WYSE Academic Challenge
Mathematics Test Solutions (Sectional) - 2015

1. Answer A: We never stated that they were even square, let alone invertible.

2. Answer B: $120{,}000\left(\frac{.08}{12}\right) = 800$. The first month interest on the loan is \$800. The principal paid in the first month is \$80.52. The second month interest on the loan is $\left((120{,}000 - 80.52)\left(\frac{.08}{12}\right) = 799.46\right)$ \$799.46. The principal paid in the second month is $\left(880.52 - 799.46 \approx 81.06\right)$ \$81.06. The combined is \$161.58.

3. Answer C: $(x+3)\left(2x^2 - 9x + 10\right) = 2x^3 - 3x^2 - 17x + 30$. Therefore

$\left(2x^3 - 3x^2 - 17x + 30\right)$ divided by $(x+3)$ is $\left(2x^2 - 9x + 10\right)$.

4. Answer D: $v = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) \Rightarrow 747{,}000 = \pi(40)^2 h + \frac{2}{3}\pi(40)^3$. Solving for h we find $h = \dfrac{747{,}000 - \frac{2}{3}\pi(64{,}000)}{1600\pi} \Rightarrow h = 121.9442614 \approx 122$.

5. Answer E: $A = \sqrt{18(5)(5)8} = 60$.

6. Answer E: By the Chain Rule,
$$\frac{d}{dx}\left[\cos(6x+7)\right] = -\sin(6x+7) \bullet \frac{d}{dx}\left[6x+7\right] = -6\sin(6x+7).$$

7. Answer A:
$A = \frac{1}{2}(50)\left(0 + 2(90) + 2(147) + 2(261) + 2(225) + 2(111) + 21\right) = 42{,}225$.

8. Answer D: If n is even, r = a cos $n\theta$ has 2n petals.

9. Answer B: $150(0.30) + x(0.10) = 50 \Rightarrow 45 + 0.10x = 50 \Rightarrow 0.10x = 5 \Rightarrow x = 50$.

10. Answer B: Two vectors are orthogonal if their dot product is 0. But the dot product of <6, 3> and <-4, -8> is -48, not 0, so the two vectors are not orthogonal.

11. Answer A: p(first round) $= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$ and

p(second round) $= \left(\frac{5}{6}\right)^3\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) \approx .06698$.

12. Answer C: If P dollars are invested at an interest rate of r (written as a decimal, not a percentage), compounded continuously for t years, the approximate total in the account after that time is $A = Pe^{rt}$. In this scenario, P = 6403, r = .063 and t = 4.5. Please note that this would give us (d), but that counts the principal as well as the interest. $8501.70 - 6403 = 2098.70$ (the interest).

13. Answer D: By matrix multiplication rules the column of the first matrix must match the number of rows in the second matrix.

14. Answer A: Since eccentricity measures how far a conic section varies from being circular, a circle would have an eccentricity of zero.

15. Answer E: The only places where the range of sec t intersects the range of sin t are –1 and 1. But if the secant is -1 or 1, that means the cosine is -1 or 1, respectively, and by the most basic of the Pythagorean identities, the sine is 0 at those places. The tangent of t, by the Pythagorean identity $1 + \tan^2 t = \sec^2 t$, will always have a smaller magnitude than sec t and will thus never equal it.

16. Answer E: Using law of cosines we have
$$x = \sqrt{45^2 + 67^2 - 2 \bullet 45 \bullet 67 \bullet \cos(35°)} \approx 39.68 \Rightarrow 39.7$$

17. Answer A: This is the definition of Hyperbolic Geometry.

18. Answer B: The double angle identity is $\sin 2\theta = 2 \sin\theta \cos\theta$.

19. Answer B: The long approach: There are two right triangles created by drawing this altitude, each of which has the altitude as a leg and each of whose hypotenuse is one of the former legs of the larger triangle. Let x be the shorter length of a side created on the former hypotenuse. Then the other side is10 – x. By the Pythagorean theorem, $x^2 + a^2 = 36$ so $a^2 = 36 - x^2$. For the other triangle, $(10 - x)^2 + a^2 = 64$ so $x^2 - 20x + 100 + a^2 = 64$ and $a^2 = -x^2 + 20x - 36$. By substitution, $36 - x^2 = -x^2 + 20x - 36$ so 20x = 72 and thus x = 3.6. So the sine of the angle in question is $\frac{3.6}{6} = 0.6$ and the arcsine of that is about 37 degrees.
The short approach: The two triangles created are similar to the original one, so we find the measures of the angles in the original triangle to be 53 degrees and 37 degrees. Since the 53 degree angle from the original is retained in the smaller triangle, the other angle is 37 degrees.

20. Answer C: Using the identity $1 + \tan^2 t = \sec^2 t$, $\frac{x+1}{3} = \sec t$ and $\frac{y-1}{2} = \tan t$. Squaring both and using the trig identity we can form the equation $\frac{(x+1)^2}{9} - \frac{(y-1)^2}{4} = 1$. This equation is in the form of a hyperbola.

21. Answer D: $\log_2\left(x^2-4\right)-\log_2\left(x+2\right)=M \Rightarrow \log_2\left(\frac{x^2-4}{x+2}\right)=M \Rightarrow \log_2\left(x-2\right)=M$

$\Rightarrow x=2^M+2$.

22. Answer A: There's 13 possible denominations that can be chosen to be tripled up and C(4,3) ways to choose the three cards of each denomination. Then there are C(12, 2) ways to choose the other two denominations and 4 ways to choose the card of the first denomination and 4 ways to choose the card of the second denomination. This comes out to 13 x C(4, 3) x C(12, 2) x 4 x 4 = 54,912 three of a kinds.

23. Answer A: Since committees have no inherent ranking system, we will use combinations. Thus we have C(16, 3) x C(8, 2) x C(7, 2) = 329,280 committees.

24. Answer D: This is a binomial distribution problem. The probability of x successes in n attempts with an individual probability of success p would be $C(n,x)\bullet p^x\bullet(1-p)^{n-x}$. In this case, the probability would be $C(5,2)\bullet.2^2\bullet.8^3=.2048\approx 20\%$.

25. Answer E: All of the above are preserved by translations, which are simply shifting of objects.

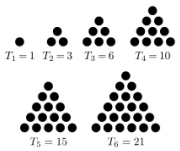
26. Answer B: If we draw a triangle from the center of the figure to two consecutive corners, it will have a side that is $\frac{1}{8}p$ opposite from a 45 degree angle. If we bisect that with the apothem, the angle will be 22.5 degrees and it will be opposite of a side with length $\frac{1}{16}p$. The other non-right angle in that triangle will be 67.5 degrees, and its tangent will be equal to the apothem divided by $\frac{1}{16}p$. So $a=\frac{p}{16}\tan(75^\circ)$, which is about 15% of the perimeter.

27. Answer A: By the Remainder Theorem, when a polynomial P(x) is divided by a simple linear function Q(x) = x – c, P(c) is the remainder. P(-3) = -13352.

28. Answer E: $\ln M=N \Rightarrow e^N=M$. Therefore, $e^{2x}=5 \Rightarrow \ln 5=2x$.

29. Answer C: We first need to multiply out the left-hand side to get $2x^2-2x-24=16$. If we subtract 16 from both sides, we get $2x^2-2x-40=0$. Factoring the left-hand side gives us $2(x-5)(x+4)=0$. Then x has to be either 5 or -4, the sum of which is 1.

30. Answer C: Triangular numbers count the number of dots that can form an equilateral triangle. The first nine triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36. See illustration below :



31. Answer D: The first two clearly have eight vertices. By f + v = e + 2, so does the third one, but the fourth one would have 10 vertices.

32. Answer D: The sine of the angle in question is one-third, so the angle is the degree equivalent of the arcsine of that number.

33. Answer A: $i^{17} - i^{15} \Rightarrow i^{15}\left(i^2 - 1\right) = -i\left(-1-1\right) = -i\left(-2\right) = 2i$

34. Answer B: $\left(\frac{1}{2}\right)\left(\frac{1}{52}\right) = \frac{1}{104}$

35. Answer C: $3x^2 + 3y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dx} \Rightarrow x^2 + y^2\frac{dy}{dx} = y + x\frac{dy}{dx}$

$\Rightarrow \left(y^2 - x\right)\frac{dy}{dx} = y - x^2 \Rightarrow \frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$. The slope of the tangent line when $x = 1$ and $y = 2$ is $\left(\frac{2-1^2}{2^2-1}\right) = \frac{1}{3}$.

36. Answer D: $\left(f \circ g\right)\left(x\right) = x^2 - 4x + 4 = \left(x - 2\right)^2$. Then $g(x) = x - 2$ and $f(x) = x^2$.

37. Answer C: (i) 10! = 3,628,800 (ii) $2^{20} = 1{,}048{,}576$ (iii) $5^{10} = 9{,}765{,}625$.

38. Answer E: $xy = 400 \Rightarrow y = \frac{400}{x}$. The perimeter is $P = 2x + 2\left(\frac{400}{x}\right) \Rightarrow P = \frac{2x^2 + 800}{x}$.

39. Answer D: Acceleration is the second derivative of position. So a(t) = 6t. We then set 6t = 40 and get (d) as our answer.

40. Answer A: From (v) we know the 23 year old is from Cape May, New Jersey. Since this clue provides information about each of the four people, the 23 year old does not struggle with English. By (i) we know the first and last name of the female, Kate Argent. By (iv) we know the first and last name of the male, Derrick Hale. The remaining first names are found in clue (ii), Stiles, and clue (v), Scott. By clue (iii) Mr. McCall isn't Stiles, so Scott must be Mr. McCall. This means Mr. Stilinski is Stiles. Looking back at

clue (iii) we see which clues can be ruled out to describe the 23 year old. "25 year old who lives in a mobile home" is out due to the age. "Mr. McCall" is out since this is Scott found in clue (v). "Palm Beach, Florida" is out since clue (v) tells us the 23 year old is from Cape May, New Jersey. Therefore, "the young man struggling in Chemistry" must describe the 23 year old.