



2015 Academic Challenge

MATHEMATICS TEST - SECTIONAL

This Test Consists of 40 Questions

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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WYSE Academic Challenge
Mathematics Test (Sectional) – 2015

1. Given four matrices with the same dimensions, which of the following operations are not guaranteed to exist?
 - a) Their inverses.
 - b) Their differences.
 - c) Their sum.
 - d) Their transposes.
 - e) All of the above can be performed.

2. Mork and Mindy are buying a house and have taken out a 30 year \$120,000 mortgage at 8% interest per year. Their monthly house payment is \$880.52. How much of the outstanding principal have they paid in the first two months?
 - a) \$161.04
 - b) \$161.58
 - c) \$440.26
 - d) \$1600
 - e) \$1761.04

3. Find the quotient of $(2x^3 - 3x^2 - 17x + 30)$ and $(x + 3)$.
 - a) $-x^2 - 17x + 10$
 - b) $2x^2 + x - 15$
 - c) $2x^2 - 9x + 10$
 - d) $x^2 + 8x - 15$
 - e) $-x^2 + 17x - 16$

4. A cylindrical grain storage building with a hemisphere top holds 747,000 cubic feet of grain. If the radius of the cylinder is 40 feet, find the height of the cylinder. Round to the nearest foot.
 - a) 62 ft.
 - b) 90 ft.
 - c) 106 ft.
 - d) 122 ft.
 - e) 150 ft.

5. Find the area of a triangular screen door with sides 13ft, 13ft, and 10ft.
 - a) 18 sq.ft
 - b) 30 sq.ft
 - c) 36 sq.ft
 - d) 49 sq.ft
 - e) 60 sq.ft

6. What is the derivative with respect to x of $y = \cos(6x + 7)$?
 - a) $y' = \sin 6$
 - b) $y' = -\sin 6$
 - c) $y' = -\sin(6x + 7)$
 - d) $y' = 6\cos(6x + 7)$
 - e) $y' = -6\sin(6x + 7)$

7. The width of the manmade lake at Tipton Park has been measured at 50 feet intervals. The results are shown in the following table:

Width	0	90	147	261	225	111	21
distance	0	50	100	150	200	250	300

Approximate the area of the lake using the trapezoidal rule. Round to nearest feet.

- a) 42,225 ft b) 42,750 ft c) 43,000 ft d) 43,225 ft e) 43,550 ft
8. How many petals would the rose described by $r = 5 \cos 8\theta$ have?
- a) 4 b) 5 c) 8 d) 16 e) 64
9. A fruit punch contains 30% pineapple juice and another fruit punch only contains 10% pineapple juice. To obtain a mixture that contains 50 gallons of pineapple juice, 150 gallons of the 30% pineapple juice fruit punch is used. How many gallons of the 10% pineapple juice fruit punch is used?
- a) 55 gal b) 50 gal c) 35 gal d) 20 gal e) 5 gal
10. Which of the following vectors are not orthogonal to $\langle 6, 3 \rangle$?
- a) $\langle 3, -6 \rangle$ b) $\langle -4, -8 \rangle$ c) $\langle 1, -2 \rangle$ d) $\langle 0, 0 \rangle$ e) They all are.
11. Suppose Sansa, Littlefinger, and Tyrion toss a six-sided fair die in turn, Sansa first, Tyrion last, until one of them wins by getting the first six. The winner gets to select the way King Joffrey is killed. What is the probability that Tyrion wins on the second round? Round to 2 decimal places.
- a) 0.07 b) 0.12 c) 0.25 d) 0.33 e) 0.40
12. If \$6,403 is invested in an account which pays 6.3% interest compounded continuously, how much interest has the account accrued after 4.5 years?
- a) \$1563.61
 b) \$2026.14
 c) \$2098.70
 d) \$8501.70
 e) \$8429.14
13. Let A represent a 3 by 5 matrix. When finding AB, the matrix B must have _____.
- a) 3 rows b) 5 columns c) 3 columns d) 5 rows e) 8 rows
14. What is the eccentricity of the unit circle? Round to two decimal places.
- a) 0.00 b) 0.56 c) 0.75 d) 1.00 e) 1.34

15. Which of the following trigonometric function(s) never take on the same value as $\sec t$?
- a) $\cos t$ b) $\sin t$ c) $\csc t$ d) $\tan t$ e) Both b and d
16. Stiles is 45 feet from Kate and 67 feet from Scott with an angle of 35.0° between Scott and Kate. How far is Kate from Scott? Round to the nearest tenth of a foot.
- a) 55.0 ft b) 52.2 ft c) 46.4 ft d) 40.6 ft e) 39.7 ft
17. In _____ geometry, within a two dimensional plane for any given line ℓ and a point A not on ℓ , there are infinitely many lines through A that do not intersect ℓ .
- a) Hyperbolic b) Elliptic c) Euclidean d) Riemannian e) Einstein
18. An identity for $\sin(2\theta)$ is _____.
- a) $\cos^2 \theta - \sin^2 \theta$
- b) $2\sin\theta\cos\theta$
- c) $2\cos^2 \theta - 1$
- d) $2\sin^2 \theta - 1$
- e) $1 - 2\sin^2 \theta$
19. In a 6-8-10 right triangle, what is the degree measure of the angle between the short leg and the altitude to the hypotenuse? Round to the nearest degree.
- a) 31° b) 37° c) 45° d) 53° e) 59°
20. Given the parametric equations $x = 3\sec t - 1$ and $y = 2\tan t + 1$, determine the rectangular graph.
- a) parabola b) circle c) hyperbola d) ellipse e) line
21. Solve for x in the equation $\log_2(x^2 - 4) - \log_2(x + 2) = M$.
- a) $x = 2^{M+2}$
- b) $x = \frac{1 + \sqrt{25 + M}}{4}$
- c) $x = \frac{1 - \sqrt{25 + M}}{4}$
- d) $x = 2^M + 2$
- e) $x = 2^M - 2$

22. In five-card draw, how many ways are there to get a three of a kind that is not a full house (5 cards with 3 of a kind and 2 of a kind)?
- a) 54,912 b) 64,896 c) 109,824 d) 129,792 e) 658,944
23. A club has 16 sophomores, 8 juniors and 7 seniors. In how many ways can a committee consisting of 3 sophomores, 2 juniors and 2 seniors be formed?
- a) 329,280
 b) 764,400
 c) 2,629,575
 d) 7,902,720
 e) 13,253,058,000
24. A goalkeeper stops 20% of the penalty shots he faces. What is the probability that he stops two of the five shots that he faces in a penalty shootout? Round to the nearest percent.
- a) 0% b) 2% c) 3% d) 20% e) 40%
25. Which of the following are not preserved by a translation?
- a) Angle measure
 b) Betweenness
 c) Collinearity
 d) Distance
 e) All of the above are preserved.
26. The apothem of a regular octagon is what percentage of the perimeter of the figure? Round to the nearest percent.
- a) 13% b) 15% c) 25% d) 30% e) 31%
27. What is the remainder for $\frac{6x^7 - 230}{x + 3}$?
- a) -13352 b) -3 c) 1 d) 3 e) 12892
28. Rewrite the exponential equation $e^{2x} = 5$ as a logarithmic equation.
- a) $\ln\left(\frac{5}{x}\right) = 2$ b) $\ln\left(\frac{2}{5}\right) = x$ c) $\ln\left(\frac{5}{2}\right) = x$ d) $\ln(2x) = 5$ e) $\ln 5 = 2x$
29. What is the sum of the solutions to $(2x + 6)(x - 4) = 16$?
- a) -4 b) -1 c) 1 d) 5 e) 10
30. Which of the following numbers are not triangular numbers?
- a) 1 b) 3 c) 9 d) 36 e) They all are.

31. Which of the following objects has a different number of vertices from the others?
- cube
 - rectangular prism
 - object with 8 faces and 14 edges
 - object with 10 faces and 18 edges
 - They all have the same number of vertices.
32. A 12 foot ladder leans up against a house. Its base is 4 feet from the edge of the foundation. What is the measure of the acute angle that it makes with the house? Round to the nearest degree.
- 11°
 - 13°
 - 18°
 - 19°
 - 71°
33. Simplify the expression $i^{17} - i^{15}$ where $i = \sqrt{-1}$.
- $2i$
 - $-2i$
 - 1
 - -1
 - 0
34. Find the probability of getting tails and the queen of hearts if you flip a fair coin once and select 1 card out of the standard deck of 52 cards.
- $\frac{27}{52}$
 - $\frac{1}{104}$
 - $\frac{1}{26}$
 - $\frac{1}{8}$
 - $\frac{2}{13}$
35. Given the equation $x^3 + y^3 = 3xy$, find the slope of the tangent line at the point $(1,2)$.
- $\frac{3}{2}$
 - $-\frac{1}{2}$
 - $\frac{1}{3}$
 - $-\frac{2}{3}$
 - 2
36. Find $f(x)$ if $(f \circ g)(x) = x^2 - 4x + 4$.
- $f(x) = x + 2$
 - $f(x) = x - 2$
 - $f(x) = x + 4$
 - $f(x) = x^2$
 - $f(x) = x^2 + 4$
37. Arrange the following from least to greatest:
- Number of possible answer forms for a 10 question matching test*
 - Number of possible answer forms for a 20 question true/false test
 - Number of possible answer forms for a 10 question multiple choice test where there are five possible choices per question.
- * We will assume that an answer form includes a 1-1 matching.
- i, ii, iii
 - i, iii, ii
 - ii, i, iii
 - ii, iii, i
 - iii, i, ii

38. A rectangle has an area of 400 square meters. Express the perimeter P as a function of the length of the base, x .

a) $P = 2x^2 + \frac{400}{x}$

b) $P = \frac{x^2 + 400}{x}$

c) $P = \frac{400}{x} + x$

d) $P = 2x + \frac{200}{x}$

e) $P = \frac{2x^2 + 800}{x}$

39. An object's position after t minutes is given by the function $s(t) = t^3 + 6t - 25$. When is the acceleration of the object equal to 40 miles per minute-squared? Round to two decimal places.

- a) 0 min b) 2.0 min c) 3.37 min d) 6.67 min e) Never

40. The clues below are given to determine the age and hometown of four students, their first and last name, the subject each is struggling with and living arrangements while in college. Find the homeland and the subject that the 23 year old is struggling with.

- i. The four college students are the student who is struggling with French who is not 21, Kate Argent, the one from Great Falls, Montana and the one who is living in an apartment.
- ii. Stiles is not 23 years old and isn't struggling with English. The youngest student is 19 years old and is from Salt Lake City.
- iii. The four college students are the 25 year old who lives in a mobile home, Mr. McCall, who isn't Stiles, the young man who is struggling with chemistry and the one from Palm Beach, Florida.
- iv. The one in a single dorm room is 2 years older than the man from Salt Lake City, Utah. The one who struggles in math is 2 years older than Derrick Hale.
- v. The four hopefuls (to be college graduates) are Mr. Stilinski, Scott, the 23 year old from Cape May, New Jersey, and the one struggling with English.

- a) Cape May; Chemistry
- b) Palm Beach; French
- c) Cape May; French
- d) Palm Beach; Math
- e) Cape May; Math