WYSE Academic Challenge
Mathematics Test Solutions (State) - 2015

1. Answer A: $\frac{d}{dx}\left[\frac{e^x}{2}+\frac{e^{-x}}{2}\right]=\frac{1}{2}e^x-\frac{1}{2}e^{-x}=\sinh x$

2. Answer A: Using $y-k=a(x-h)^2$ form of the parabola where (h,k) is the vertex and (x,y) a point on the parabola, we can find a. $0-1225=a(70-35)^2 \Rightarrow -1225=a(1225)$. So $a=-1$.

3. Answer D: $S(L)=4\pi L^2$ and $L(t)=3t$. $S(t)=4\pi(3t)^2=36\pi t^2$. So $S'(t)=72\pi t$ and in this scenario, we would set t = 5.

4. Answer B: Using similar triangles we can find the opposite boundary length for lot B. $\frac{120}{140}=\frac{x}{224}\Rightarrow x=192$. The area of the larger triangle made from lots A and B together minus the area of lot A produces the area of lot B alone $\left(\frac{224(192)}{2}-\frac{140(120)}{2}=13104\right)$. Lot B is $13{,}104\,ft^2$.

5. Answer B: The area under the curve is the answer to A, but that doesn't take into account that the area between the curve and the x-axis from x = -3 to x = 4 would be counted as negative. So we must use the following sum of integrals: $\int_{-4}^{-3}(x^2-x-12)dx+\int_{-3}^{4}(-x^2+x+12)dx$.

6. Answer B: $\log_{xy} x^6y^6=\log_{xy}(xy)^6=6$

7. Answer C: Let *d* represent the number of dimes, *n* represent the number of nickels and *q* represent the number of quarters. Then $d=n+5$, $q=\frac{d}{2}=\frac{n+5}{2}$.
We have the equation $.05n+.10(n+5)+.25\left(\frac{n+5}{2}\right)=8$. Simplified, we have $55n+225=1600$. Therefore, n = 25.

8. Answer D: He travels for 220 miles in one direction and 110 in the other one. To get from one direction to the other, we've got an angle of 185 degrees between the two paths. (63 degrees to get from the first bearing to east, 90 degrees from north to east and 32 degrees from east to the second bearing.) We can then use the law of cosines to find the distance: $d^2=220^2+110^2-2\cdot110\cdot220\cos185^\circ$.

9. Answer D: After simplifying the radical, we have $\sqrt{x^7+9x^4}\Rightarrow x^2\sqrt{x^3+9}$ Let $u=x^3+9$ and $du=3x^2\,dx$. Then $\frac{1}{3}du=x^2\,dx$. The lower bound is

$(0^3 + 9 = 9)9$ and the upper is $(3^3 + 9 = 36)36$. Substituting with u, we get $\frac{1}{3}\int_9^{36} \sqrt{u}\, du$.

10. Answer E: $7i = (x - 2y)i \Rightarrow 7 = x - 2y$. Then $y + 2 = 0$. So y is -2. This means $x - 2y = 7 \Rightarrow x - 2(-2) \Rightarrow x + 4 = 7$. Therefore x must be 3 to make the equation true.

11. Answer E: The rule is $\cot^2 x + 1 = \csc^2 x$, so $\csc^2 x - \cot^2 x = 1$ which is always positive.

12. Answer A: The cardioid is a heart shaped graph which is not one of the graphs you obtain by slicing a cone in different manners.

13. Answer D: I and III are equivalent by change of base. By change of base, II is $\frac{\ln 6}{\ln 7}\frac{\ln 8}{\ln 6}$.

14. Answer B: $3\log_2 3 - \log_2 x = \log_2 45 \Rightarrow \log_2\left(\frac{3^3}{x}\right) = \log_2 45 \Rightarrow \frac{27}{x} = 45 \Rightarrow x = \frac{27}{45}$

15. Answer A : $50000 = P\left(1 + \frac{.06}{365}\right)^{365 \cdot 26} \Rightarrow 10{,}508.15$

16. Answer C: The surface area of a cylinder is given by $A = 2\pi r(h + r)$. Here the radius is 2 inches and the height in terms of inches is 96 inches. So $A = 2\pi(2)(96 + 2) \Rightarrow A = 4\pi(98) \approx 1231.5$. The total surface area of 1231.5 square inches divided by 350 square inches gives the outcome of 3.52 approximate cans. You must have 4 cans to complete the job.

17. Answer D: $2^{15} = 32768$. $i^{15} = -i$.

18. Answer D: If two matrices have the same number of rows and the same number of columns then their sum can be determined. If the number of rows or the number of columns of the two matrices is not equal, they cannot be added.

19. Answer C: To complete the square, we add 9 and 36 to both sides, giving us a radius of 0 and a center of (-3, 6).

20. Answer E: This parametric equation is the graph of the unit circle. Except the endpoints, the graph is non self-intersecting, so the graph is simple. When $t = 0$ and $t = 2\pi$ the points coincide at $(1,0)$, the endpoints, which means the graph is closed.

21. Answer E: Since a = 1, 4p = 1, so p, the focal distance, is $\frac{1}{4}$. The focal distance is the same as the distance from the vertex (-1, -4) to the directrix—it is just applied in a southward direction rather than a northward direction. (The directrix is a line that runs perpendicular to the axis of symmetry, so a clever student will note that E is the only possible answer without making a single calculation.)

22. Answer A: The equation of the line is $y = -x + 1$. Changing to polar we have $r\sin\theta = -r\cos\theta + 1 \Rightarrow r\sin\theta + r\cos\theta = 1 \Rightarrow r = \dfrac{1}{\sin\theta + \cos\theta}$.

23. Answer B: The circle has a radius of 1 unit and thus a diameter of 2 units. That would be the diagonal of the square. Since two sides of the square and the diagonal form an isosceles right triangle, the sides have measure $\sqrt{2}$ units and the area is thus 2 square units.

24. Answer B: Since the standard period (the x distance between a point and the next corresponding point for which the value of y repeats) for cosine is $0 \le x \le 2\pi$, we trap the angle $\left(\pi x + \dfrac{2\pi}{3}\right)$ between 0 and 2π and solve for x.

$0 \le \left(\pi x + \dfrac{2\pi}{3}\right) \le 2\pi \Rightarrow \dfrac{-2}{3} \le x \le \dfrac{4}{3}$. From this we know the phase shift value is $\dfrac{-2}{3}$ and the next time the y value repeats will be at $x = \dfrac{4}{3}$.

25. Answer C: The vertical shift for the function g is 0 if it is to reflect about the line $y = 1$ The parabola cannot be concaving up, but rather down. This makes the function $g(x) = -x^2$. Then $g(-2) = -(-2)^2 = -4$.

26. Answer C: $\dfrac{1}{4}x^2 = \dfrac{1}{4}(3+2t)^2 = \dfrac{1}{4}(4t^2 + 12t + 9) = t^2 + 3t + \dfrac{9}{4}$ So we must remove $\dfrac{57}{4}$

$\dfrac{1}{4}x^2 + x = t^2 + 5t + \dfrac{21}{4}$

to get the desired result.

27. Answer B: There are $5^{10} = 9{,}765{,}625$ possible answer sets. Out of these, there are a total of $C(10,2)C(8,2)C(6,2)C(4,2)C(2,2) = 113{,}400$ answer sets with two of each letters as answers. The probability of randomly selecting that would be $\dfrac{113{,}400}{9{,}765{,}625} \approx 0.01161216$

28. Answer D: $A \cup B = \{2,4,6,8,10,12\}$. There are 6 members in the set.

29. Answer A: By the 68-95-99.7 rule, there's only 0.3% of the population that would be more extreme than three standard deviations away from the mean. This student is **six** standard deviations above the mean, so the percentage will be considerably smaller than even that, which already rounds to 0%.

30. Answer C: p(first round) $= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$ and p(second round) $= \left(\frac{5}{6}\right)^3\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$.

Continuing this process we find the probability for Barbossa can be represented by

$p(\text{Barbossa wins}) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\sum_{n=0}^{\infty}\left[\left(\frac{5}{6}\right)^3\right]^n \Rightarrow \frac{25}{216}\bullet\frac{1}{1-\left(\frac{5}{6}\right)^3} = \frac{25}{91}$.

31. Answer E: Even with two square matrices, we are not guaranteed that they are invertible. Therefore the difference of their inverses is not guaranteed to be defined. Also, if one is not invertible, the product is not invertible and the transpose thereof is not invertible.

32. Answer D: This is a permutation since the order matters for the 3 different parts. ${}_8P_3 - {}_7P_3 = 336 - 210 = 126.$

33. Answer E: $\sin x \tan x + \cos x \Rightarrow \frac{\sin^2 x}{\cos x} + \cos x \Rightarrow \frac{\sin^2 x + \cos^2 x}{\cos x} \Rightarrow \frac{1}{\cos x} = \sec x$.

34. Answer C: As this is a polynomial with odd degree and a negative leading coefficient, we can consider the end behavior of y = -x, which has “up/down” end behavior.

35. Answer A: Since the denominator is not a factor of the numerator, there is a vertical asymptote at x = -6.

36. Answer A: Let c represent the side opposite the included angle. Then $c = \sqrt{45^2 + 67^2 - 2(45)(67)\cos(35°)} \approx 39.7$. Using Hero’s formula we have $A = \sqrt{75.85(36.15)(30.85)(8.85)} = 865.22$.

37. Answer D: If a real-valued polynomial has a root at a complex number, it also has a root at the complex number’s conjugate, so this also has a root of -3 – 4i. That means that the function can be written as (x + 3 - 4i)(x + 3 + 4i).
So we can write this as the difference of squares $(x+3)^2 - (4i)^2$.
So this is $x^2 + 6x + 9 + 16 = x^2 + 6x + 25$.

38. Answer B: Let vector A be 500N with $\theta = 0°$. Vector B is 350N with $\theta = 40°$. Writing each into their component vectors A_x, A_y, B_x, B_y. We can find vector R, the resultant force. $A_x = 500\cos(0°) = 500$; $A_y = 500\sin(0°) = 0$;
$B_x = 350\cos(40°) \approx 268.12$; $B_y = 350\sin(40°) \approx 224.98$. The components for vector R are $R_x = 500 + 268.12 = 768.12$ and $R_y = 0 + 224.98 = 224.98$. Then
$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(768.12)^2 + (224.98)^2} = 800.39 \approx 800.$

39. Answer D: By the first statement, each giraffe takes eight hours to denude a tree. So in five hours, each giraffe will denude five-eighths of a tree. So

$\frac{5}{8}g = 1000$ and thus 1,600 giraffes will denude a forest of 1,000 acacia trees in five hours.

40. Answer C: Grippos serves potato chips (i and ii) and George buys pretzels there (i). LaRosa's is where George orders Ziti Alfredo (iii) and pizza (iii) is what LaRosa's is noted for. We now have Grippos potato chips, LaRosa's pizza, and since the hot fudge cake is not at Skyline (iii), Frisch's must serve the hot fudge cake. This leaves Skyline as the establishment that serves cheese cake.