



Worldwide Youth in Science and Engineering

2014 Academic Challenge

MATHEMATICS TEST - STATE

This Test Consists of 40 Questions

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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WYSE Academic Challenge
Mathematics Test (State) – 2014

- In a certain town it was found that approximately 24% of the residents participated in the 2 year crime watch program, 43% participated in the 4 year program and the remaining 33% did not participate at all. Of those that participated in the two year program, 49% of the homes were vandalized, while those who participated in the 4 year program only had 28% of their homes vandalized. For those who did not participate in the crime watch program, 61% of their homes were vandalized. Find the probability that a resident who was vandalized did not participate in the crime watch program. Round to the nearest percent.

a) 20% b) 27% c) 44% d) 46% e) 73%
- Given $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, and $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Which of these is $\frac{d}{dx}(\sinh x)$?

a) $\sinh x$ b) $-\sinh x$ c) $\cosh x$ d) $-\cosh x$ e) $\tanh x$
- The letters of the word "Thumper" are arranged in all possible ways. If an arrangement is picked at random, what is the probability that the arrangement will start with a consonant and end with a vowel?

a) $\frac{1}{5040}$ b) $\frac{1}{720}$ c) $\frac{1}{7}$ d) $\frac{1}{21}$ e) $\frac{5}{21}$
- A cubic watermelon grows three inches in length per month. If one starts to grow at the beginning of April, at what rate is its volume increasing at the beginning of October? (Each answer is in cubic inches per month.)

a) 848 b) 1027 c) 2916 d) 3429 e) 5832
- A vehicle traveling at 60 feet per second applies brakes that supply a constant deceleration of -2 feet per second squared. Determine how far the vehicle travels after applying the brakes before coming to a complete stop.

a) 60 feet b) 120 feet c) 300 feet d) 600 feet e) 900 feet
- Identify the curve represented by $7y^2 + 28(x + y) - 14x^2 = 0$.

a) Hyperbola b) Ellipse c) Circle d) Parabola e) Cone
- What is the area of the region between $x^2 - x - 56$, the x-axis, and the lines $x = -10$ and $x = 16$?

a) $\frac{494}{3}$ b) 562.5 c) 130 d) 184 e) $\frac{3869}{3}$

8. A population had 10000 people at the start of 2000 and 11000 at the start of 2001. If the population maintains exponential growth, how many people will there be in the population at the start of 2015? Round to the nearest whole number.
- a) 25000 b) 35500 c) 41579 d) 41772 e) 44817
9. Find the area of the parallelogram determined by $\mathbf{u} = \langle 6, 1, 5 \rangle$ and $\mathbf{v} = \langle -1, 4, -3 \rangle$. Round the answer the nearest unit.
- a) 4 b) 11 c) 17 d) 25 e) 36
10. Find the rectangular coordinate of the polar equation $7r = 14 - 7 \cos \theta$.
- a) $x^2 + y^2 - y = 0$
 b) $x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - 3x^2 - 4y^2 = 0$
 c) $x^4 + y^4 - 4x^3 + 2x^2y^2 - 4xy^2 - 4y^2 = 0$
 d) $(x^2 + y^2) - 2xy = 0$
 e) $x^4 + y^4 + 2x^2y^2 + 2x^2y + 2y^3 - 9x^2 - 8y^2 = 0$
11. If it is known that the inverse of matrix A is $A^{-1} = \begin{bmatrix} \frac{7}{2} & \frac{-3}{2} \\ 2 & k \end{bmatrix}$ and $\det A = -2$, find the value a_{11} in matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.
- a) $a_{11} = -2$
 b) $a_{11} = -1$
 c) $a_{11} = 2$
 d) $a_{11} = 1$
 e) cannot be determined
12. Which of the following is $\log_x x^3 \cdot \log_y (y^2)^5$?
- a) $\log_{xy} (x^2y^3)^5$
 b) $\log_x x^3 (y^2)^5$
 c) $\log_{x+y} (x^3 + (y^2)^5)$
 d) $\log_{xy} (x^3)^{(y^2)^5}$
 e) 30

13. The number of people in a school who have heard a rumor after t days can be modeled by the function $R(t) = \frac{500}{0.25 + e^{-t}}$. At the moment when the number of people who have heard the rumor is growing the fastest, how many people have heard the rumor? Round your answer to the nearest whole number.

a) 400 b) 500 c) 1000 d) 1200 e) 2000

14. Simplify $\cos 3x \cos x + \sin 3x \sin x$ to a one-termed expression.

a) $3 \sin x$
 b) $3 \sin^2 x$
 c) $\cos^2(3x)$
 d) $\cos(2x)$
 e) $3 \sin(2x) \cos(2x)$

15. To the nearest degree, which of the following is a solution to the equation $12 \cot x = 5 \sin x$?

a) 22 b) 123 c) 158 d) 202 e) 429

16. Simplify the expression $\cot(\csc^{-1}(x))$ into a form with no trig functions.

a) $\frac{\sqrt{x^2 - 1}}{x}$
 b) $\frac{\sqrt{1 - x^2}}{x}$
 c) $\sqrt{x^2 + 1}$
 d) $\sqrt{x^2 - 1}$
 e) $\frac{x}{\sqrt{x^2 - 1}}$

17. Find the first derivative y' of the function $y = y(x)$ if $x^2 + y^2 = 9 + xy$.

a) $y' = (2y - x)^3$ b) $y' = \frac{3x - 3y}{(2x - y)^2}$ c) $y' = \frac{(2x - y)^2}{3x - 3y}$
 d) $y' = \frac{x^2 - x + 2y}{(2y - x)^2}$ e) $y' = \frac{y - 2x}{2y - x}$

18. In which quadrant(s) is $\tan^2 x - \sec^2 x$ negative?

a) I b) II c) III d) I and III e) All four

19. Evaluate $\int \frac{2x+5}{4x} dx$.

- a) $\frac{1}{2}x + \frac{5}{4}\ln|x| + c$ b) $\frac{5}{4} + \frac{1}{2}x^{-1} + c$ c) $\frac{5}{4}x + 2\ln|x| + c$
 d) $\frac{5}{4}x + \ln\left|\frac{1}{2}x\right| + c$ e) $\frac{5}{4}\left(2x - \frac{1}{x^2}\right) + c$

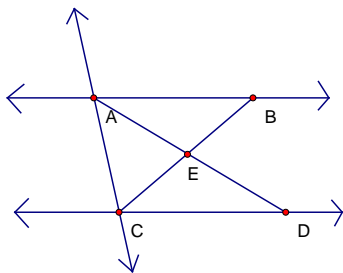
20. A traveling salesman must visit six cities and return home. He wants to do so in the cheapest manner possible, so he sets his computer to the task. The computer can check one route per 30 seconds, so for how long must it work to exhaustively search all possible routes?

- a) 24 min b) 36 min c) 3 hr d) 6 hr e) 12 hr

21. A group of ten individuals has a mean value of 4, a median value of 5, and a mode value of 6. What is the value of the standard deviation of the group? Round your answer to the nearest tenth.

- a) 0.5 b) 1.0 c) 1.4 d) 5.0 e) Not enough information

22. In the diagram below, parallel lines AB and CD are cut by transversal AC. Segments BC and AD intersect at point E. $m\angle BAC < m\angle ACD$. Segment CB bisects $\angle ACD$ and segment AD bisects $\angle BAC$. Diagram below is not drawn to scale.



Which statement(s) are true?

1. Line segment AC is the same length as line segment AB.
2. Line segment AE is greater than line segment CE.
3. The measure of angle DAC is greater than the measure of angle BCD.

- a) 1 b) 1 and 2 c) 2 d) 2 and 3 e) 3

23. ChimpsMatchYouUp.com is the hottest new dating website, and using trained apes to randomly match users is touted as “more accurate than (redacted name of rival site)”. If the Chimp Executive Officer is given 30 people to match up with 30 others, what is the probability that each will be matched with their most perfect partner of the group? Assume each user has been paired with only one other user, and round to the nearest percent.

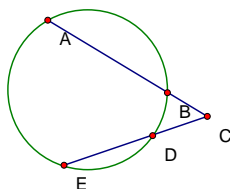
- a) 0% b) 2% c) 5% d) 10% e) 25%

24. Given the equation $\sum_{n=1}^{\infty} x^n = 3 - \sum_{m=0}^{\infty} 2x^m$, solve for x.
- a) 1/4 b) 1/3 c) 1/2 d) 1 e) No solution

25. Fill in the blank with the appropriate word choice below for the following statement: “In an (a) _____ triangle, the angle bisector of the base angle cuts the opposite side into two equal parts.
- a) Scalene
 b) Obtuse
 c) Equilateral
 d) Isosceles
 e) All of the above.

26. Which of the following is true of $f(x) = 2^x - 1$ for natural numbers?
- a) If x is even, f(x) cannot be prime.
 b) If x is prime, f(x) is prime.
 c) Primes of the form f(x) where x is prime are called Gaussian primes.
 d) f(4) is composite.
 e) f(7) – f(4) is prime.

27. In the diagram below AB=9, BC=3, and CD=2. Find DE. Diagram is not drawn to scale.



- a) 10 b) 12 c) 14 d) 16 e) 18
28. Geese of a certain species have eight goslings per clutch, all of whom should be named Ryan (Source: I made it up w/help from Nicholas Sparks.) Of these, three fourths will learn to fly and two thirds of those will reach maturity the year after they are born. (The rest die.) If a pair of geese “double clutches” (has two clutches per year) for ten years and “single clutches” for three years after that, how many goslings will reach maturity by the end of this 13-year period? If necessary, round to the nearest silly goose.
- a) 88 b) 92 c) 123 d) 132 e) 138
29. If the polynomial function f(x) crosses through the x-axis at -2 and 2 and touches the x-axis at -1 and 1, what is the minimum possible degree of f(x)?
- a) 4 b) 5 c) 6 d) 7 e) 8

30. $g(x) = 3x + 5$ and $(g \circ f)(x) = 3x^2 + 2$. Find $f(x)$.
- a) $f(x) = x^2 - 3$
 - b) $f(x) = (x - 1)^2$
 - c) $f(x) = (1 - x)^2$
 - d) $f(x) = \frac{1}{3}x^2 - 3$
 - e) $f(x) = x^2 - 1$
31. A square is inscribed in another square, and then, in a fashion calling to mind Russian nesting dolls, three successive inscriptions of squares take place, one inside the next. What percentage of the area of the initial square does the terminally-inscribed square contain? Round to the nearest percent.
- a) 3%
 - b) 6%
 - c) 13%
 - d) 25%
 - e) Insufficient Information
32. If two of the altitudes of a triangle are perpendicular to each other, which of the following must be true about the triangle?
- a) The triangle must be a right triangle
 - b) The triangle must be an isosceles triangle
 - c) The triangle must be both right and isosceles
 - d) All triangles have this property
 - e) No triangle can ever have this property
33. A chemist has 10,000 ml of a solution containing 20% acid. How many ml should be removed and replaced by pure acid in order to obtain a solution containing 50% acid?
- a) 3750 ml
 - b) 3875 ml
 - c) 4000 ml
 - d) 4125 ml
 - e) 4250 ml
34. What is the magnitude of the fourth power of $2 + i$? Round to the nearest unit.
- a) 2
 - b) 5
 - c) 9
 - d) 25
 - e) 81
35. Lacamedia, a dinner theater, charges the following group rates: \$60 per person for the first 30 patrons. For larger groups, no more than 90, each person receives a 50 cent discount for each patron in excess of 30. Determine the amount of money the dinner theater makes from the group size that produces the maximum amount of money.
- a) \$3,750.00
 - b) \$2,812.50
 - c) \$2,762.50
 - d) \$2,700.00
 - e) \$2,612.50
36. If $x = 3t$ and $y = 2t + 7$, what is the slope of the line created by this set of equations?
- a) $\frac{2}{3}$
 - b) 1.5
 - c) 0
 - d) 3
 - e) 2

37. How many x-intercepts does the function $g(x) = \frac{1}{x} + \frac{1}{x+1} - \frac{2}{x-1}$ have?
- a) 0 b) 1 c) 2 d) 3 e) 4
38. A person recently bought a car at a dealership which advertised averages of 25 mi/gal in the city and 40 mi/gal on the highway. A recent trip covered 1625 miles using 51 gallons of gasoline. By the advertisement the car should have traveled 1800 miles. The owner of this vehicle is confident that the car averages 25 mi/gal in the city but is very skeptical about the highway average of miles per gallon. Find the highway average miles/gallon that the person got on the recent trip.
- a) 31 mi/gal
b) 33 mi/gal
c) 35 mi/gal
d) 37 mi/gal
e) 39 mi/gal
39. An ice cream cone has roughly the shape of a cone with radius 2" and height 10". The ice cream takes on the shape of a hemisphere with radius 2". (Yes, it just lies on top.) What percentage of the total volume is taken up by the ice cream? Round to the nearest percentage.
- a) 8% b) 23% c) 30% d) 67% e) 70%
40. Three sets of twins, Anne (f), Brady (m), Calvin (m), Dana (f), Erin (f), and Felicia (f), recently played a game at a local corn maze. Two of them started with golden charms, four with clear charms. There were two rules: 1) When you meet someone who is not your twin, you must exchange charms. 2) The first set of twins to have both gold charms at the same time wins. As they played, the following happened:
- I – Anne and Brady started the game with the gold charms
II – Each person met up with each other person exactly once (15 meetings).
III – Of the 12 total exchanges, 5 involved one gold charm and 2 involved both
IV – Calvin got a gold charm from Brady but ended with a clear charm he got from Dana
V – The only time Erin even saw a gold charm was when she met her twin sister
VI – All of Felicia's exchanges involved at least one gold charm
- Who was the winning set of twins?
- a) Anne and Dana
b) Brady and Dana
c) Anne and Felicia
d) Brady and Felicia
e) Situation cannot exist as written