



2016 Academic Challenge

MATHEMATICS TEST – REGIONAL

– This Test Consists of 30 Questions –

Mathematics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader

Linda Wiggins, Illinois State University – Author

Matthew Childers, Illinois State University – Author

Sandra Cox, Kaskaskia College (ret.) – Reviewer

Sahid L. Rosado Lausell, WYSE – Coordinator of Test Production

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** Time: 40 Minutes *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

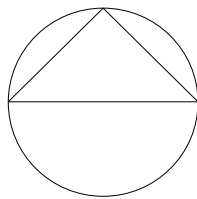
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WYSE – Academic Challenge
Mathematics Test (Regional) – 2016

1. The Loki kite makes an angle of 28° with the level ground and 73 meters of string is out. The Thor kite makes an angle of 45° with the level ground and 50 meters of string is out. Which kite is higher and by how much? Round to the nearest tenth of a meter.
- a) Loki; 29.1m b) Loki; 1.1m c) Thor; 1.1m
d) Thor; 29.1m e) Loki; 20m
2. A car moves across a coordinate grid and its location is described by $x = 5 + 30t$ miles and $y = 14 - 40t$ miles after t hours. How fast is the car moving? Round to the nearest mile per hour.
- a) 10 mph b) 44 mph c) 50 mph d) 61 mph e) 70 mph
3. The border of a garden in feet can be defined as a curve with polar equation $r = 4 \cos \theta$, where r is in feet. If you would like to create a fancy mesh fence border for this garden, how many linear feet of mesh fencing material will be needed? Round to the nearest foot.
- a) 25 ft b) 13 ft c) 8 ft d) 6 ft e) 4 ft
4. When multiplying an identity matrix by the inverse of its transpose, what is the result?
- a) A different identity matrix.
b) The same identity matrix.
c) A zero matrix.
d) A matrix not listed above.
e) The result does not exist.
5. A right isosceles triangle is inscribed inside a circle as shown below. If the area of the triangle is 1 m^2 , determine the area of the circle. Round to the nearest tenth of a square meter.



- a) 1.6 m^2 b) 2.2 m^2 c) 3.1 m^2 d) 4.4 m^2 e) 6.2 m^2

6. A college student decides to save \$40 of her first monthly paycheck. During each succeeding month she saves \$2.00 more than she did the month before. If she continues this pattern for 2 years, how much money will she have saved over the 2 year period?
- a) \$1008 b) \$1140 c) \$1272 d) \$1512 e) \$1600
7. What is the magnitude of $(22.5 + 22.5\sqrt{3}i)^3$? Round to the nearest whole number.
- a) 1 b) 45 c) 180 d) 70,578 e) 91,125
8. Find the eccentricity of $9(x - 4)^2 + 5(y + 3)^2 = 45$.
- a) $\frac{9}{5}$ b) $\frac{3}{2}$ c) $\frac{\sqrt{5}}{2}$ d) $\frac{2}{\sqrt{5}}$ e) $\frac{2}{3}$
9. Which of the following is the domain of $y = \log_5 x^{3/2}$?
- a) All reals b) $(-\infty, 0) \cup (0, \infty)$ c) $(0, \infty)$ d) $(-\infty, 0)$ e) None of these
10. In 2000, a town had a population of 10,000 people. After ten years of exponential growth, the town had 15,000 people. If the town continues to grow exponentially, in what year should the town reach 30,000 people? Round to the nearest year.
- a) 2022 b) 2027 c) 2030 d) 2040 e) 2060
11. A paddle ball is whirled around in a circle at the end of a string that is 28 inches long. If it makes 5 revolutions in 2 seconds, what is the ball's linear speed? Round to the nearest whole inch per second.
- a) 440 in./sec b) 400 in./sec c) 340 in./sec
d) 140 in./sec e) 40 in./sec
12. In how many distinguishable ways can the letters from the word "illegibility" be reordered?
- a) 120 b) 5,040 c) 3,326,400 d) 39,916,800 e) 479,001,600
13. Form the converse for each of the following statements below. Then, select the converse statement(s) that is(are) false.
1. If two angles are congruent then the two angles are vertical angles.
 2. If two angles are adjacent then they have the same vertex.
 3. If two lines are parallel, then their alternate interior angles are congruent.
- a) Converse 1 b) Converse 2 c) Converse 3
d) Converse 1 and 2 e) Converse 1 and 3

14. Bob does a job in 12 hours. Nate does that same job in 16 hours. How long does it take for both of them to do the job simultaneously? Round to the nearest hour.
- a) 6 hr b) 7 hr c) 8 hr d) 9 hr e) 10 hr
15. Three children start with different weights of candy. Anne ate $\frac{1}{3}$ of hers, Blaise ate $\frac{3}{4}$ of his, and Cherise ate $\frac{3}{5}$ of hers, but each of them had 1 pound left over. How many total pounds of candy were eaten?
- a) $1\frac{19}{60}$ lb b) $1\frac{41}{60}$ lb c) 5 lb d) 6 lb e) 8 lb
16. Mark leaves his home and jogs 5 miles east. He then heads 12 miles north. What is the shortest distance that Mark must jog now to get back home?
- a) 9 mi b) 11 mi c) 13 mi d) 15 mi e) 17 mi
17. $f(x) = \frac{3x-1}{4x+6}$. What is $f^{-1}(5)$?
- a) $-\frac{31}{17}$ b) $\frac{7}{13}$ c) 1 d) $\frac{13}{7}$ e) None of these
18. Given the function $f(x) = \frac{3(x-2)^3(x+1)^2}{12(x-2)^2(x+1)}$, find all vertical asymptotes.
- a) $x = -1$ b) $x = 2$ c) $x = -1; x = 2$ d) $x = \frac{1}{4}$ e) none
19. How many of the following four functions are even?
- I) $\sin(\cos x)$ II) $\cos x + \sec x$ III) $\sin x + \tan x$ IV) $\cos(\tan x)$
- a) 0 b) 1 c) 2 d) 3 e) 4
20. What is the formal term for a four sided polygon that has one adjacent pair of sides of length 'a' and the other sides length 'b' ($a \neq b$)?
- a) kite b) rhombus c) rectangle d) trapezoid e) parallelogram
21. Simplify the expression: $\frac{(x+5)^{-1} - (x-3)^{-1}}{(x+5)^{-1} + (x-3)^{-1}}$
- a) $\frac{-4}{x+1}$ b) $\frac{4}{x+1}$ c) 0 d) $\frac{-8}{(x+5)(x-3)}$ e) $\frac{2x+2}{(x+5)(x-3)}$

22. What is $\lim_{x \rightarrow -\infty} (e^{-2x} + 5x - 1)$?
- a) $-\infty$ b) -1 c) 0 d) ∞ e) undefined
23. A sociology study for the city of Chicago found that of the new married couples that move into the city, 40% of them move out within 5 years. Assuming that the new married couples move out of Chicago exponentially, find the half life of the new married couples living in the city of Chicago. Round to the nearest year.
- a) 4 years b) 5 years c) 6 years d) 7 years e) 8 years
24. What is the period of $y = 4 + \tan(3x + 2\pi)$?
- a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) π d) 2π e) 5
25. A bag of marbles has one red, three blue, and six green. If we reach into the bag and grab a handful of three marbles (assume each marble is equally likely of being picked), what is the probability that all three marbles are the same color?
- a) $\frac{1}{6}$ b) $\frac{1}{5}$ c) $\frac{1}{3}$ d) $\frac{3}{40}$ e) $\frac{7}{40}$
26. Find the sum of the coordinates of the point of intersection of the lines having equations $10x + 5y = -10$ and $\frac{7}{2}x + y = \frac{5}{2}$.
- a) -8 b) -5 c) -3 d) 3 e) 5
27. If $f(x) = x^2 - 3x + 2$, which of the following is $f(x - 2)$?
- a) $x^2 - 3x + 12$ b) $x^2 - 7x$ c) $x^2 - 3x + 4$
 d) $x^2 - 7x + 12$ e) $x^2 - 3x$
28. If the $\log_b 4 = \frac{2}{3}$, what is the value of b ?
- a) $b = 4^{\frac{2}{3}}$ b) $b = \left(\frac{2}{3}\right)^4$ c) $b = \left(\frac{3}{2}\right)^4$ d) $b = 2^{\frac{4}{3}}$ e) $b = 4^{\frac{3}{2}}$
29. If $f(0) = 8$ and $f(3) = 6$, what is the average rate of change of $f(x)$ from $x = 0$ to $x = 3$?
- a) $-\frac{2}{3}$ b) 1 c) $\frac{3}{2}$ d) 8 e) $\frac{14}{3}$

30. A flower warehouse employs three drivers, Amy, Beth, and Carol. On one particular day, each driver made hourly deliveries from 10AM until 2PM to the four stores they supply, Embry's, Flowers 'R Us, GrowLots, and Hamm's (a total of twelve deliveries).

- I. One of the stores received only morning deliveries.
- II. No driver ever went to the same store on back-to-back deliveries.
- III. Three times, Carol had to go to the store Amy had just visited to fix a mistake.
- IV. At 10AM, Amy went to Flowers 'R Us, Beth to GrowLots, and Carol to Embry's.
- V. At noon, Amy went to GrowLots, Beth went to Flowers 'R Us, and Carol to Hamm's.

Determine whether or not any of the drivers delivered to all four stores.

- a) Only Amy delivered to all four stores.
- b) Only Beth delivered to all four stores.
- c) Only Carol delivered to all four stores.
- d) Amy and Beth delivered to all four stores.
- e) Beth and Carol delivered to all four stores.