a}_{original} = (5.00\text{ kg})(2.00\ \mathbf{i})\text{ m/s}^2 = 10.0\ \mathbf{i}\text{ N}$$
$$\boldsymbol{F}_{original} + \boldsymbol{F}_{additional} = m\boldsymbol{a}_{final}$$
$$10.0\ \mathbf{i}\text{ N} + \boldsymbol{F}_{additional} = (5.00\text{ kg})(1.50\ \mathbf{i} + 1.82\ \mathbf{j})\text{ m/s}^2 = (7.50\ \mathbf{i} + 9.10\ \mathbf{j})\text{ N}$$
$$\boldsymbol{F}_{additional} = (-2.50\ \mathbf{i} + 9.10\ \mathbf{j})\text{ N}$$

12. **Correct answer: C**

$$V_f^2 = V_i^2 + 2ad$$
$$a = \frac{V_f^2 - V_i^2}{2d} = \frac{\left(\frac{42000\text{ m}}{3600\text{ s}}\right)^2 - \left(\frac{78000\text{ m}}{3600\text{ s}}\right)^2}{2(92.0\text{ m})} = -1.81\text{m/s}^2$$

13. **Correct answer: B**



$$d_{vert} = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2d_{vert}}{g}}$$

$$V_{horiz} = \frac{d_{horiz}}{t} = \frac{7.50\text{ m}}{\sqrt{\frac{2d_{vert}}{g}}} = \frac{7.50\text{ m}}{\sqrt{\frac{2(3.50\text{ m})}{9.80\text{ m/s}^2}}} = 8.87\text{ m/s}$$

14. **Correct answer: B**

Let { } denote "mass of"

$$\{{}^2_1H\} + \{{}^3_1H\} + \frac{energy_{init}}{c^2} = \{{}^4_2He\} + \{{}^1_0n\} + \frac{energy_{final}}{c^2}$$

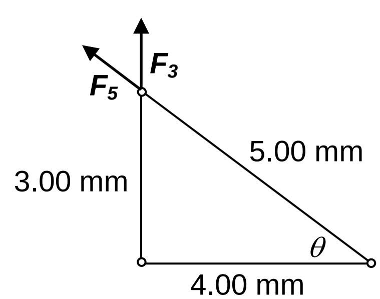
$$\{{}^2_1H\} + \{{}^3_1H\} = \{{}^4_2He\} + \{{}^1_0n\} + \frac{energy_{final}}{c^2} - \frac{energy_{init}}{c^2} = \frac{energy_{released}}{c^2}$$

$$\begin{aligned} energy_{released} &= c^2\left(\{{}^2_1H\} + \{{}^3_1H\} - \{{}^4_2He\} - \{{}^1_0n\}\right) \\ &= \left(2.9979\times10^8\text{m/s}\right)^2(2.0135532 + 3.0160493 - 4.0026032 - 1.0086649)u \\ &= \left(1.6478\times10^{15} u\cdot m^2/s^2\right)\left(1.6605389\times10^{-27}\text{ kg/u}\right) = 2.7632\times10^{-12}\text{ J} \\ &= \left(2.7632\times10^{-12}\text{ J}\right)/\left(1.602\times10^{-19}\text{ J/eV}\right) = 17.1\text{MeV} \end{aligned}$$

15. **Correct answer: E**

$$E_{average} = f\left(\frac{1}{2}kT\right) = 6\left(\frac{1}{2}\right)\left(1.381\times10^{-31}\text{ J/K}\right)(154 + 273.15)\text{K} = 1.77\times10^{-20}\text{ J}$$

16. **Correct answer: C**



$$\boldsymbol{F_5} = \frac{ke^2}{\left(5\times10^{-3}\text{ m}\right)^2}\left(\cos(\theta)\,\textbf{left} + \sin(\theta)\,\textbf{up}\right)$$

$$= \frac{ke^2}{\left(5\times10^{-3}\text{ m}\right)^2}\left(\frac{4}{5}\,\textbf{left} + \frac{3}{5}\,\textbf{up}\right)$$

$$\boldsymbol{F_3} = \frac{ke^2}{\left(3\times10^{-3}\text{ m}\right)^2}\,\textbf{up}$$

$$\boldsymbol{F_3} + \boldsymbol{F_5} = \frac{ke^2}{\left(1\times10^{-6}\text{ m}^2\right)}\left(\frac{1}{9}\,\textbf{up} + \frac{3}{125}\,\textbf{up} + \frac{4}{125}\,\textbf{left}\right)$$

$$\boldsymbol{F_3} + \boldsymbol{F_5} = \frac{ke^2}{\left(1\times10^{-6}\text{ m}^2\right)}\left(\frac{152}{(9)(125)}\,\textbf{up} + \frac{4}{125}\,\textbf{left}\right)$$

$$\left|\boldsymbol{F_3} + \boldsymbol{F_5}\right| = \frac{e^2}{4\pi\varepsilon_o\left(1\times10^{-6}\text{ m}^2\right)}\left(\left[\frac{152}{(9)(125)}\right]^2 + \left[\frac{4}{125}\right]^2\right)^{1/2} = 3.20\times10^{-23}\text{ N, where}$$

$e = 1.602\times10^{-19}$ C and $\varepsilon_o = 8.854\times10^{-12}\text{ C}^2/(N\cdot m^2)$.

17. **Correct answer: E**

$$y = 0.0352\text{ m}\cos\left[\left(\frac{1.45}{\text{m}}\right)x + \left(\frac{28.4}{\text{s}}\right)t\right]$$

$$y = A\cos\left[\left(\frac{2\pi}{\lambda}\right)x + (2\pi f)t\right]$$

$$\frac{2\pi}{\lambda} = \frac{1.45}{\text{m}}, \quad \lambda = \frac{2\pi}{1.45}\text{ m}$$

$$2\pi f = \frac{28.4}{\text{s}}, \quad f = \frac{28.4}{2\pi}\text{ s}^{-1}$$

$$v = f\lambda = \frac{28.4}{1.45}\text{ m/s}$$

$$\text{v} = \sqrt{\frac{\text{T}}{\rho}} \quad \rightarrow \quad T = v^2\rho = \left(\frac{28.4}{1.45}\text{ m/s}\right)^2\left(1.67\times10^{-4}\text{ kg/m}\right) = 64.1\text{mN}$$

18. **Correct answer: B**

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$\frac{1}{65.0\text{ mm}} = \frac{1}{d_i} + \frac{1}{12000\text{ mm}}$$

$$d_i = 65.354\text{ mm}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -h_o\left(\frac{d_i}{d_o}\right) = -(1750\text{ mm})\left(\frac{65.354}{12000}\right) = -9.53\text{ mm}$$

19. **Correct answer: D**

The motion is constant accelerated motion with an acceleration 9.8 m/s^2 vertically downward. Using upward as the positive y direction and the fact that the greatest height is reached when the vertical velocity is zero:

$$v_y^2 - v_{y0}^2 = 2a_y(y - y_0) \quad \Rightarrow \quad y = \frac{v_y^2 - v_{y0}^2}{2a_y} + y_0 = \frac{0^2 - (30.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} + 2.0\text{ m} = 47.9\text{ m}$$

20. **Correct answer: B**

The motion is constant accelerated motion with an acceleration 9.8 m/s^2 vertically downward. Using upward as the positive y direction,

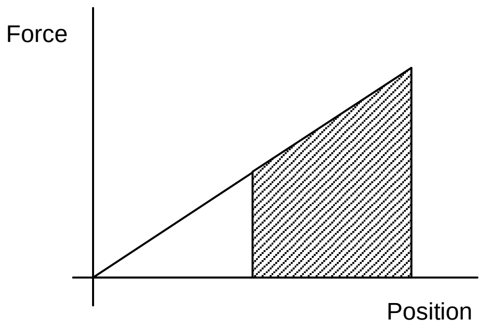
$$v_y = v_{y0} + a_y t = 30.0\text{ m/s} - 9.8\text{ m/s}^2(2.0\text{ s}) = 10.4\text{ m/s}$$

21. **Correct answer: D**

There are two perpendicular components to the car's acceleration, the centripetal acceleration, a_c, and the tangential acceleration, a_t:

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\frac{v^2}{R}\right)^2 + a_t^2} = \sqrt{\left(\frac{(9.00 \text{ m/s})^2}{270.\text{ m}}\right)^2 + (0.400 \text{ m/s})^2} = 0.500 \text{ m/s}^2$$

22. **Correct answer: E**



Work is the area under the force versus position graph between the limits of the motion. The force at position x = 2.00 m is F = (3.00 N/m)(2.00 m)=6.00 N. The force at position x = 4.00 m is F = (3.00 N/m)(4.00 m)=12.00 N. The work is the area of the trapezoid with base 2.00 m and the first height 6.00 N and second height 12.00 N. The work is

$$W = \tfrac{1}{2}(12.00\,\text{N} + 6.00\,\text{N})(2.00\,\text{m}) = 18.0\,\text{J}$$

23. **Correct answer: C**

Applying impulse equals change in momentum and using a coordinate system with positive x toward the north:

$$\text{Impulse} = mv_2 - mv_1 \quad \Rightarrow \quad m = \frac{\text{Impulse}}{v_2 - v_1} = \frac{-80.0\,\text{N}\cdot\text{s}}{(-40.0\,\text{m/s}) - (10.0\,\text{m/s})} = 1.60\,\text{kg}$$

24. **Correct answer: B**

As the velocity of the block is constant, the net force on the object is zero. Considering the horizontal components of force:

$$F_{friction} - 60\,\text{N}\cos 20° = 0 \quad \Rightarrow \quad F_{friction} = 60\,\text{N}\cos 20° = 56.4\,\text{N}$$

Considering the vertical components of force:

$$F_{normal} - mg + 60\,\text{N}\sin 20.0° = 0 \quad \Rightarrow \quad F_{normal} = mg - 60\,\text{N}\sin 20.0° = (50.0\,\text{kg})(9.8\,\text{m/s}^2) - 60\,\text{N}\sin 20.0° = 469.4\,\text{N}$$

$$F_{friction} = \mu_k F_{normal} \quad \Rightarrow \quad \mu_k = \frac{F_{friction}}{F_{normal}} = \frac{56.4\,\text{N}}{469.4\,\text{N}} = 0.120$$

25. **Correct answer: B**

Applying Newton's Second Law and using vertically upward as the positive y direction:

$$F_{net\,y} = ma_y \quad \Rightarrow \quad a_y = \frac{F_{net\,y}}{m} = \frac{100\,\text{N} - 20\,\text{N}\cdot\sin 30.0° - (8.00\,\text{kg})(9.8\,\text{m/s}^2)}{8.00\,\text{kg}} = 1.45\,\text{m/s}^2$$

26. **Correct answer: C**

$$\alpha = \frac{\tau}{I} = \frac{(9.00\,\text{N}\cdot\text{m})}{\left(5.00\,\text{kg}\cdot\text{m}^2\right)} = 1.80\,\text{rad/s}^2$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \text{ and } \omega_0 = 0 \quad \Rightarrow \quad t = \sqrt{\frac{2(\theta - \theta_0)}{\alpha}} = \sqrt{\frac{2(\pi/2 - 0)}{1.80\,\text{rad/s}^2}} = 1.32\,\text{s}$$

27. **Correct answer: E**

$$\text{N}\cdot\text{s/kg} \neq \text{J}$$

28. **Correct answer: E**

Applying conservation of energy:

$$\tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \quad \Rightarrow \quad k = \frac{mv_2^2 - mv_1^2}{x_1^2 - x_2^2} = \frac{(6.00\,\text{kg})\left((2.00\,\text{m/s})^2 - (3.00\,\text{m/s})^2\right)}{0 - (0.400\,\text{m})^2} = 188.\,\text{N/m}$$

29. **Correct answer: A**

Applying the parallel axis theorem:

$$I = I_{cm} + md^2 \quad \Rightarrow \quad d = \sqrt{\frac{I - I_{cm}}{m}} = \sqrt{\frac{\left(28.0\text{ kg}\cdot\text{m}^2\right) - \left(20.0\text{ kg}\cdot\text{m}^2\right)}{2.00\text{ kg}}} = 2.00\text{ m}$$

30. **Correct answer: C**

Applying conservation of angular momentum:

$$L_{rotor\,f} + L_{body\,f} = L_{rotor\,i} + L_{body\,i} \quad \Rightarrow \quad I_{rotor}\,\omega_f + I_{body}\,\omega_f = I_{rotor}\,\omega_{rotor\,i} + I_{body}\,\omega_{body\,i}$$

$$\Rightarrow \quad \omega_f = \frac{I_{rotor}\,\omega_{rotor\,i} + I_{body}\,\omega_{body\,i}}{I_{rotor} + I_{body}} = \frac{\left(1.00\times10^2\text{ kg}\cdot\text{m}^2\right)\left(2.00\times10^2\text{ rad/s}\right) + \left(2.00\times10^3\text{ kg}\cdot\text{m}^2\right)(0)}{\left(1.00\times10^2\text{ kg}\cdot\text{m}^2\right) + \left(2.00\times10^3\text{ kg}\cdot\text{m}^2\right)} = 9.52\,\text{rad/s}$$

31. **Correct answer: B**

Reactance

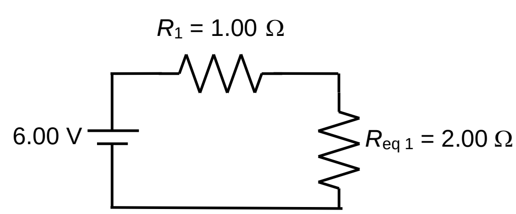
32. **Correct answer: A**

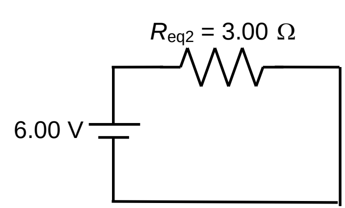
The equivalent resistance of the 6.00 Ω and the 3.00 Ω resitors is

$$R_{eq1} = \left(\frac{1}{6.00\,\Omega} + \frac{1}{3.00\,\Omega}\right)^{-1} = 2.00\,\Omega$$

This equivalent resistance is in series with the 1.00 Ω, resulting in a net resistance

$$R_{eq\,2} = 2.00\,\Omega + 1.00\,\Omega = 3.00\,\Omega$$





Using Ohm's Law, the current flowing through this net equivalent resistance is

$$I = \frac{6.00\,\text{V}}{3.00\,\Omega} = 2.00\,\text{A}$$

This will also be the current through equivalent resistance R_{eq1}, resulting in a voltage across R_{eq1}, $V_{eq1} = I\,R_{eq1} = (2.00\,\text{A})(2.00\,\Omega) = 4.00\,\text{V}$

This will also be the voltage across the original 3.00 Ω resistor, resulting in a current through the 3.00 Ω resistor

$$I_{3.00\,\Omega} = \frac{(4.00\,\text{V})}{(3.00\,\Omega)} = 1.33\,\text{A}$$

33. **Correct answer: E**

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \Rightarrow \quad n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1.00)\sin(55.0°)}{\sin(25.0°)} = 1.94$$

34. **Correct answer: C**

$$P = \sigma A T^4 \quad \Rightarrow \quad \frac{P_2}{P_1} = \frac{\sigma A T_2^4}{\sigma A T_1^4} \quad \Rightarrow \quad P_2 = \frac{T_2^4}{T_1^4} P_1 = \frac{(273.15 + 273.15)^4}{(0 + 273.15)^4}(1.00\ \text{W}) = 16.0\ \text{W}$$

35. **Correct answer: C**

proton