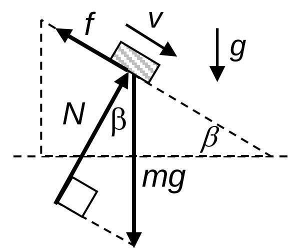
WYSE Academic Challenge
2015 Regional Physics Exam
SOLUTION SET

1. **Correct answer: D**

Note: [*quantity*] denotes: units of *quantity*

$$[\vec{\tau}] = [\vec{r} \times \vec{F}] = [r][m][a] = [length][mass][length / time^2] = [mass][length]^2 / [time]^2$$

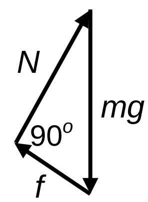
2. **Correct answer: A**



Since the block slides down the incline at constant speed, the vector sum of the Normal force, the gravitational force, and the frictional force acting on the block must be zero.

Parallel to the incline's surface:

a) $-f + mg\sin(\beta) = 0 \quad \Rightarrow \quad \mu_K N = mg\sin(\beta)$



Perpendicular to the incline's surface:

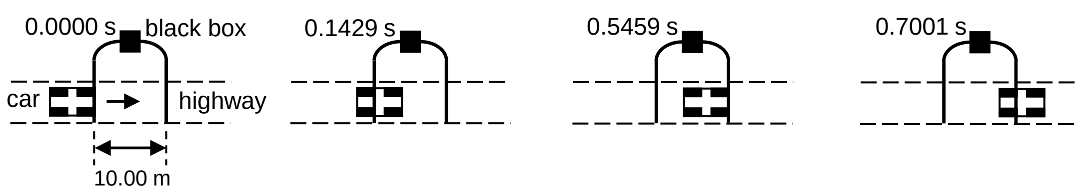
b) $N - mg\cos(\beta) = 0 \quad \Rightarrow \quad N = mg\cos(\beta)$

Dividing equation a) by equation b):

c) $$\frac{\mu_K N}{N} = \frac{mg\sin(\beta)}{mg\cos(\beta)}$$
$$\mu_K = \tan(\beta)$$

When the block slides at constant velocity, the coefficient of kinetic friction depends only upon the angle the incline makes with the horizontal.

3. **Correct answer: E**



Average speed according to the times recorded for the rear wheels:

$$v_{ave} = \frac{length}{time} = \frac{10.00\,\text{m}}{(0.7001 - .1429)s} = 17.95\,\text{m/s}$$

4. **Correct answer: B**

v_{ave}(between 0.5459 s and 0.7001 s) / v_{ave}(between 0.0000 s and 0.1429 s)?

Let L represent the distance between front wheels and back wheels.

$$\frac{v_{ave(from\ 0.5459\ s\ to\ 0.7001\ s)}}{v_{ave(from\ 0.0000\ s\ to\ 0.1429\ s)}} = \frac{L/(0.7001 - 0.5459)}{L/(0.1429 - 0.0000)} = \frac{0.1429}{0.1542} = 0.9267$$

5. **Correct answer: E**

Using conservation of momentum:

$$m_1 v_{1,initial} + m_2 v_{2,initial} = m_1 v_{1,final} + m_2 v_{2,final}$$
$$m_1(+23.5) + m_2(0) = m_1(-4.00) + m_2(15.0)$$
$$27.5 m_1 = 15.0 m_2$$
$$m_1/m_2 = 15.0/27.5$$
$$m_1/m_2 = 0.545$$

6. **Correct answer: B**

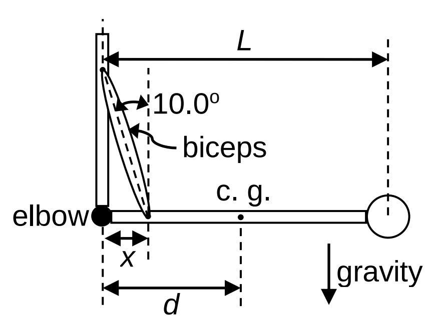
$$t = \frac{v - v_o}{a} = \frac{\left(\frac{2.9979 \times 10^8}{100}\right) - 0}{9.80}\left(\frac{\text{m/s}}{\text{m/s}^2}\right) = 3.06 \times 10^5 \text{ s}$$

7. **Correct answer: A**

$$v^2 - v_o^2 = 2ad$$

$$d = \frac{v^2 - v_o^2}{2a} = \frac{\left(\frac{2.9979 \times 10^8}{100}\right)^2 - 0}{2 \times 9.80}\left(\frac{\text{m}^2/\text{s}^2}{\text{m/s}^2}\right) = 4.59 \times 10^{11} \text{m}$$

8. **Correct answer: A**



Ball weight = 154.0 N
Forearm weight = 22.0 N
Elbow to biceps connection: x = 0.0480 m.
Elbow to center of gravity (c. g.) of forearm: d = 0.150m.
Elbow to ball's center of gravity: L = 0.340 m.

The sum of torques about about any point is zero.

Using the elbow joint to calculate torques acting on the forearm:

$$T_{biceps}\cos(10.0^o)(x)(ccw) + W_{forearm}d(cw) + W_{ball}L(cw) = 0$$

$$T_{biceps}\cos\left(10.0^o\right)\left(x\right)\left(-cw\right)+W_{forearm}d\left(cw\right)+W_{ball}L\left(cw\right)=0$$

$$T_{biceps}=\frac{W_{forearm}d+W_{ball}L}{x\cos\left(10.0^o\right)}=\frac{\left(22.0\right)\left(0.150\right)+\left(154.0\right)\left(.340\right)}{0.0480\cos\left(10.0^o\right)}\left(\frac{\text{Nm}}{\text{m}}\right)=1180\text{N}$$

9. **Correct answer: B**

$$\boldsymbol{a}=\frac{\boldsymbol{F}_{net}}{m}=\frac{\left\{-\left(369\,\text{N/m}\right)x+\left(93.7\text{N}\right)\right\}\hat{x}}{m}$$

$$=\frac{\left\{-\left(369\,\text{N/m}\right)\left(0.800\,\text{m}\right)+\left(93.7\text{N}\right)\right\}\hat{x}}{0.168\,\text{kg}}$$

$$=-1199\,\hat{x}\ \text{m/s}^2=-1.20\,\hat{x}\ \text{km/s}^2$$

10. **Correct answer: D**

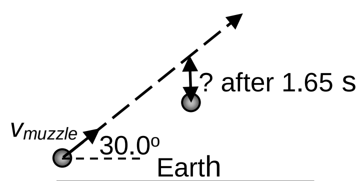
The maximum speed occurs where the acceleration (and therefore the force) is zero.

$$-\left(369\,\text{N/m}\right)x+\left(93.7\text{N}\right)=0$$

$$x=\frac{93.7\text{N}}{369\,\text{N/m}}=0.254\,\text{m}$$

11. **Correct answer: B**

Letting down be the positive direction:



$$y-y_o=v_{oy}t+\tfrac{1}{2}a_yt^2$$

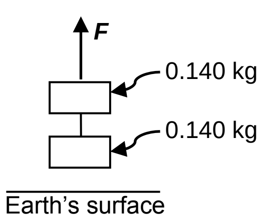
$$y-y_o=\left(0\right)+\tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(1.65\,\text{s}\right)^2=13.3\,\text{m}$$

12. **Correct answer: C**

$$v_{muzzle}\cos(30.0^o)=\text{v}_{horizontal}=\frac{248\,\text{m}}{1.65\,\text{s}}=150.3\,\frac{\text{m}}{\text{s}}$$

$$v_{muzzle}=\frac{150.3\,\frac{\text{m}}{\text{s}}}{\cos\left(30.0^o\right)}=174\,\frac{\text{m}}{\text{s}}$$

13. **Correct answer: A**



$$F-m_1g-m_2g=m_{total}a$$

$$F=\left(m_1+m_2\right)g+\left(m_1+m_2\right)a$$

$$F=\left(m_1+m_2\right)\left(g+a\right)=\left(0.280\,\text{kg}\right)\left(9.80+2.2\right)\text{m/s}^2=3.36\,\text{N}$$

14. **Correct answer: B**

Since the boy is moving in a circular orbit, his direction of motion is constantly changing. Even though the magnitude of his velocity (i.e., his speed) is constant, the boy's velocity is changing because of the change in direction. The change in the velocity vector is toward the center of the circular orbit. Therefore the boy has an acceleration toward the center of the circular orbit.

Therefore, the net force acting on the boy is radially inward.

15. **Correct answer: D**

$$\mathrm{F} = \frac{\mathrm{Gm}_{Earth}\, m_{ISS}}{R_{orbit}^2} = m_{ISS} a_{ISS}$$

$$a_{ISS} = \frac{\mathrm{Gm}_{Earth}}{R_{orbit}^2} = \frac{\left(6.673\times10^{-11}\right)\left(5.98\times10^{24}\right)}{\left(6.38\times10^{6}+2.50\times10^{5}\right)^2}\,\frac{\left(\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}\right)(\mathrm{kg})}{\left(\mathrm{m}^2\right)}$$

$$a_{ISS} = 9.08\,\frac{\mathrm{N}}{\mathrm{kg}} = 9.08\,\mathrm{m/s^2}$$

16. **Correct answer: C**

$$W = \boldsymbol{F}\bullet\boldsymbol{d} = |\boldsymbol{F}||\boldsymbol{d}||\cos(\theta)|$$

θ is the angle between the force and the displacement.

$\boldsymbol{F} = 3.50\,\mathrm{N}, \textbf{\textit{east}} + 6.00\,\mathrm{N}, \textbf{\textit{south}}$

$\boldsymbol{d} = 4.00\,\mathrm{m}, \textbf{\textit{west}}$

The work done by the 3.50 N component is:

$$W_{3.50N,east} = (3.50\,\mathrm{N})(4.00\,\mathrm{m})\cos(180^o) = -14.0\,\mathrm{J}$$

The work done by the 6.00 N component is:

$$W_{6.00N,south} = (6.00\,\mathrm{N})(4.00\,\mathrm{m})\cos(90^o) = 0$$

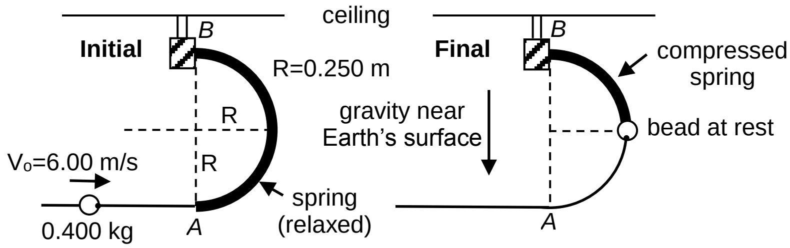
The total work done by the push is therefore –14 J.

17. **Correct answer: E**

The work done = the change in energy of the object.

$$W = \Delta E = E_{final} - E_{init} = 0 - \frac{1}{2}mv_{init}^2 = -\frac{1}{2}(125)(49.0)^2\left(\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2}\right) = -1.50\times10^5\,\mathrm{J}$$

18. **Correct answer: E**



Using conservation of energy:

$$E_{final} = E_{initial}$$
$$\frac{1}{2}kx_{final}^2 + mgh_{final} + \frac{1}{2}mv_{final}^2 = \frac{1}{2}kx_{init}^2 + mgh_{init} + \frac{1}{2}mv_{init}^2$$
$$\frac{1}{2}kx_{final}^2 + mgR + \frac{1}{2}m(0)^2 = \frac{1}{2}k(0)^2 + mg(0) + \frac{1}{2}mv_{init}^2$$
$$\frac{1}{2}kx_{final}^2 + mgR = \frac{1}{2}mv_{init}^2 \qquad (\leftarrow \text{With practice, one would probably start here.})$$
$$k = \frac{mv_{init}^2 - 2mgR}{x_{final}^2} = \frac{m\left(v_{init}^2 - 2gR\right)}{\left(\frac{2\pi R}{4}\right)^2} = \frac{(0.400)\left((6.00)^2 - 2(9.80)(0.250)\right)}{\left(\frac{2\pi(0.250)}{4}\right)^2}\left(\frac{\text{kg}\,\text{m}^2/\text{s}^2}{\text{m}^2}\right) = 80.7\,\text{N/m}$$

19. **Correct answer: A**

The momentum of the cannon-sled-cannon ball system is conserved (constant) in the Earth frame of reference.

Let: B=ball, CS=cannon & sled, E=Earth, r.t.=relative to

Using the relative velocity formula to convert the ball's velocity to the Earth frame:

$$v_{\text{B r.t. E}} = v_{\text{B r.t. CS}} + v_{\text{CS r.t. E}}$$
$$v_{\text{B r.t. E}} = 63.0\,\text{m/s} + v_{\text{CS r.t. E}}$$

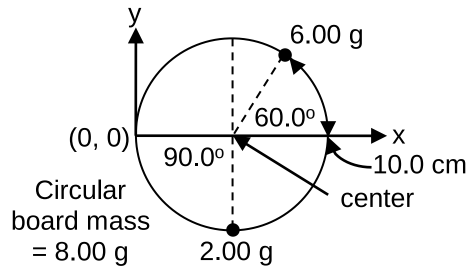
Inserting this into the momentum equation:

Final momentum of the B-CS system = Initial momentum of the B-CS system = 0

$$m_{\text{CS}}v_{\text{CS r.t. E}} + m_B v_{\text{B r.t. E}} = 0$$
$$m_{\text{CS}}v_{\text{CS r.t. E}} + m_B\left(63.0\,\text{m/s} + v_{\text{CS r.t. E}}\right) = 0$$
$$\left(m_{\text{CS}} + m_B\right)v_{\text{CS r.t. E}} = -m_B(63.0\,\text{m/s})$$
$$v_{\text{CS r.t. E}} = \left(\frac{-m_B}{m_{\text{CS}} + m_B}\right)(63.0\,\text{m/s}) = \left(\frac{-76.3}{1450 + 76.3}\right)(63.0\,\text{m/s}) = -3.15\,\text{m/s}$$
$$\left|v_{\text{CS r.t. E}}\right| = 3.15\,\text{m/s}$$

20. **Correct answer: C**

Find the weighted average of x:



$$x_{avg} = \frac{\sum_i m_i x_i}{m_{total}} = \frac{(2.00\,\text{g})(5.00\,\text{cm}) + (8.00\,\text{g})(5.00\,\text{cm}) + (6.00\,\text{g})\left(5.00\,\text{cm}+5.00\,\text{cm}\cos\left(60.0^o\right)\right)}{16.0\,\text{g}}$$

$$x_{avg} = \frac{(10.0 + 40.0 + 45.0)\,\text{g cm}}{16.0\,\text{g}} = 5.94\ \text{cm}$$

21. **Correct answer: D**

$$\Delta\theta = 8.50\,\text{rev}\left(2\pi\,{}^{\text{rad}}\!/_{\text{rev}}\right) = 17.0\pi\,\text{rad}$$

$$\omega_o = 0$$

Using a formula from rotational kinematics:

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\alpha = \frac{2\Delta\theta}{t^2} = \frac{2(17.0\pi)}{(15.0\,\text{s})^2} = 0.475\,\text{rad/s}^2$$

22. **Correct answer: D**

The direction of the force is in the same direction as the acceleration. Since the acceleration has a negative x-component and a positive y-component, the acceleration vector lies in the 2nd quadrant.

$$\vec{a} = \left(-12.0\hat{i} + 16.0\hat{j}\right)\ \text{m/s}^2$$

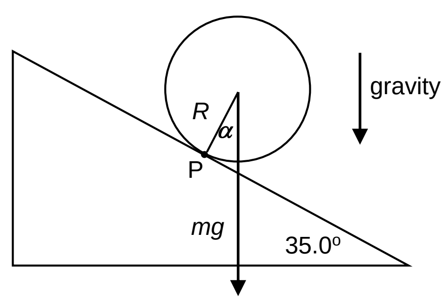
$$\theta = \tan^{-1}(16.0 / -12.0) = -53.13^o$$

$$\text{angular direction} = 180^o - 53.13^o = 127^o$$

23. **Correct answer: E**

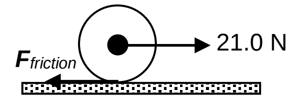
$$|\boldsymbol{F}| = m|\boldsymbol{a}| = (4.50\,\text{kg})\sqrt{(-12.0)^2 + (16.0)^2}\ \text{m/s}^2 = 90.0\,\text{N}$$

24. **Correct answer: A**



$$|\boldsymbol{Torque}| = |\boldsymbol{F}|(\textit{lever arm})$$
$$= mgR\sin(\alpha)$$
$$= (2.45\,\text{kg})\left(9.80\,\text{m/s}^2\right)(0.180\,\text{m})\sin\left(35.0^o\right)$$
$$= 2.48\,\text{Nm}$$

25. **Correct answer: B**



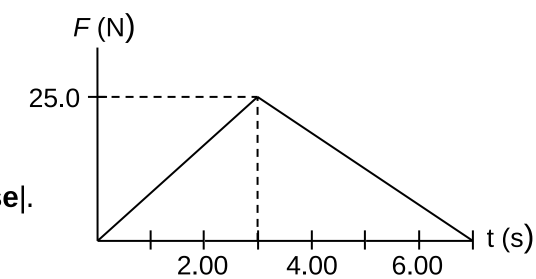
The frictional force acts at the contact point between the wheel and the surface. The direction of the frictional force is to the left. Applying Newton's 2nd Law:

$$21.0\,\text{N}(\boldsymbol{right}) + \boldsymbol{F}_{friction} = (15.0\,\text{kg})(0.800\,\text{m/s}^2)(\boldsymbol{right})$$
$$\boldsymbol{F}_{friction} = (15.0\,\text{kg})(0.800\,\text{m/s}^2)(\boldsymbol{right}) - 21.0\,\text{N}(\boldsymbol{right})$$
$$\boldsymbol{F}_{friction} = -9.00\,\text{N}(\boldsymbol{right}) = 9.00\,\text{N}(\boldsymbol{left})$$
$$|\boldsymbol{F}_{friction}| = 9.00\,\text{N}$$

26. **Correct answer: A**

$$\omega = \frac{v}{R} = \frac{at}{R} = \frac{(0.800\,\text{m/s}^2)(3.00\,\text{s})}{0.250\,\text{m}} = 9.60\,\text{rad/s}$$

27. **Correct answer: C**



Using:
Impulse = change in **momentum**
and
The area under a force versus time graph =|**impulse**|.

$$\text{area} = |\boldsymbol{impulse}| = |\Delta\boldsymbol{momentum}|$$
$$\frac{1}{2}(3.00\,\text{s})(25.0\,\text{N}) + \frac{1}{2}(4.00\,\text{s})(25.0\,\text{N}) = 87.5\,\text{Ns} = |m\boldsymbol{v}_f - m\boldsymbol{v}_i| = (18.0\,\text{kg})(v_f)$$
$$v_f = 4.86\,\text{m/s}$$

28. **Correct answer: D**

Gauss' Law relates the net flux of an electric field through a closed surface to the net charge enclosed by that surface.

29. **Correct answer: B**

An adiabatic process is a process in which the system does not exchange heat with its surroundings.

30. **Correct answer: B**

$$v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda$$
$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(365\,\text{m/s})}{2.80\,\text{m}} = 819\,\text{rad/s}$$

31. **Correct answer: E**

$$P_{crawler} = \frac{18\times10^6\ lb}{(8)(10.ft)(41.ft)} = 5488\,\text{lb/ft}^2$$

$$P_{person} = \frac{180\,lb}{(2)(0.75\,ft)(0.25\,ft)} = 480\,\text{lb/ft}^2$$

$$\frac{P_{crawler}}{P_{person}} = 11$$

32. **Correct answer: D**

$$P_{top} + \tfrac{1}{2}\rho v_{top}^2 = P_{bottom} + \tfrac{1}{2}\rho v_{bottom}^2$$

$$\cancel{P_{top}} + \tfrac{1}{2}\rho v_{top}^2 = \left(\cancel{P_{top}} + 1210\,\text{Pa}\right) + \tfrac{1}{2}\rho v_{bottom}^2$$

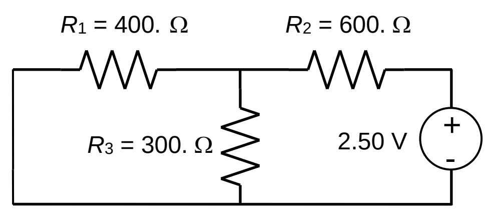
$$\tfrac{1}{2}\rho v_{top}^2 = (1210\,\text{Pa}) + \tfrac{1}{2}\rho v_{bottom}^2$$

$$\tfrac{1}{2}\rho\left(v_{top}^2 - v_{bottom}^2\right) = 1210\,\text{N/m}^2$$

$$\tfrac{1}{2}\left(1.28\,\text{kg/m}^3\right)\left((145\,\text{m/s})^2 - v_{bottom}^2\right) = 1210\,\text{N/m}^2$$

$$v_{bottom} = 138\,\text{m/s}$$

33. **Correct answer: E**



In the given circuit, the 300. Ω resistor and the 400. Ω resistor are in parallel, and those two are in series with the 600. Ω resistor.

$$R_{equiv} = 600.\Omega + [300.\Omega \,||\, 400.\Omega]$$

$$= 600.\Omega + \frac{(300.)(400.)}{300. + 400.}\Omega = 600.\Omega + 171.4\Omega = 771.4.\Omega$$

$$P = \frac{v^2}{R_{equiv}} = \frac{(2.50)^2}{771.4}\text{W} = 8.10\text{ mW}$$

Note: Series resistances add, while two parallel resistances combine as the product divided by the sum. (This does not extend to three parallel resistances.)

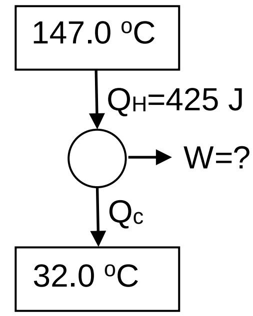
34. **Correct answer: C**

The speed of a wave on a wire under tension depends upon the tension in the wire and the linear density of the material making up the wire.

$$v = \sqrt{\frac{T}{\lambda}} = \sqrt{\frac{454\,\text{N}}{0.332\,\text{kg}\ \ /15.0\,\text{m}}} = 143.2\,\text{m/s}$$

$$\text{t}_{round\ trip} = \frac{2L}{v} = \frac{2\times15.0\,\text{m}}{143.2\,\text{m/s}} = 0.209\,\text{s}$$

35. **Correct answer: D**



Conservation of energy: $Q_H = W + Q_C$

Carnot relationship: $\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$

$$Q_H = W + Q_H\left(\frac{T_C}{T_H}\right)$$

Therefore:

$$W = Q_H\left(1 - \frac{T_C}{T_H}\right)$$

$$W = (425)\left(1 - \frac{32.0 + 273.15}{147 + 273.15}\right)\text{J}$$

$$W = 116\,\text{J}$$