WYSE – Academic Challenge
Math Test (Sectional) - 2016

1. **Correct Answer: A**

Area of square ABCD is 100 cm^2. Since the inscribed square has half the area of the outer square, square EFGH has an area of 50 cm^2. Continuing this process to find the area of square MNOP, we find the area is 12.5 cm^2.

2. **Correct Answer: B**

$5^{-3x+1} = 5 \cdot 5^{-3x} = 5 \cdot \left(\frac{1}{125}\right)^x$. This is a vertically-stretched decreasing exponential.

3. **Correct Answer: B**

$k^2 - 3k = 10 \Rightarrow k = -2; k = 5$. If $a_{12} = -2$, then the sum is 1. But, if $a_{12} = 5$, then the sum is 8. Therefore $k = -2$.

4. **Correct Answer: C**

There are only two classes to worry about—the folks who pass the first test while failing the second, and the ones who fail the first test and yet pass the second. We would expect 30 to pass the first test (so 60 fail it). Four fifths of 30 is 24, so six fail the second test after passing the first. Of the 60 who fail the first test, 40 will likewise fail the second test, so 6 + 20 = 26 will pass exactly one test.

5. **Correct Answer: B**

Split the hexagon into six equilateral triangles, each with sides of 1/6 meter and altitudes of $\frac{\sqrt{3}}{12}$ meter. That means each triangle has an area of $\frac{1}{2} \cdot \frac{1}{6} \cdot \frac{\sqrt{3}}{12} = \frac{\sqrt{3}}{144}$ square meters, and six of them is $\frac{\sqrt{3}}{24} \approx 0.072$ square meters.

6. **Correct Answer: E**

$\left(\sqrt{5} + \sqrt{-2}\right)\left(2 + \sqrt{-10}\right) \Rightarrow 2\sqrt{5} + \sqrt{-50} + 2\sqrt{-2} - 1\sqrt{20} \Rightarrow 5\sqrt{2}\,i + 2\sqrt{2}\,i \Rightarrow 7\sqrt{2}\,i$

7. **Correct Answer: B**

B is actually a contradiction.

8. **Correct Answer: C**

$2x\left(24 + 2x\right) + 40x + 40x = 740 \Rightarrow x^2 + 32x - 185 = 0$ where x represents the width. Therefore $x = 5$ or $x = -37$. Then the width has to be 5 inches.

9. **Correct Answer: D**

If you add you add together the three rows, you get x + y + z = 12. (Incidentally, the solution was x = 3, y = 4 and z = 5.)

10. **Correct Answer: A**

20 triangle faces, 5 edges meeting at each vertex, 20 * 3 / 2 * 2 / 5 = 12 vertices.

11. **Correct Answer: E**

$$\left(-2^2x^2\right)^3\left(\left(-2x\right)^2\right)^{-3} \Rightarrow \frac{\left(-1\right)^3 2^6 x^6}{\left(-2\right)^6 x^6} \Rightarrow -1.$$

12. **Correct Answer: E**

This is a parabola opening upward, so it has no max value. Its min value is A.

13. **Correct Answer: C**

$$\frac{\binom{6}{4}\binom{12}{2}}{\binom{18}{6}} \Rightarrow 0.0533$$

14. **Correct Answer: C**

Let x = the number of pygmies. Then x = the number of goldens, and the number of Juliis is 26 – 2x. 0.75x + 2.5(26 – 2x) + 2.75x = 56, so x = 6 and 26 – 2x = 14.

15. **Correct Answer: D**

The ball hits at h = 0, $t = \sqrt{\frac{20}{16}} \approx 1.118$ seconds. The velocity is given by $h'(t) = -32t$, so the speed is $-32 \cdot 1.118 \approx -35.78$ feet per second

16. **Correct Answer: D**

$8a^3 - 36a^2 + 54a - 27$ written in prime form is $\left(2a\right)^3 - 3^2\left(2a\right)^2 + 2 \cdot 3^3 a - 3^3$. This is a cube factor form $\left(u - v\right)^3$ where $u = 2a$ and $v = 3$.

17. **Correct Answer: E**

By the chain rule, the derivative in question is $3(4x + 7)^2 \cdot 4$.

18. **Correct Answer: B**

$9\cos^2(t)\left(1+\tan^2(t)\right) \Rightarrow \frac{9\cos^2(t)}{\cos^2(t)} \Rightarrow 9$.

19. **Correct Answer: D**

Contrary to specious YouTube videos, the answer is infinity. The summed numbers continue to get larger, creating an infinite sum.

20. **Correct Answer: C**

$\sin^2 x+\cos^2 x=1$ and $\tan x=\frac{\sin x}{\cos x}$, so $\tan x=\frac{\sin x}{\sqrt{1-\sin^2 x}}=\frac{0.6}{\sqrt{1-0.6^2}}=0.75$. Or using a calculator, simply use the inverse sine then the tangent.

21. **Correct Answer: C**

The miles traveled east is given by $\sqrt{16^2+10^2-2(16)(10)\cos(110°)} \approx 21.6$. The detour is 10 + 21.6 =31.6. The difference is approximately 16 miles.

22. **Correct Answer: E**

It's the single point (4, 3), to be exact. If we complete the square, we need to rewrite the equation as $2(x^2-8x)+3(y^2-6y)=-59$ and then we would have to add 32 and 27 to each side to complete each of the squares, which leaves us with the following: $2(x^2-8x+16)+3(y^2-6y+9)=2(x-4)^2+3(y-3)^2=0$. The only way this is possible is if each of the squares is equal to 0, hence the single point (4, 3).

23. **Correct Answer: D**

The parametric equations are $x=(140\cos(60°))t$; $y=(140\sin(60°))t-16t^2$. We solve for the height y to be 0. Here we find $t=0$ and $t=\frac{140\sin(60°)}{16} \approx 7.5777$. The max height occurs halfway between these two times. Therefore the needed value is $t=3.78885$. $y=(140\sin(60°))(3.78885)-16(3.78885)^2=229.6875 \approx 230$.

24. **Correct Answer: D**

8 times 360 is 2880. 6 times 180 is 1080. (Remember that the total interior angle measure of a polygon is the number of sides, less two, times 180 degrees.) So the difference is 1800 degrees.

25. **Correct Answer: E**

Al reads 1/4 page per minute for A minutes and Bill 1/6 per minute for B minutes, so the equation 1/4 A + 1/6 B = 100 models the solution. Substitute A = B + 20 and solve to get B = 228. This number of minutes after 12:20 corresponds to the time 4:08PM

26. **Correct Answer: C**

For an average score of 66 we have $66 = 77 - 10\ln(t+1) \Rightarrow t \approx 2$. When the average score is 59 we have $59 = 77 - 10\ln(t+1) \Rightarrow t \approx 5$. The difference is 3 weeks.

27. **Correct Answer: A**

Let the length of the rectangle = 3x and its width be 2x. Then 10x = 60 and x = 6, so the length is 18 and the width is 12. Those form the legs of a right triangle whose hypotenuse is the diagonal, so by the Pythagorean Theorem, the length of the diagonal is $\sqrt{468} = 6\sqrt{13} \approx 21.6$.

28. **Correct Answer: A**

$V = \frac{4}{3}\pi r^3; \frac{dv}{dr} = 4\pi r^2; \ \frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt}$. The radius here is 25 cm. Substituting in derivative values for Matt we have $150 = 4\pi(25)^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} \approx 0.019$. For Thomas we have $200 = 4\pi(25)^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} \approx 0.025$. Matt is closer without going over the 0.02 cm/s warning. Thomas is bursting his balloon at his fill rate. Matt is the efficient employee.

29. **Correct Answer: C**

If $r = a - b\sin\theta$ or $r = a - b\cos\theta$ and a and b are both nonzero and $\left|\frac{a}{b}\right| \geq 2$, the graph is of a convex limacon. (If $\left|\frac{a}{b}\right| < 1$, the limacon has an inner loop and if $1 < \left|\frac{a}{b}\right| < 2$, the limacon has a dimple. If $\left|\frac{a}{b}\right| = 1$, the graph is of a cardioid, not a limacon.)

30. **Correct Answer: E**

We should first notice that Dana must be selling the candles and Bethany must be selling the clothes. Eric cannot be selling jewelry since he has the same number of customers as Bethany. If we let Aaron sell jewelry, then he must be selling a minimum of 3, which would cause Craig to have 9 customers (too many!). This must mean Craig is selling the jewelry. Eric cannot be selling the pillows. If he were, then either Dana would have to sell 1 candle, Eric and Bethany 2 each, leaving 10 for Aaron and Craig (which would make the 3x impossible), or Diana would have to sell 2, making Eric and Bethany sell 4, leaving 5 for Aaron and Craig (also impossible due to the 3x). This means Eric must be selling the sports memorabilia and Aaron is selling pillows. Dana must sell 1 candle, Aaron 2 pillows, and Craig 6 jewelry, to get the multiples to work out. This means Bethany sells 3 clothing items and Eric sells 3 sports memorabilia.