2016 Academic Challenge

MATHEMATICS TEST – SECTIONAL

– This Test Consists of 30 Questions –

Mathematics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader
Linda Wiggins, Illinois State University – Author
Matthew Childers, Illinois State University – Author
Sandra Cox, Kaskaskia College (ret.) – Reviewer
Sahid L. Rosado Lausell, WYSE – Coordinator of Test Production

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ◐ , ◑ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

WYSE – Academic Challenge
Mathematics Test (Sectional) – 2016

1. Square MNOP is inscribed in square IJKL. Square IJKL is inscribed in square EFGH. Square EFGH is inscribed in square ABCD. If square ABCD has a side length of 10 cm, find the area of square MNOP. Round to one decimal place.

a) $12.5\,cm^2$ b) $25\,cm^2$ c) $37.5\,cm^2$ d) $42.3\,cm^2$ e) $50\,cm^2$

2. The graph of $y = 5^{-3x+1}$ takes on what form?

a) Increasing exponential b) Decreasing exponential c) Constant
d) Increasing linear e) Other shape

3. Given $A = \begin{bmatrix} 1 & k \\ 3 & k^2 \end{bmatrix}$, find k if the det(A) = 10 and the sum of a_{12} and a_{21} is odd:

a) $k = -5$ b) $k = -2$ c) $k = 2$ d) $k = 4$ e) $k = 5$

4. One third of the applicants for a job pass the first test. Of the ones who pass the first test, four fifths pass the second test. Of the ones who fail the first test, two thirds proceed to fail the second test as well. Of 90 "typical" applicants, how many would you expect to fail <u>exactly</u> one test?

a) 6 b) 24 c) 26 d) 46 e) 50

5. Determine the area of a regular hexagon that has a perimeter of 1 meter. Round to the nearest hundredth of a square meter.

a) $0.03\ m^2$ b) $0.07\ m^2$ c) $0.17\ m^2$ d) $0.29\ m^2$ e) $0.43\ m^2$

6. Multiply $\left(\sqrt{5}+\sqrt{-2}\right)\left(2+\sqrt{-10}\right)$. Write the result in standard $a+bi$ form.

a) $4\sqrt{5}+10i$ b) $7\sqrt{2}-4\sqrt{5}\,i$ c) $4\sqrt{5}-3\sqrt{2}\,i$
d) $7\sqrt{2}-0\,i$ e) $0+7\sqrt{2}\,i$

7. Which of the following is not a trig identity?

a) cot x tan x = 1 b) $\csc^2 x + 1 = \cot^2 x$ c) $\sin x \cos x = \dfrac{\tan x \cos x}{\sec x}$

d) $\sin^2 2x = \sec^2 2x - \tan^2 2x - \cos^2 2x$ e) All of the above are identities.

8. Mia wants to create a uniform border around a rectangular poster that measures 40 by 24 inches. She has enough border material to cover 740 square inches. How wide should the border be to use up all of the border material on this poster?

a) 4.5 in. b) 4.8 in. c) 5.0 in. d) 5.5 in. e) 5.8 in.

9. What is the sum of x, y and z in the solution to the system $\begin{cases} x + 6y + 2z = 37 \\ -3x + 4y + 3z = 22 \\ 3x - 9y - 4z = -47 \end{cases}$?

a) 7 b) 8 c) 9 d) 12 e) 24

10. How many vertices are there on an icosahedron?

a) 12 b) 20 c) 24 d) 30 e) 60

11. Simplify $\left(-2^2 x^2\right)^3 \left(\left(-2x\right)^2\right)^{-3}$. Leave answer in exponential form.

a) 1 b) $2^2 x^6$ c) $2^6 x^4$ d) $\frac{1}{2^4 x^6}$ e) -1

12. What is the maximum value for $f(x) = x^2 - 5x + 2$?

a) -4.25 b) -2.5 c) 2 d) $\frac{5 + \sqrt{17}}{2}$ e) There is no maximum

13. There are 18 assorted cookies in a cookie jar: 6 chocolate chip, 4 sugar, 5 peanut butter, and 3 oatmeal. If you pick 6 cookies at random, what is the probability that you picked 4 chocolate chip cookies? Round answer to 4 decimal places.

a) 0.0049 b) 0.0222 c) 0.0533 d) 0.0613 e) 0.0735

14. A train of 26 maximally-sized *Corydoras* catfish is lined up head to tail. It contains some pygmy cories (*C. pygmaeus*), whose maximum size is 0.75", some Juliis (*C. julii*, maximum size 2.5") and some goldens (*C. aeneus*), which max out at 2.75". The train is 56" long, and contains as many pygmies as goldens. How many Juliis are there?

a) 6 b) 12 c) 14 d) 20 e) 26

15. The function $h(t) = -16t^2 + 20$ takes t, the number of seconds since a ball was dropped off a twenty foot tall wall, and gives h, the height of the ball off the ground in feet. How fast is the ball moving when it hits the ground? Round to the nearest foot per second.

a) $16 \frac{ft}{s}$ b) $20 \frac{ft}{s}$ c) $32 \frac{ft}{s}$ d) $36 \frac{ft}{s}$ e) $40 \frac{ft}{s}$

16. Completely factor $8a^3 - 36a^2 + 54a - 27$.

a) $4a^2\left(2a - 9\right) + 27\left(2a - 1\right)$ b) $\left(4a^2 - 27\right)\left(2a - 1\right)$ c) $\left(4a^2 + 27\right)\left(2a - 1\right)$

d) $\left(2a - 3\right)^3$ e) $\left(4a^2 + 9\right)\left(2a - 3\right)$

17. Which of the following is $\frac{d}{dx}(4x+7)^3$ when x = 1?

a) 64 b) 256 c) 363 d) 1331 e) 1452

18. Simplify $9\cos^2 t + (3\cos(t)\cdot\tan(t))^2$. Assume $\cos(t) \neq 0$.

a) $9\cos^2(t)$ b) 9 c) $9\sin^2(t)$ d) $9\tan(t)$ e) 0

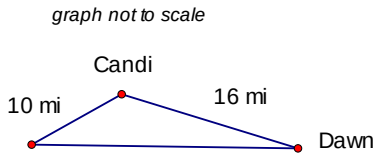
19. Which of the following is **mathematically** equivalent to $\sum_{n=1}^{\infty} n$?

a) $-\infty$ b) $-\frac{1}{12}$ c) 0 d) ∞ e) None of these

20. If an angle's sine value is 0.6, what is its tangent? Round to the nearest hundredth.

a) 0.60 b) 0.63 c) 0.75 d) 0.80 e) 1.50

21. Candi usually travels to Dawn's home on a direct southern route that is 16 miles long. Due to flash flood, the direct route is no longer available and she must take a detour that is $110°$ SW of the direct route. She follows this road for 10 miles and then turns directly east to get to Dawn's home. How much longer is the detour than the direct route to Dawn's home? See the diagram below. Round to the nearest mile.



a) 32 mi b) 22 mi c) 16 mi d) 12 mi e) 6 mi

22. Which of the following is created by $2x^2 + 3y^2 - 16x - 18y = -59$?

a) Circle b) Ellipse c) Hyperbola d) Parabola e) Single Point

23. A bottle rocket is placed on the ground at an angle of $60°$ with the horizontal. If the rocket's initial velocity is 140 feet per second, what is the maximum height of the bottle rocket during its flight? Round to the nearest foot.

a) 140 ft. b) 155 ft. c) 200 ft. d) 230 ft. e) 265 ft.

24. What is the sum of the **external** angle measures of an octagon, measured in degrees?

a) 180 b) 225 c) 1080 d) 1800 e) 2160

25. When proofreading, Al takes four minutes per page, and Bill 6 minutes per page. At noon, Al starts proofreading a stack of 100 pages. At 12:20, Bill joins in and helps until all the pages are proofread once. When do they finish? Round to the nearest minute.

a) 2:20 PM b) 2:40 PM c) 3:28 PM d) 3:48 PM e) 4:08 PM

26. A group of friends are participating in a science fact recall study. After they complete the semester science course materials, they are tested at the end and weekly thereafter on the same material. The average score on the exam taken after t weeks is given by the function $a(t) = 77 - 10\ln(t+1)$. How many weeks does it take for the average score to drop from 66 to 59?

a) 1 week b) 2 weeks c) 3 weeks d) 4 weeks e) 5 weeks

27. What is the length of the diagonal of a rectangle whose sides are in a 3:2 ratio and whose perimeter is 60? Round to the nearest tenth.

a) 21.6 b) 24.0 c) 30.0 d) 43.3 e) 60.0

28. Party City balloons are filled with helium by two employees, Matt and Thomas. They are told to blow up some spherical balloons of the same type as quickly as possible, but are warned that the balloons will burst if the radius grows at a rate any faster than 0.02 cm/s when the diameter of the balloon reaches 50 cm. Matt sets his helium tank so the volume of air in the balloon increases by 150 cm^3/s while Thomas sets his tank at 200 cm^3/s. Which employee is most efficient blowing up these balloons and at what rate is the radius of the balloon increasing when the diameter of the balloon reaches 50 cm?

a) Matt; 0.019 cm/s b) Thomas; 0.013 cm/s c) Matt; 0.013 cm/s
d) Thomas; 0.025 cm/s e) Matt; 0.022 cm/s

29. Is $r = 5 - 2\sin\theta$ the equation of a limacon? If so, is it one of the three types listed?

a) Inner-looped limacon b) Dimpled limacon c) Convex limacon
d) Other type of limacon e) Not a limacon

30. At a recent flea market, Aaron (m), Bethany (f), Craig (m), Dana (f), and Eric (m) all had stalls. Each one sold a different type of item, and the five types were clothes, jewelry, candles, pillows, and sports memorabilia. The sellers had a total of fifteen customers, each seller getting at least one customer. The following statements are also true:

i. Bethany had more customers than the woman selling candles, but fewer customers than the man selling jewelry.
ii. Eric had the same number of customers as the woman selling clothes.
iii. The man selling pillows had twice as many customers as Dana.
iv. Craig had three times as many customers as Aaron.

Who was selling the sports memorabilia and how many customers did they have?

a) Aaron – one customer b) Aaron – two customers c) Craig – three customers
d) Eric – two customers e) Eric – three customers