2016 Academic Challenge

MATHEMATICS TEST – STATE

– This Test Consists of 30 Questions –

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ◑ , ◐ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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1. Simplify the expression $\sec\theta\csc\theta\left(1+\dfrac{1}{\sec^2\theta}\right)-\dfrac{2}{\tan\theta}$.

   a) $\sec\theta$ b) $\csc\theta$ c) $\sec^2\theta$ d) $\cot\theta$ e) $\tan\theta$

2. Which of the following is the indefinite integral of $f(x)=\dfrac{1}{x}+e^{2x}+e$?

   a) $F(x)=x^0+e^{2x}+\dfrac{e^2}{2}+C$ b) $F(x)=x^0+\dfrac{e^{2x}}{2}+\dfrac{e^2}{2}+C$ c) $F(x)=\ln|x|+\dfrac{e^{2x}}{2}+ex+C$

   d) $F(x)=\ln|x|+\dfrac{e^{2x}}{2}+e^x+C$ e) None of these

3. A surveyor lays out a baseline segment AB of length 150 m. An inaccessible point C forms the third vertex of a triangle ABC. If the surveyor determines that angle ACB is $83°$ and angle CBA is $61°$, then how far is point C from A? Round to the nearest meter.

   a) 36 m   b) 43 m   c) 89 m   d) 111 m   e) 132 m

4. If it costs Nick $30,000 to produce 5,000 cups and the cost increases by C'(x) = 5x + 2 thousand dollars per thousand cups, how much will it cost him to make 6,000 cups?

   a) $27,000   b) $36,000   c) $57,000   d) $59,500   e) $87,000

5. A population currently has 20,000 people and is growing at a rate of 1,000 people per year. If this population continues to grow exponentially, determine how many years it should take to reach 50,000 people. Round to the nearest whole year.

   a) 8 years   b) 18 years   c) 20 years   d) 24 years   e) 30 years

6. In the story of *Aladdin*, the "Cave of Wonders" has an isosceles trapezoidal opening with the top measuring 7 yards and the height 8 yards. The area of this opening is 104 square yards. How long is the lower base of this opening?

   a) 19 yards   b) 21 yards   c) 22 yards   d) 25 yards   e) 26 yards

7. If you know that a 30-question multiple choice exam has 6 of each possible answer (A, B, C, D, E), what is the probability that you can perfectly guess the location of at least 5 of the 6 A's with no prior knowledge? Round to the nearest hundredth of a percent.

   a) 0.00%   b) 0.01%   c) 0.02%   d) 0.03%   e) 2.44%

8.   Choose the quadrilateral that has the following characteristics: Diagonals are not always congruent nor perpendicular. Diagonals always bisect each other but do not always bisect vertex angles.

a) Rectangle   b) Parallelogram   c) Rhombus
d) Square   e) Trapezoid

9.   In how many ways can the letters from "tattooed lion" be rearranged into distinct "words"? Please disregard the space.

a) 18,480   b) 2,217,600   c) 13,305,600
d) 79,833,600   e) 479,001,600

10.   A bolt machine correctly identifies properly made bolts 95% of the time and correctly identifies improperly made bolts 98% of the time. If 99% of bolts are made properly, what percentage of bolts that the machine identifies as improperly made are in fact properly made? Round to the nearest whole percent.

a) 0%   b) 5%   c) 7%   d) 71%   e) 83%

11.   Florida's Disney World has offered a special 7 day tour at a rate of $800 per person for a group of 50 people. The company also gives a discount of $15 for every additional 2 people in the group. Find the revenue function R(x) for this company.

a) $R(x) = \dfrac{-15}{2}x + 1175$ b) $R(x) = -15x + 2350$ c) $R(x) = -30x^2 + 2350x$

d) $R(x) = \dfrac{-15}{2}x^2 + 1175x$ e) $R(x) = -30x^2 - 2350x$

12.   If x = 3 + 2 cos t and y = 4 - 3 sin t, what is the area of the shape created by letting t go from 0 to 2π ? Round to the nearest tenth of a square unit.

a) 6.0   b) 18.8   c) 36.0   d) 37.7   e) 113.1

13.   Given the function $f(x) = \dfrac{x^3 - x^2 - 2x}{2x^2 - 4x}$, determine the types of asymptotes that the function obtains.

a) vertical   b) horizontal   c) oblique
d) vertical and horizontal   e) vertical and oblique

14.   On a long, straight stretch of highway, Jake is stopped at mile marker 242, and John at mile marker 247. A balloon launches from somewhere between them. After a while, they simultaneously measure their angles of elevation to the balloon — Jake records 45 degrees and John 56 degrees. How high is the balloon, to the nearest tenth of a mile?

a) 1.9 miles   b) 2.0 miles   c) 2.9 miles   d) 3.0 miles   e) 3.1 miles

15.   A circular track is a quarter mile around. At noon, Josh begins jogging counterclockwise from the starting line at 5 mph. At 12:20 PM, Kelly begins running counterclockwise from the starting line at 10 mph. At what time does Kelly pass Josh for the third time? Round to the nearest whole minute

a) 12:26 PM b) 12:27 PM   c) 12:28 PM   d) 12:29 PM   e) 12:30 PM

16.   An air traffic controller spots two airplanes at the same altitude converging to a point as they fly at right angles to each other. One airplane is 180 miles from the point and has a speed of 270 miles per hour. The other is 240 miles from the point and has a speed of 360 miles per hour. At what rate is the distance between the planes changing?

a) decrease by 300 mi/hr   b) decrease by 360 mi/hr   c) decrease by 390 mi/hr
d) decrease by 420 mi/hr   e) decrease by 450 mi/hr

17.   If $3x^2 - 5x + c = 0$, what value of c makes it so that there's one and only one solution?

a) 25/12   b) 5/3   c) 3/5
d) No such c exists   e) There is such a c, but it's not listed

18.   Determine an equation of the line tangent to the graph of $f(x) = \left(x^3 - 4x + 2\right)^4$ at $(1,0)$.

a) $y = 4x - 4$ b) $y = 4x + 4$ c) $y = -4x - 4$
d) $y = -4x + 4$ e) $y = 4x$

19.   Ping has three types of animals, dogs, cats and lizards. There's a combined 200 legs in his menagerie. There are as many dogs as cats. There are 6 more lizards than dogs and cats combined. How many dogs are there?

a) 11   b) 22   c) 28   d) 44   e) 112

20.   If the four equilateral triangle sides of a square pyramid have a total surface area of 100 m^2, what is the pyramid's volume? Round to the nearest cubic meter.

a) 42 m^2 b) 83 m^2 c) 103 m^2 d) 118 m^2 e) 146 m^2

21.   A sea captain hires you as a deck hand and offers to pay you $2 the first day, $4 the second day, $8 the third day, and so forth, so that your daily earnings form a geometric sequence. How much will you earn in 2 weeks?

a) $65,532   b) $32,766   c) $16,384   d) $8192   e) $4096

22.   What is the sum of the values of x for which <x, 3, 5> and <x, x, -8> are orthogonal to one another?

a) -3   b) 0   c) 3   d) 5   e) 40

23. Using Kramer's Rule, provide the set up necessary to solve for x in the system of equations given below:

$$\begin{array}{rrrrrrrr} x & - & 4y & - & 2z & = & -7 \\ -x & + & 5y & + & 5z & = & 18 \\ 3x & - & 7y & + & 10z & = & 38 \end{array}$$

a) $\dfrac{\begin{vmatrix} -4 & -2 & -7 \\ 5 & 5 & 18 \\ -7 & 10 & 38 \end{vmatrix}}{\begin{vmatrix} 1 & -4 & -2 \\ -1 & 5 & 5 \\ 3 & -7 & 10 \end{vmatrix}}$

b) $\dfrac{\begin{vmatrix} -2 & -7 & 1 \\ 5 & 18 & -1 \\ 10 & 38 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -4 & -2 \\ -1 & 5 & 5 \\ 3 & -7 & 10 \end{vmatrix}}$

c) $\dfrac{\begin{vmatrix} 1 & -4 & -2 \\ -1 & 5 & 5 \\ 3 & -7 & 10 \end{vmatrix}}{\begin{vmatrix} -7 & -4 & -2 \\ 18 & 5 & 5 \\ 38 & -7 & 10 \end{vmatrix}}$

d) $\dfrac{\begin{vmatrix} -7 & -4 & -2 \\ 18 & 5 & 5 \\ 38 & -7 & 10 \end{vmatrix}}{\begin{vmatrix} 1 & -4 & -2 \\ -1 & 5 & 5 \\ 3 & -7 & 10 \end{vmatrix}}$

e) $\dfrac{\begin{vmatrix} 1 & -4 & -2 \\ -1 & 5 & 5 \\ 3 & -7 & 10 \end{vmatrix}}{\begin{vmatrix} -4 & -2 & -7 \\ 5 & 5 & 18 \\ -7 & 10 & 38 \end{vmatrix}}$

24. What is the quadrupling time for an investment at a 4% annual rate compounded continuously? Round to the nearest year.

a) 17 years   b) 18 years   c) 30 years   d) 34 years   e) 35 years

25. Factory line A produces one part every 5 seconds and line B one part every 4 seconds. A packing machine can wrap one part every 2 seconds. At 8:00 AM, line A begins filling a hopper that can hold 1000 parts. Line B joins in filling the hopper at 8:15 AM and the packing machine begins emptying it at 8:30 AM. All of these run until the hopper is either empty or full. What happens, and at what time? Round to the nearest minute.

a) Empty at 9:10 AM   b) Empty at 10:25 AM   c) Empty at 11:45 AM
d) Full at 10:13 AM   e) Full at 10:25 AM

26. Find the equation of the parabola with its focus at (4,3) and the directrix, the line y = -1.

a) $y = \dfrac{x^2 - 8x + 24}{8}$ b) $y = -\dfrac{1}{2}x^2 + 3$ c) $y = 3x^2 - \dfrac{1}{2}$

d) $y = x^2 - x + 3$ e) $y = x^2 - x + 24$

27. Which of the following is 2 + 2i raised to the 4th power?

    a) -16 + 16i   b) 32   c) -16 – 16i   d) -64   e) $16\sqrt{2} + 16i\sqrt{2}$

28. A radioactive material has a half-life of 25 years. If a scientist has a sample of 500 grams of this material, when will 400 grams of this material be left? Round to the nearest year.

    a) 6 yrs.   b) 8 yrs.   c) 10 yrs.   d) 12 yrs.   e) 14 yrs.

29. Which of the following is the intersection of the angle bisectors of a triangle?

    a) Centroid   b) Incenter   c) Barycenter d) Excenter   e) Orthocenter

30. Four women, Amelia, Bethany, Carol, and Deborah, recently participated in a charity competition. The competition consisted of four events, a head-to-head obstacle course, a ranked breath holding contest, a head-to-head wall climb, and a ranked tricycle race. Points were awarded as follows: 3 points for a ranked victory, 2 points for a head-to head victory or ranked second place, and 1 point for a head-to-head tie or third place in a ranked event.

    i.   Amelia held her breath longer than her older sister and teenage daughter, but not as long as her twenty year old niece.
    ii. Bethany tied her mom in the obstacle course, then tied her aunt in the wall climb.
    iii. Carol beat her mom in the obstacle course, then lost to her aunt in the wall climb.
    iv. Deborah didn't score any points until the wall climb.
    v.   The tricycle race finished in order of age, with the oldest woman winning.

    Did anyone have the same overall score at the end?

    a) Yes, Amelia and Carol tied for first
    b) Yes, Bethany and Carol tied for second
    c) Yes, Carol and Deborah tied for third
    d) Yes, Bethany and Deborah tied for third
    e) No