



2015 Academic Challenge

MATHEMATICS TEST - REGIONAL

This Test Consists of 40 Questions

Mathematics Test Production Team

Linda Wiggins, Illinois State University – Author/Team Leader

Matthew Childers, Illinois State University – Author

Sandra Cox, Kaskaskia College (ret.) – Reviewer

Kathryn Torrey, WYSE – Coordinator of Test Production

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

© 2015 Worldwide Youth in Science and Engineering

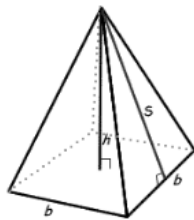
“WYSE”, “Worldwide Youth in Science and Engineering” and the “WYSE Design” are service marks of and this work is the Copyright © 2015 of the Board of Trustees of the University of Illinois at Urbana - Champaign. All rights reserved.

WYSE Academic Challenge
 Mathematics Test (Regional) - 2015

1. What is the quantity which results when the number of diagonals in an octagon is subtracted from the number of diagonals in a decagon?
- a) 2 b) 10 c) 15 d) 17 e) 38
2. When the following two matrices are added, what are the dimensions of the resultant matrix?

$$\begin{bmatrix} 1 & 3 & 4 \\ -7 & 6 & x \\ 8 & y & 3 \end{bmatrix} + \begin{bmatrix} 1 & z & 3 \\ 4 & 2 & 8 \end{bmatrix}$$

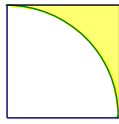
- a) 2 x 3 b) 3 x 2 c) 3 x 3 d) 2 x 2 e) Not a matrix
3. In a newly developed town an election is being held to form a town council that will consist of 6 members. At least 3 of the council members must be clergymen and at least 2 of the council members must be IT-technicians. If 8 clergymen, 5 IT-technicians, 7 teachers and 10 farmers are running for seats on this council, how many ways can this council be chosen?
- a) 10,780 b) 14,000 c) 62,510 d) 114,240 e) 168,000
4. Which of the following polar coordinates correspond to the rectangular coordinates $(1, \sqrt{3})$?
- i) $\left(2, \frac{\pi}{3}\right)$ ii) $\left(-2, \frac{4\pi}{3}\right)$ iii) $\left(2, \frac{7\pi}{3}\right)$ iv) $\left(-2, \frac{7\pi}{3}\right)$
- a) only i b) i ,ii ,iii c) only ii d) i, iii, iv e) only iv
5. What is $i^{504} + i^{507}$?
- a) $-2i$ b) $1-i$ c) $1+i$ d) -2 e) 0
6. Find the volume of a floored square based pyramid shaped tent with a surface area of 360 square feet and a base of 100 square feet. See Diagram below. Round your answer to the nearest cubic feet.



(Diagram is not drawn to scale)

- a) 120 ft³
 b) 360 ft³
 c) 400 ft³
 d) 780 ft³
 e) 1200 ft³

7. Find the area of the shaded region in the diagram below where the length of the side of the square is 2 cm. A quarter circle is inscribed in the square.



- a) $4\pi^2 \text{ cm}^2$ b) $(\pi + 4)\text{cm}^2$ c) $\pi \text{ cm}^2$ d) $(4 - \pi)\text{cm}^2$ e) $4\pi \text{ cm}^2$
8. Find the perimeter of a parallelogram with vertices $(-4, 1)$, $(-1, 5)$, $(7, 5)$ and $(4, 1)$.
- a) 34 units b) 32 units c) 30 units d) 28 units e) 26 units
9. What is the sum of the coefficients of $(x - 2y)^5$?
- a) -32 b) -1 c) 6 d) 32 e) 243
10. Find the radius of a circle where the perimeter of a regular hexagon inscribed in a circle is 60 inches.
- a) 6 in b) $5\sqrt{2}$ in c) $5\sqrt{3}$ in d) 9 in e) 10 in
11. Simplify the quotient: $\frac{54m^3 - 18m^2n - 6mn^2 + 2n^3}{9m^2 - n^2}$.
- a) $6m - 2n$
 b) $2n - 6m$
 c) $-6m - 2n$
 d) $6n - 2m$
 e) $-6m - 12mn^2$
12. A certain number of X-Men shirts cost \$1000. If they cost \$5 less per shirt, 10 additional shirts could be purchased for the same amount of money. What is the cost of each X-Men shirt?
- a) \$20 b) \$25 c) \$30 d) \$35 e) \$40
13. In a geometric sequence, we have $a_1 = 43$ while the 15th term is $a_{15} = 14$. What is $\sum_{n=1}^{\infty} a_n$? Round to the nearest integer if necessary.
- a) 303 b) 558 c) 605 d) ∞ e) undetermined

14. In how many distinguishable ways can the letters from the word "pyrotechnic" be reordered?
 a) 10 b) 3,628,800 c) 9,979,200 d) 19,958,400 e) 39,916,800
15. A gondolier rows 4.5 mph in still water and 2.5 mph against the current. How long will it take him to row 65 miles upstream and subsequently make the return trip? Round to the nearest hour.
 a) 20 hrs b) 25 hrs c) 30 hrs d) 36 hrs e) 41 hrs
16. Find the intersection of the circle $x^2 + y^2 = 8$ and the hyperbola $3x^2 - y^2 = 4$ where $x > 0, y > 0$.
 a) (3,5) b) $(\sqrt{3}, 5)$ c) $(\sqrt{3}, \sqrt{5})$ d) $(\sqrt{5}, \sqrt{3})$ e) (5,3)
17. How many diagonals does a rectangular prism have?
 a) 7 b) 16 c) 20 d) 21 e) 28
18. Multiply radicals and combine them under one radical sign: $\sqrt[3]{5}\sqrt{2}$
 a) $\sqrt[6]{10}$ b) $\sqrt[3]{10}$ c) $\sqrt[5]{100}$ d) $\sqrt[6]{200}$ e) $\sqrt[3]{200}$
19. Find the value k when the $\det \begin{pmatrix} -5 & 4 & 5 \\ 2 & k & 5 \\ 1 & k & 2 \end{pmatrix} = -136$.
 a) $K = 5$ b) $K = 4$ c) $K = 0$ d) $K = -4$ e) $K = -7$
20. Find x given $\log_2 12x^2 - \log_2 3x = 3$.
 a) $x = 2$ b) $x = 1$ c) $x = 0$ d) $x = -1$ e) $x = -2$
21. One adult can mow a lawn in 12 minutes. His kid can mow the same lawn in 48 minutes. How many minutes would it take them to mow the lawn if they worked together? Round to the nearest minute.
 a) 5 min b) 9 min c) 10 min d) 12 min e) 24 min
22. How many squares can be drawn with all of their vertices arranged in a 5 x 5 grid of points?
 a) 16 b) 25 c) 50 d) 55 e) 65

23. Which of the following is not an even function?
 a) $\sin 4x$
 b) $2 \cos 5x$
 c) $3 \cos x \cos 2x$
 d) $\sin^2 2x - \cos^2 2x$
 e) $4 \sec 6x$
24. The _____ is a measure of central tendency.
 a) deviation b) mode c) percentile d) range e) variance
25. Select the statement below that best describes the curve of the equation $4y^2 - x^2 - 4x - 8y = 4$.
 a) Axis parallel to x-axis.
 b) Major axis parallel to x-axis.
 c) Major axis parallel to y-axis.
 d) Transverse axis parallel to x-axis.
 e) Transverse axis parallel to y-axis.
26. How many asymptotes (horizontal, vertical or oblique) are there for the rational function $r(x) = \frac{x^4 - 6}{x^3 - 8}$?
 a) 0 b) 1 c) 2 d) 3 e) 4 or more
27. Find $\csc \theta$ for the angle θ in standard position with its terminal side passing through the point (5, 12).
 a) $\frac{13}{5}$ b) $\frac{12}{5}$ c) $\frac{13}{12}$ d) $\frac{5}{12}$ e) $\frac{5}{13}$
28. If the coversine of t is defined as $1 - \sin t$ and the versine of t is defined as $1 - \cos t$, which of the following is equivalent to the sum of squares of these two trig functions?
 a) -1
 b) 1
 c) $3 - 2\cos t - 2\sin t$
 d) $3 - \sin^2 t - \cos^2 t$
 e) 4
29. If $x = 3 - 2t$ and $y = 5 + 4t$, what is the distance traveled from $t = 3$ to $t = 10$? Round to the nearest unit.
 a) 7 b) 31 c) 33 d) 42 e) 65

30. The top of a water slide above the water level is 67 feet high. The slide is inclined at an angle of 52° with the horizontal. How long is the water slide above the water level? Round to the nearest foot.
- a) 52 ft. b) 53 ft. c) 84 ft. d) 85 ft. e) 109 ft.
31. Which of the following is equivalent to $\log_3(3x + 1)$?
- a) $\log_3(3x) + \log_3 1$
 b) $3\log_3(x + 1)$
 c) $3\log(3x + 1)$
 d) $\frac{\ln(3x + 1)}{\ln 3}$
 e) None of the above
32. Given the numbers 5,2,6,4,7,4,7,2,8,9,4,10,9,1,3 find the probability of randomly selecting two numbers without replacement whose sum is 11. Round to the nearest percent.
- a) 5% b) 6% c) 7% d) 8% e) 9%
33. Which of the following rules is necessary in order to rationalize the denominator of $\frac{x}{\sqrt[3]{x} + \sqrt[3]{25}}$?
- a) Difference of squares
 b) Sum of cubes
 c) Conjugation of square root expressions
 d) Multiplying by a single square root
 e) None of the above
34. A certain tournament involves a group stage which is a round robin between four teams. Each game either results in one team winning (3 standings points) and one team losing (0) or both teams earning a draw (1 point each). What is the smallest number of points that a team could get and still win the group (via a tiebreaker)?
- a) 1 b) 2 c) 3 d) 4 e) 5
35. Jack marked up a sweater 38% at his work. Later, he had to mark it down 40% in order to sell it. In aggregate, what percent markup or markdown did he wind up making? Round to the nearest percent.
- a) Down 17% b) Up 17% c) Down 2% d) Down 1% e) Up 5%

36. A decagon is inscribed in a circle. What percent of the circle's area is covered by the decagon? Please round to the nearest percent.
- a) 6% b) 9% c) 42% d) 58% e) 94%
37. Megan lives directly north from the center of Central Park. Robin lives 15 kilometers directly east from the center of Central Park. The bearing from Megan to Robin's house is 62°SE . Find the shortest distance from Megan to Robin's house. Round to one decimal place.
- a) 17.0 km b) 28.2 km c) 32.0 km d) 36.5 km e) 40 km
38. 4, 5 and 8 are the lengths (in no particular order) of three of the four segments formed by chords of a circle which intersect one another. Which of the following could **not** be the length of the fourth chord?
- a) 2.5 b) 6.4 c) 9.6 d) 10 e) None of these
39. Timothy enters a lottery that pays \$1000. A ticket cost \$1 per drawing. The probability of a 50% chance of winning in x drawings with a \$1 ticket is given by $1 - 0.999^x = 0.5$. How much money will Timothy spend to have a 50% chance of winning?
- a) \$615 b) \$650 c) \$673 d) \$693 e) \$700
40. Maria, Jane and Rose are spending the day at Clinton Lake participating in different sport activities (horseback riding, hiking and swimming). Each spends a different amount of time at their activity. Use the clues below to determine the sport activity that Rose participates in and the time that she took to complete her activity.
1. The least amount of time was spent horseback riding. One activity took as long as the other two activities combined. The combined total of all 3 activities is 200 minutes.
 2. Jane spent time hiking. Rose's activity took 1.5 times as long as one of the other activities. Ten times the shortest timed activity is the same as 4 times the longest timed activity.
- a) Horseback riding; 40 min
b) Hiking; 100 min
c) Swimming; 33 min
d) Hiking; 67 min
e) Swimming; 60 min