

WYSE – Academic Challenge
Math Test (Sectional) – 2016

1. **Correct Answer: E**

$$\sec \theta \csc \theta \left(1 + \frac{1}{\sec^2 \theta} \right) - \frac{2}{\tan \theta} \Rightarrow \frac{1 + \cos^2 \theta - 2 \cos^2 \theta}{\sin \theta \cos \theta} \Rightarrow \frac{1 - \cos^2 \theta}{\sin \theta \cos \theta} \Rightarrow \frac{\sin \theta}{\cos \theta} \Rightarrow \tan \theta .$$

2. **Correct Answer: C**

An antiderivative of $1/x$ is $\ln|x|$. An antiderivative of e to the kx power is e to that same power divided by k . Since e is a constant, its antiderivative is ex , not e raised to a power.

3. **Correct Answer: E**

$$AC = \sin 61^\circ \frac{(150)}{\sin 83^\circ} \approx 132$$

4. **Correct Answer: D**

The net change from 5,000 ($x = 5$) to 6,000 ($x = 6$) is given by the integral expression

$$\int_5^6 (5x + 2) dx = \left[2.5x^2 + 2x \right]_5^6 = 102 - 72.5 = 29.5 \text{ thousand dollars. So it costs Nick the original } \$30,000 \text{ and the extra } \$29,500, \text{ so he spent } \$59,500 \text{ on 6,000 cups.}$$

5. **Correct Answer: B**

If we model this as $y = ae^{kx}$, then $y' = ke^{kx}$, $y(0) = 20000$, and $y'(0) = 1000$, giving us $a = 20000$ and $k = 0.05$. Solve $50000 = 20000e^{0.05x}$, giving us $x = 18.32$.

6. **Correct Answer: A**

$$104 = \frac{1}{2}(8)(7 + x) \text{ where } x \text{ is the lower base length. } 26 = 7 + x \Rightarrow x = 19 .$$

7. **Correct Answer: C**

There are $C(30, 6) = 593,775$ ways to place the A's. There is only 1 way to place all six of them accurately and $C(6, 5) C(24, 1) = 144$ ways to place 5 of the six accurately. When rounded to two decimal places, this comes out to a stunningly improbable 0.02%.

8. **Correct Answer: B**

A rectangle is a parallelogram whose diagonals are always congruent. A rhombus is a parallelogram whose diagonals are always perpendicular but not always congruent. A square is a special type of rhombus. A trapezoid is not a type of parallelogram. Diagonals of a trapezoid never bisect each other. The parallelogram is the only quadrilateral that can possess the different characteristics described here.

9. **Correct Answer: C**

There are 12 letters to move about, and 3 of them are t's and three are o's. (The rest are unique.) There are $C(12,3)$ ways to place the t's, which leaves only nine slots in which to place the o's, so there are $C(9,3)$ ways to place those. And then there are $6!$ ways to place the remaining six letters. So there are $C(12,3) C(9,3) 6! = 13,305,600$ ways to move them around and get unique words.

10. **Correct Answer: E**

If A is the event a bolt is properly made and B a bolt is identified as properly made, then we have $P(B|A)=0.95$, $P(B'|A')=0.98$, and $P(A)=0.99$, and want $P(A|B')$.

$$P(A|B') = \frac{P(B' \cap A)}{P(B')} = \frac{P(B'|A) \cdot P(A)}{P(B'|A) \cdot P(A) + P(B|A') \cdot P(A')} = \frac{0.05 \cdot 0.99}{0.05 \cdot 0.99 + 0.98 \cdot 0.01} \approx 0.8347$$

11. **Correct Answer: D**

The slope is $m = \frac{-15}{2}$ and the point is $(50, 800)$ where x is the number of people and y

is the price per person. We find that $y = -\frac{15}{2}x + 1175 \Rightarrow R(x) = x \left(-\frac{15}{2}x + 1175 \right)$.

12. **Correct Answer: B**

This is an ellipse with semimajor axis 3 and semiminor axis 2, so the area is given by $\pi \cdot 2 \cdot 3 = 6\pi \approx 18.8$ square units.

13. **Correct Answer: C**

$f(x) = \frac{x^3 - x^2 - 2x}{2x^2 - 4x} = \frac{x(x-2)(x+1)}{2x(x-2)}$. There are no vertical asymptotes, just holes at

$x = 0$ and $x = 2$. The degree of the numerator is 1 degree larger than the denominator. Therefore there is an oblique asymptote. No horizontal asymptote exists.

14. **Correct Answer: D**

Let x = the height of the balloon in miles. Then, because the angle from Jake to the balloon is 45 degrees, his horizontal distance to the balloon is also x miles. So John's

horizontal distance to the balloon is $5 - x$ miles and thus $\tan 56^\circ = \frac{x}{5 - x}$. Thus

$(5 - x) \tan 56^\circ = x$ and $5 \tan 56^\circ - x \tan 56^\circ = x$. Then $5 \tan 56^\circ = x \tan 56^\circ + x$. If we factor,

$5 \tan 56^\circ = x(\tan 56^\circ + 1)$. After division, $\frac{5 \tan 56^\circ}{\tan 56^\circ + 1} = x \approx 3.0$ miles.

15. **Correct Answer: C**

Start by converting Josh's rate to $1/3$ lap per minute and Kelly's to $2/3$ lap per minute by multiplying by 4 laps per mile and $1/60$ hours per minute. Next determine that Josh has jogged $20 \cdot 1/3$ or $6 \frac{2}{3}$ laps by 12:20. This means Kelly will catch him the first time after t minutes, where $\frac{1}{3}t + \frac{2}{3} = \frac{2}{3}t$. Solve to get $t = 2$. From there, she gains two laps after x

minutes, where $\frac{1}{3}x + 2 = \frac{2}{3}x$. Solve to get $x = 6$, or 8 total minutes after 12:20.

16. **Correct Answer: E**

Using Pythagorean Theorem, we set $x = 240$ and $y = 180$. Then z , the hypotenuse,

must be 300. Differentiating with respect to time we have $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$.

Solving for $\frac{dz}{dt}$, we find $\frac{dz}{dt} = \frac{2x \frac{dx}{dt} + 2y \frac{dy}{dt}}{2z} \Rightarrow \frac{2(240) \cdot (-360) + 2(180)(-270)}{2(300)} = -450$.

17. **Correct Answer: A**

To have only one solution, the discriminant, $b^2 - 4ac$ must be equal to 0. In this case, b is -5 and a is 3. So $25 - 12c = 0$ and $c = 25/12$.

18. **Correct Answer: A**

$f'(1) = 4(1^3 - 4(1) + 2)^3 \cdot (3(1)^2 - 4) = 4$. Therefore our slope is 4. Using the given point we find the equation of the line $y = 4x - 4$.

19. **Correct Answer: A**

Let c = the number of cats, d = the number of dogs and l = the number of lizards. we know $4c + 4d + 4l = 200$, $d = c$, and $l = d + c + 6$. So $l = 2d + 6$ and $8d + 4l = 200$. This means that $8d + 4(2d + 6) = 200$, and $16d + 24 = 200$. Thus, $d = 11$. Though it doesn't matter, there are 11 cats and 28 lizards as well.

20. **Correct Answer: C**

If the base of each face is x , then the height is $\frac{\sqrt{3}}{2}x$, giving us a triangle area of $\frac{1}{2}x \frac{\sqrt{3}}{2}x$ and $x \approx 7.598$. The height of the pyramid is $\frac{\sqrt{2}}{2}x$ and volume $\frac{1}{3}x^2 \frac{\sqrt{2}}{2}x \approx 103.4$

21. **Correct Answer: B**

Here $a_1 = 2$, $n = 14$. We have $2 \left(\frac{1 - 2^{14}}{1 - 2} \right) = 32,766$.

22. **Correct Answer: A**

Orthogonal vectors are ones whose dot product is 0. So $x^2 + 3x - 40 = 0$ and $(x + 8)(x - 5) = 0$, and then the x -values are -8 and 5, which add to -3.

23. **Correct Answer: D**

The home determinant is $\begin{vmatrix} 1 & -4 & -2 \\ -1 & 5 & 5 \\ 3 & -7 & 10 \end{vmatrix}$. To find x , we replace the first column

coefficients with $\begin{bmatrix} -7 \\ 18 \\ 38 \end{bmatrix}$ giving us $\begin{vmatrix} -7 & -4 & -2 \\ 18 & 5 & 5 \\ 38 & -7 & 10 \end{vmatrix}$. We find this determinant and then

divide this result by the home determinant.

24. **Correct Answer: E**

We need to set $4P = Pe^{.04t}$, so $4 = e^{.04t}$. Then $.04t = \ln 4$ and $t = 25 \ln 4$, which is approximately 35 years.

25. **Correct Answer: C**

Line A produces 12 parts per minute, so by 8:15 there are $12 \cdot 15 = 180$ parts. Line B produces 15 parts per minute, so by 8:30 the total is $180 + 12 \cdot 15 + 15 \cdot 15 = 585$ parts. Since the packing machine empties the hopper at 30 parts per minute, the hopper will be emptying at a rate of 3 parts per minute, which takes $585/3 = 195$ minutes. 195 minutes (3 hours 15 minutes) after 8:30 is 11:45.

26. **Correct Answer: A**

$$\sqrt{(x-4)^2 + (y-3)^2} = y+1 \Rightarrow (x-4)^2 + (y-3)^2 = y^2 + 2y + 1. \text{ Solving for } y \text{ we find}$$

$$y = \frac{x^2 - 8x + 24}{8}.$$

27. **Correct Answer: D**

$$(2 + 2i)^4 = 2^4 + 4 \cdot 2^3 \cdot 2i + 6 \cdot 2^2 \cdot (2i)^2 + 4 \cdot 2 \cdot (2i)^3 + (2i)^4 = 16 + 64i - 96 - 64i + 16 \text{ which equals } -64.$$

28. **Correct Answer: B**

$$0.5 = e^{25r} \Rightarrow \frac{\ln(0.5)}{25} = r. \text{ Then the rate of decay is approximately } -0.0277. \text{ We can now}$$

$$\text{solve for the time. } 400 = 500e^{-0.0277t} \Rightarrow \frac{4}{5} = e^{-0.0277t} \Rightarrow t = \frac{\ln\left(\frac{4}{5}\right)}{-0.0277} \approx 8.$$

29. **Correct Answer: B**

That's one way how the incenter is defined. (The other is the center of a circle drawn to be internally tangent to all three sides thereof.) The centroid and barycenter are the same—the centroid is the intersection of the medians and the barycenter is the center of gravity of the triangle. The orthocenter is the intersection of all of the altitudes. The excenter is not unique—there are three excenters of a triangle, each of which is the center of an excircle.

30. **Correct Answer: E**

Deborah must be Carol's mom, so Amelia must be Bethany's mom. Deborah is oldest, followed by Amelia, then Carol, then Bethany. The results per person are as follows. Amelia tied Bethany in the obstacle course, got second in breath holding, won against Carol in the wall climb, then second in tricycle race for 7 points total. Bethany tied in the obstacle course, got third in breath holding, tied Deborah in the wall climb, and then got fourth in the tricycle race for a total of 3 points. Carol won against Deborah in the obstacle course, got first in the breath holding, lost to Amelia in the wall climb, and then got third in the tricycle race for a total of 6 points. Deborah lost to Carol in the obstacle course, got fourth in the breath holding, tied Bethany in wall climb, and then won the tricycle race for a total of 4 points. No ties in the final scoring.