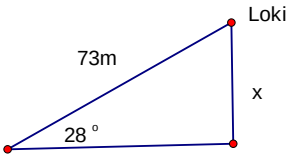
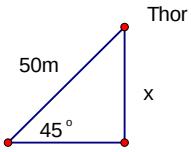
WYSE – Academic Challenge
Math Test (Regional) - 2016

1. **Correct Answer: C**

For the height of the Loki kite we have $x = 73\sin 28° \approx 34.271$. For the height of the Thor kite we have $x = 50\sin 45° \approx 35.355$. The Thor kite is higher by 1.1 meter. See the diagrams below:





2. **Correct Answer: C**

The car is moving 30 mph to the east and 40 mph to the south. Using the Pythagorean Theorem, that means it's moving 50 mph in total.

3. **Correct Answer: B**

Using $\cos\theta = \frac{x}{r}$ and $r^2 = x^2 + y^2$, $r = 4\cos\theta \Rightarrow r = 4\left(\frac{x}{r}\right) \Rightarrow r^2 = 4x$. Substituting for r^2 we have $x^2 + y^2 = 4x \Rightarrow x^2 - 4x + y^2 = 0$. Completing the square we have $(x-2)^2 + y^2 = 4$. The garden is circular with a radius of 2 feet. The circumference is what we need to find the linear feet for the border. $2\pi(2) = 4\pi = 12.5664 \approx 13$.

4. **Correct Answer: B**

An identity matrix is its own transpose. An identity matrix is its own inverse. An identity matrix multiplied by itself is the same identity matrix.

5. **Correct Answer: C**

Let x be the leg length of the triangle. $\frac{1}{2}x^2 = 1$, $x = \sqrt{2}$. The hypotenuse/ diameter is therefore $\sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = 2$, the radius = 1, and area = $\pi 1^2 \approx 3.1$

6. **Correct Answer: D**

This describes an arithmetic series with $a_1 = 40$, $d = 2$, and $n = 24$. Then $\sum_{k=1}^{24} a_k = 24(40) + \frac{24(23)}{2}2 = 1512.$

7. **Correct Answer: E**

The magnitude of $22.5+22.5\sqrt{3}i$ is $\sqrt{22.5^2+\left(22.5\sqrt{3}\right)^2}=45$. By DeMoivre's Theorem, the magnitude of a complex number x raised to the nth power is the magnitude of that complex number, r, raised to the nth power. 45 cubed is 91,125.

8. **Correct Answer: E**

$9\left(x-4\right)^2+5\left(y+3\right)^2=45\Rightarrow\frac{\left(x-4\right)^2}{5}+\frac{\left(y+3\right)^2}{9}=1$. The semi-major axis is $a=3$. The semi-minor axis is $b=\sqrt{5}$ and the distance from the center to the foci is $c=\sqrt{9-5}=2$. The eccentricity is $e=\frac{c}{a}=\frac{2}{3}$.

9. **Correct Answer: C**

The domain is dependent upon where $x^{3/2}$ is positive. That is equivalent to the positive real numbers.

10. **Correct Answer: B**

Using x = years after 2000, the population can be modeled by $y=10000\cdot1.5^{\frac{x}{10}}$, or the equivalent $y=10000\cdot e^{0.04055x}$. Solving $30000=10000\cdot1.5^{\frac{x}{10}}$ gives $x\approx27.1$.

11. **Correct Answer: A**

$\omega=\frac{\theta}{t}$; The angular speed. $\omega=\frac{5\left(2\pi\right)}{2}=5\pi$. The linear speed $v=r\omega=28\left(5\pi\right)=439.8229715\approx440$.

12. **Correct Answer: C**

There are 12 letters in illegibility, one of which is repeated four times and one of which is repeated thrice. So the number of distinguishable strings is $\frac{12!}{4!3!}=3,326,400$.

13. **Correct Answer: B**

Converse of 1 states that if two angles are vertical angles then the angles are congruent (true). Converse of 2 states that if two angles have the same vertex then the two angles are adjacent. This is false. The angles could be vertical. Converse of 3 states if alternate interior angles are congruent then the two lines are parallel. This is true (Postulate 4.1). Only the Converse of 2 is false.

14. **Correct Answer: B**

Bob does $\frac{1}{12}$ of the job every hour and Nate does $\frac{1}{16}$ of the job every hour. That means that together, they do $\frac{1}{12}+\frac{1}{16}=\frac{7}{48}$ of the job each hour. This means that it will take the two of them $\frac{1}{7/48}=\frac{48}{7}=6\frac{6}{7}$ hours to finish the job.

15. **Correct Answer: C**

Let the starting amounts be A, B, and C. $\frac{2}{3}A=1$, so $A=1.5$, $\frac{1}{4}B=1$, so $B=4$, and $\frac{2}{5}C=1$, so $C=2.5$. The eaten amounts are $\frac{1}{3}A=0.5$, $\frac{3}{4}B=3$, and $\frac{3}{5}C=1.5$, for a total amount eaten of $0.5+3+1.5=5$.

16. **Correct Answer: C**

The jog represents a right triangle with the legs of the triangle represented by 5 and 12. The shortest distance then would be the hypotenuse side to get back home. This is the typical 5,12,13 right triangle $\left(\sqrt{5^2+12^2}=13\right)$.

17. **Correct Answer: A**

This is easiest to solve when we set f(x) to be equal to 5 and then solve for x.
$\frac{3x-1}{4x+6}=5$, so $3x-1=5(4x+6)$. So $3x-1=20x+30$ and then $-17x=31$.

18. **Correct Answer: E**

Since there are factors of $(x-2)$ and $(x+1)$ in both the numerator and denominator, there are no vertical asymptotes.

19. **Correct Answer: D**

Even functions are functions for which f(-x) = f(x) is an identity. cos x and sec x are the two basic trig functions which are even. This makes both I and II even. Odd functions are functions for which f(-x) = -f(x) is an identity. The other four basic trig functions are odd. This makes III odd, and since tan x is odd, cos (tan x) would be even.

20. **Correct Answer: A**

Kite, by definition.

21. **Correct Answer: A**

$$\frac{(x+5)^{-1}-(x-3)^{-1}}{(x+5)^{-1}+(x-3)^{-1}} \Rightarrow \frac{\frac{1}{(x+5)}-\frac{1}{(x-3)}}{\frac{1}{x+5}+\frac{1}{x-3}} \Rightarrow \frac{x-3-x-5}{x-3+x+5} \Rightarrow \frac{-8}{2x+2} \Rightarrow \frac{-4}{x+1}$$

22. **Correct Answer: D**

The exponential portion goes to infinity much, much faster than the linear portion goes to negative infinity, so the answer is infinity.

23. **Correct Answer: D**

$0.6p = pe^{-5r} \Rightarrow r = \frac{\ln(0.6)}{-5} = 0.102165$, $0.5 = e^{-0.102165t} \Rightarrow t = \frac{\ln(0.5)}{-0.102165} \approx 6.78$. The half-life of a new married couple in the city is approximately 7 years.

24. **Correct Answer: A**

The period of tan x is π, so the period of tan 3x would be A. The other transformations of the function are merely shifts, so the period does not change.

25. **Correct Answer: E**

The overall number of possible draws is given by C(10,3) = 120. The ways we can get all three the same color is all three blue C(3,3)=1 or all three green C(6,3)=20. This makes the overall probability 21/120=7/40.

26. **Correct Answer: B**

Solving the system of equations we have $7x+2y-5=10x+5y+10 \Rightarrow y=-x-5$. Then $7x+2(-x-5)=5 \Rightarrow 5x-10=5 \Rightarrow x=3$. Then $y=-8$. $3+(-8)=-5$.

27. **Correct Answer: D**

$f(x-2)=(x-2)^2-3(x-2)+2=x^2-4x+4-3x+6+2=x^2-7x+12.$

28. **Correct Answer: E**

$$\log_b 4 = \frac{2}{3} \Rightarrow b^{\frac{2}{3}} = 4 \Rightarrow b = 4^{\frac{3}{2}}$$

29. **Correct Answer: A**

The average rate of change of a function is the slope of the secant line between the two described points. In this case, the points are (0, 8) and (3, 6) and the slope is thus A.

30. **Correct Answer: E**

We can start by filling out the 11AM deliveries as Amy to Hamms (since Carol follows), Carol to Flower's R Us (to follow Amy), and Beth to Embry's (so she doesn't repeat). We then fill out the 1PM as none to Embry's (to satisfy I), Carol to GrowLots (to follow Anne), Beth to Hamm's (so she doesn't repeat) and Amy to Flowers R' Us (by default). We can now list deliveries by person. Amy: F-H-G-F, Beth: G-E-F-H, and Carol: E-F-H-G. Beth and Carol went to all four.